Dirac Monopoles and the Wu-Yang Variant[1]

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Dirac's considerations of magnetic monopoles [1, 2] were to provide a possible explanation of the observed quantization of electric charge.

A recent paper [3] considers a classical model of a Dirac monopole, without mention of its quantum context, as a semi-infinite tube of magnetic flux of infinitesimal cross section, often called a Dirac string. The lines of magnetic field (which obeys $\boldsymbol{\nabla}\cdot\mathbf{B}=0$) that emerge from the end of the string when at rest at the origin take on the Coulomb form outside the string,

$$\mathbf{B} = \mu_0 g\,\frac{\hat{\mathbf{r}}}{r^2} \tag{1}$$

(in SI units), where $\mu_0 g$ is the magnetic flux inside the string and g is the effective magnetic charge of the system, as shown in Fig. 1 of [3]. See also, for example, Fig. 2 of [4].

The magnetic field inside the Dirac string is maintained by azimuthal currents on the surface of the string/flux tube, or equivalently by a string of magnetic dipoles.

The calculation of the force between two Dirac strings can be modified to find the self force $\mathbf{F}_{2,2}$ on string 2 by changing index 1 to 2 in eq. (8) of [3] and defining $\mathbf{x}'$ as the vector from the end of string 2 (at $\mathbf{x}=0$) to a point along it. Then, $\mathbf{F}_{2,2}$ is given by eq. (10) with $r=0$, so the self force on a semi-infinite Dirac string is infinite.

A variant of Dirac's vision was proposed by Wu and Yang [5] (reviewed in sec. 4.1 of [4] and Sec. 7 of [6]), which avoids the issue of the self force on a string monopole at rest. For a magnetic monopole g at rest at the origin, its magnetic field can be described by the vector potential $\mathbf{A} = A_\phi\,\hat{\boldsymbol{\phi}}$ in spherical coordinates (r,θ,ϕ) with

$$A_\phi = \frac{\mu_0 g}{r\sin\theta}\begin{cases} (1-\cos\theta) & (0 \le \theta \le \pi/2), \\ -(1+\cos\theta) & (\pi/2 \le \theta \le \pi). \end{cases} \tag{2}$$

This corresponds to supposing there exist two semi-infinite Dirac strings, each with internal magnetic flux of $\mu_0 g/2$, one along the positive z-axis with downward magnetic flux inside, and another along the negative z-axis with upward flux inside.[2] See, for example, Figs. 5 and 6 of [6]. The magnetic field $\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}$ is then the Coulomb form (1) except along the strings, and the force between two such monopoles at rest obeys Coulomb's law, as can be confirmed by a variant of the argument in [3].

The self force on each string of a Wu-Yang monopole at rest points away from the origin/monopole, so that the total self force is zero.

An issue is that the string of an accelerated Dirac monopole could not be straight (as rigid bodies are inconsistent with special relativity), so the magnetic field of the monopole

[1] C.N. Yang passed away at age 103 on Oct. 18, 2025.

[2] It seems that a Dirac monopole of strength g could be associated with any number n (a positive integer) of strings, each with internal flux $\mu_0 g/n$ that points to the position of the monopole.

would exert nonzero self forces and torques on the curved Dirac string, even for the Wu-Yang variant. Furthermore, accelerated string monopoles would emit radiation from all accelerated points along the string, which radiation would make the string observable.

While neither Dirac's monopole nor the variant of Wu and Yang are very plausible, already in 1974 't Hooft [7] and Polyakov [8] had proposed massive magnetic monopoles (without Dirac strings) in the context of so-called grand-unified field theories, in which particles have additional fundamental interactions beyond those presently known. Such magnetic monopoles have not yet been observed,

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