Negative Resistance in \(p-n\) Junctions Under Avalanche Breakdown Conditions, Part I

T. MISAWA

Abstract—A one-dimensional, small-signal analysis of the space-charge region of a \(p-n\) junction in which avalanche occurs uniformly is presented. The impedance is found to have a negative real part. The impedance is well represented by a parallel connection of the depletion layer capacitance, an inductance, and a negative resistance. The admittance magnitude of the negative resistance is proportional to the bias current. The impedance is well represented by a parallel connection of the depletion layer capacitance, an inductance, and a negative resistance. The admittance magnitude of the negative resistance is proportional to the bias current. The impedance is found to have a negative real part. The impedance is well represented by a parallel connection of the depletion layer capacitance, an inductance, and a negative resistance. The admittance magnitude of the latter two is proportional to the bias current. The admittance magnitude of the latter two is proportional to the bias current.

The negative resistance is due to an intrinsic instability in the avalanching electron-hole plasma. A discussion of the instability and a traveling-wave tube-like amplification is given.

INTRODUCTION

In this paper, Part I, many simplifying assumptions are made in order to bring out the essential principles. A more realistic and therefore more complex analysis is given in a later paper, Part II, this issue.

INSTABILITY IN AVALANCING ELECTRON-HOLE PLASMA AND NEGATIVE RESISTANCE

When the electric field is high enough, electrons and holes in semiconductors drift at saturated velocities (scattering limited velocities) which are independent of field [2]. Suppose we have an electron density perturbation which varies sinusoidally in space in a field high enough to produce scattering limited velocities. The density perturbation produces the charge and hence, the field perturbation. Since electrons drift at the scattering limited velocity regardless of field, the perturbation does not spread out but propagates without attenuation in the direction of electron drift. The situation is illustrated in Fig. 1. The field wave lags the electron density wave by 90°. We see that the “stiffened” electron-hole plasma can convey a non-attenuating space-charge wave.

Now it will be described how the above space-charge wave grows in the presence of avalanche multiplication. The generation rate of electron-hole pairs is larger both when the electric field is stronger and when there are more carriers [3]. Therefore, in the above case of an electron density perturbation the generation rate peaks somewhere between the place where the field is strongest and the place where the density is largest. This means that the generation rate leads the electron density wave by less than 90°, as shown in Fig. 1. Note that, since in the case of Fig. 1 the de field is in the negative \(x\) direction, the field becomes strongest at its negative peak. This increased generation rate gives rise to an excess electron density which lags the rate by 90°. In the positive half cycle of the rate the excess electron density keeps decreasing, and in the negative half cycle the excess density decreases. Now the resultant total electron density gives a current which lags the rate by 90°. As for holes, although individual holes created by the avalanche drift in the direction opposite to electrons, the hole density wave is dragged by the electron density wave. The hole density wave also lags the field by more than 90°. The situation is shown in Fig. 1. Thus, the ohmic losses due to both electron and hole currents produced by avalanche are negative. This means that the above space-charge wave keeps growing. This is an instability.

One of the easiest ways to produce an avalanching electron-hole plasma is to break down a \(p-n\) junction. Since we have boundaries in this case, the situation is a little bit different. However, we expect a negative resistance due to the above mentioned instability of the plasma. The negative-resistance property, or the instability of this confined plasma can be discussed in the following way. The model to be discussed is shown in Fig. 2. Electrons enter from the left side and holes from the right side. We consider the voltage transient after an impulsive current is applied to the avalanching \(p-n\) junction. First the applied impulsive current produces charge...
These charge spikes raise the field in the space-charge region by a constant amount (Fig. 3). This increased field causes more avalanche and generated electrons and holes neutralize the charge spikes. However, the situation is such that we still have a supply of electrons and holes even after the spikes are neutralized. Subsequently, we have too many holes at the left end and too many electrons at the right end. This is a momentary narrowing of the space-charge region. Now these extra charges decrease the field in the space-charge region and we have a situation just opposite to what has just been considered. Since we now have a negative, or decreased, supply, these spikes die away and again the "pendulum" swings beyond the steady state. The situation is repeated over and over again. We again have an instability.

**ANALYSIS**

**Fundamental Equations**

The following equations govern the dynamics of the avalanching electron-hole plasma.

Poisson’s equation,

$$\frac{\partial E}{\partial x} = \frac{q}{\varepsilon} (N_D - N_A + p - n).$$

(1)

Continuity equations for electrons and holes,

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} + g,$$

(2)

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + g.$$  

(3)

where the generation rate [3] $g$ is

$$g = \alpha |v_n| n + \beta |v_p| p.$$  

(4)

We neglect diffusion current; then

$$J_n = -q v_n n,$$  

(5)

$$J_p = q v_p p.$$  

(6)

Here $E$ is the electric field, $q$ the absolute value of the electronic charge, $\varepsilon$ the dielectric constant, $N_D$ and $N_A$ donor and acceptor densities, $n$ and $p$ electron and hole densities, $J_n$ and $J_p$ electron and hole current densities, $\alpha$ and $\beta$ the ionization rates of electrons and holes, $v_n$ and $v_p$ the velocities of electrons and holes.

**Plane Wave Case**

We consider a very simple case in which both electrons and holes have the same ionization rate and the same drift velocity, and the field is so high that the drift velocity $v$ is independent of the field. We divide the solutions of (1) through (6) into the time-dependent part and the constant part. The time-dependent part is assumed to vary like $e^{i\omega t}$ and to be very small compared to the dc part.
When we assume that the dc electric field is independent of \( z \) so that the avalanche occurs uniformly all over, the ac solution of the form \( e^{\lambda(z-kt)} \) is possible. This is a plane wave with the wave number or the propagation constant \( k \). The dispersion relation for the plane wave is found to be given by

\[
\frac{k^2 + 2\alpha'J - \beta_{2\alpha}x - \omega^2}{\omega^2} = 0, \quad (7)
\]

where \( \alpha' = \frac{\partial\omega}{\partial E} \) and \( J \) is the total dc current, i.e., the sum of electron and hole currents. Although electron and hole currents increase downstream, the sum \( J \) is constant. Note that since \( \alpha \) is an increasing function of \( |E| \), \( \alpha'J \) is always positive regardless of the sign of \( J \). Details of the derivation of (7) are shown in the Appendix. We used the various units given in Table I. From (7)

\[
\omega = \alpha \pm \sqrt{\alpha^2 - 2\alpha'J + k^2}. \quad (8)
\]

With real \( k \) we have a spatially sinusoidally varying perturbation. Equation (8) shows that the perturbation grows exponentially with time. This means that the perturbation is unstable. When the quantity in the square root sign of (8) is negative, \( \alpha'J \) is always positive regardless of the sign of \( J \). Details of the derivation of (7) are shown in the Appendix. We used the various units given in Table I. From (7)

\[
\omega = \alpha \pm \sqrt{\alpha^2 - 2\alpha'J + k^2}. \quad (8)
\]

We see that, since the imaginary part of the quantity in the square root sign in (9) is positive, one of the \( k's \) is in the first quadrant and the other in the third quadrant for positive \( \omega \). When \( k \) is in the first quadrant, the wave propagates in the positive \( x \) direction because the real part of \( k \) is positive and its amplitude increases toward the positive \( x \) axis because the imaginary part is positive. When \( k \) is in the third quadrant the wave propagates and grows in the negative \( x \) direction.

This means that, if it is possible to excite this growing wave by an input probe, an output probe, which is placed in a position distant from the input probe, picks up the amplified wave. This is a traveling-wave tube-type amplification. We are not going to investigate this possibility however, but consider instead the space-charge region of a \( p-n \) junction, in which avalanche occurs uniformly.

**Space-Charge Region**

We assume that the \( p \) side region is on the negative side of the \( x \) axis as shown in Fig. 2. Electrons enter the space-charge region from the left end and holes from the right end. We designate the currents associated with these primary electrons and holes by \( J_{es} \) and \( J_{hs} \). The Appendix shows that the field and carrier currents are composed of three terms, i.e., the terms proportional to \( J_{es}, J_{hs}, \) and \( J \). Here \( \sim \) sign means the ac part. The ac total current \( J \) now contains both carrier currents and displacement current. For example, the ac field is given by

\[
E = \rho_i J_{es} + \rho_p J_{hs} + \rho J, \quad (10)
\]

where the proportionality constants \( \rho_i, \rho_p, \) and \( \rho \) are determined by the structure parameters and bias conditions and are dependent on the position \( x \). An important quantity is the voltage \( V_s \) across the diode, which is obtained by integrating (10):

\[
V_s = Z_n J_{es} + Z_p J_{hs} + ZJ. \quad (11)
\]

Since we assumed the same property for both electrons and holes, the integrated coefficients of \( J_{es} \) and \( J_{hs} \) are the same.

\( J_{es} \) and \( J_{hs} \) are due to thermally generated carriers and depend on the voltages \( V_s \). However, the relevant admittances are on the order of the admittance of the reverse biased \( p-n \) junction and their product with \( Z \) is much less than unity. Therefore, the first two terms in (11) are negligible. The impedance of the diode is given by \( Z \).

We consider a specific numerical example. The width of the region is \( 5 \mu \). The dielectric constant is 12 as in Si. The ionization rate is taken from Fig. 3 of [1], which is for silicon. Figure 4 shows the impedance as a function of frequency and bias current. Normalization is in the units given in Table I. The three sets of curves are shown there which correspond to three different bias currents, 1, 0.1, and 0.01. The real part shown is all negative. Its magnitude increases rapidly from low frequency, has a maximum value at the resonance frequency, and decreases. The reactance is inductive at lower frequencies and changes to capacitive after the resonance. The resonance frequency increases approximately proportionally to the square root of the bias current.

| Table I |
|---|---|---|---|
| Unit | Expression | Numerical Value | Meaning |
| Length | \( \mu \) | 5 | width of the region |
| Time | \( \tau = \frac{\mu}{\nu} \) | 5.88 \( \times 10^{-11} \) sec | transit time |
| Frequency | \( \nu \) | 2.71 \( \times 10^9 \) c/s | |
| Density | \( N_0 \) | \( 10^9 \) cm\(^{-3} \) | |
| Current | \( qN_0 \) | 1.36 \( \times 10^7 \) A/cm\(^2 \) | |
| Field | \( \omega qN_0 \) | 7.54 \( \times 10^4 \) volt/cm | |
| Impedance | \( \omega \mu / \pi \) | 2.77 \( \times 10^8 \) ohm/cm | impedance of the capacitance with width \( \mu \) at unit frequency |
| Charge | \( q \) | \( 1.6 \times 10^{-9} \) coulomb | |

* \( \nu \) is scattering limited velocity, which is assumed as 8.5 \( \times 10^4 \) cm/sec.
This type of behavior of the impedance can be represented by a parallel connection of an inductance, a capacitance, and a negative resistance as shown in Fig. 5. If the capacitance is chosen to be equal to the depletion layer capacitance, the inductance and the negative resistance are fairly frequency independent. Figure 6 shows the admittance of the diode after the admittance of the depletion layer capacitance is subtracted. The negative resistance is fairly constant over a decade of frequency around resonance. The straight line behavior and the slope of the susceptance plot show that the inductance is also fairly frequency independent. It is seen that the admittance is approximately proportional to the bias current. The inductance is inversely proportional to the bias current; hence, it gives, along with the frequency-independent capacitance, a resonance frequency proportional to the square root of the bias current.

The small-signal $Q$ of the device is plotted in Fig. 7. The $Q$ is defined as the angular frequency times the ratio of the average stored energy to the average energy dissipation per unit time. Its calculation is explained in the Appendix. The $Q$ gives information about threshold and the build-up rate of oscillation when the negative resistance is used as an oscillator. A smaller magnitude of negative $Q$ is preferable. From Fig. 7 we see that the $Q$ is better at lower frequencies for a given bias current and at higher currents for a given frequency.

It is illustrative to see how the carrier densities and the electric field behave in the space-charge region. The space-charge wave in the avalanching electron-hole plasma has a wave number given by (9). In the space-charge region this plasma wave propagates back and forth so as to satisfy the appropriate boundary conditions. Although the nature of the wave and the boundary conditions are quite different from those in the ordinary electromagnetic wave in the hollow cavity, we expect, and really have, a resonance phenomenon at a certain frequency as in the latter case. Figure 8 is the phasor or vector diagram\(^3\) of the electric field (a) and the electron current (b). Here the phasors of the field and the current at 17 equally spaced points are shown in complex planes for the resonance frequency at a normalized dc current of 1. The phasor of the total current through the space-charge region $J$ is on the positive real axis. The

\(^3\) For general information about the vector diagram, see, for example, Harris A. Thompson, Alternating-Current and Transient Circuit Analysis. New York: McGraw-Hill, 1955, p. 79.
from left to right although in the left half of the region it looks almost like a standing wave. Its amplitude increases while propagating. The hole current behaves in the same way, but now from right to left. These distributions of field and currents give the power dissipation shown in Fig. 8(c). Electrons dissipate energy in the first one-third of the passage, but their energy dissipation becomes negative after that. Holes behave in the same way and the total energy loss is negative all over the region.

**DISCUSSION**

We have shown that the space-charge region of a p-n junction in which avalanche occurs uniformly shows a negative resistance, which is due to the intrinsic nature of the avalanching electron-hole plasma. The obtained negative resistance is fairly large in magnitude and the Q of the device is very good.

This negative resistance may be contrasted to another negative resistance utilizing avalanche, which was proposed by Read [1]. The Read diode is understandable by following the same line of reasoning as Shockley’s minority carrier delay diode or transit-time mode operation of the transistor structure [4]. Carriers are generated at a “cathode” plane and drift through a “drift space.” When the transit time is properly chosen, the phase relation between current and voltage is such that a negative resistance results. Read investigated the dynamics of avalanche multiplication, and proposed using a well-localized avalanche region as the “cathode” and the avalanche-free part of the space-charge region as the “drift space,” as Shockley used the emitter of the transistor as a “cathode” and the base region and the collector depletion layer as a “drift space.” In Read’s structure avalanche is confined to one end of the space-charge region.

**APPENDIX**

**Fundamental Equations**

When equal ionization rate and drift velocity are assumed for both electrons and hole, (1) through (6) in the text become

\[
\frac{\partial E}{\partial x} = N_D - N_A + p - n, \quad (12)
\]

\[
\frac{\partial n}{\partial t} = \frac{\partial J_n}{\partial x} + \alpha(n + p), \quad (13)
\]

\[
\frac{\partial p}{\partial t} = -\frac{\partial J_p}{\partial x} + \alpha(n + p), \quad (14)
\]

\[
J_n = -n, \quad (15)
\]

\[
J_p = -p, \quad (16)
\]
in the units given in Table I. The electric field was assumed to be in the negative $x$ direction as in Figs. 1 and 2.

**LINEARIZATION**

The quantities are divided in dc parts and small ac parts. The latter changes as $e^{i\omega t}$, $\omega$ being angular frequency. We replace $\partial/\partial t$ by $j\omega$ and retain only the first order terms in the ac part. Then we have

$$
\frac{\partial E}{\partial x} = J_n - J_p, \quad \text{(17)}
$$

$$
\frac{\partial J_n}{\partial x} = \alpha' E + (\alpha - j\omega)J_n + \alpha J_p, \quad \text{(18)}
$$

$$
\frac{\partial J_p}{\partial x} = -\alpha' E - \alpha J_n - (\alpha - j\omega)J_p, \quad \text{(19)}
$$

where $\alpha' = \partial \alpha / \partial E$ and $J$ is the total dc current. Since $\alpha$ is an increasing function of $|E|$, $\alpha'J$ is always positive regardless of the sign of $J$.

**SOLUTIONS**

One of the integrals is

$$
J = J_n + J_p + j\omega E, \quad \text{(20)}
$$

in the dimensionless form.

In the case of constant avalanche, $\alpha$ and $\alpha'$ are independent of $x$. Then the solution of (17) through (19) is given by a superposition of solutions of the form $e^{\pm ikx}$ with three different values of $k$. We know, by inserting (20) into (17) through (19), that one of the $k$'s is zero and the other two are the roots of the following equation,

$$
k^2 + 2\alpha'J - j2\omega \alpha - \omega^2 = 0, \quad \text{(21)}
$$

which is (7) in the text.

In the $p$-$n$ junction case, we have the following boundary conditions:

$$
J_n(0) = J_{ns}, \quad \text{(22)}
$$

$$
J_n(1) = J_{ns}, \quad \text{(23)}
$$

We obtain the following solutions:

$$
\tilde{E} = C_1 e^{ikx} + C_2 e^{-ikx} + \frac{2\alpha - j\omega}{k^2} J, \quad \text{(24)}
$$

$$
J_n = \frac{i}{2} (k - \omega)C_1 e^{ikx} - \frac{i}{2} (k + \omega)C_2 e^{-ikx} - \frac{\alpha'J}{k^2} J, \quad \text{(25)}
$$

$$
J_p = \frac{-i}{2} (k + \omega)C_1 e^{ikx} + \frac{i}{2} (k - \omega)C_2 e^{-ikx} - \frac{\alpha'J}{k^2} J, \quad \text{(26)}
$$

where

$$
k = \sqrt{\omega^2 - 2\alpha'J + j2\omega \alpha}, \quad \text{(27)}
$$

$$
C_1 = \frac{1}{\Delta} \begin{vmatrix}
J_{ns} + \frac{\alpha'J}{k^2} J & -\frac{i}{2} (k + \omega) \\
J_{ns} + \frac{\alpha'J}{k^2} J & \frac{i}{2} (k - \omega) e^{-i\omega}
\end{vmatrix}, \quad \text{(28)}
$$

$$
C_2 = \frac{1}{\Delta} \begin{vmatrix}
\frac{i}{2} (k - \omega) J_{ns} + \frac{\alpha'J}{k^2} J & -\frac{i}{2} (k + \omega) e^{i\omega} \\
\frac{i}{2} (k + \omega) J_{ns} + \frac{\alpha'J}{k^2} J & \frac{i}{2} (k - \omega) e^{-i\omega}
\end{vmatrix}, \quad \text{(29)}
$$

and

$$
\Delta = \frac{1}{4} \{ (k + \omega)^2 e^{i\omega} - (k - \omega)^2 e^{-i\omega} \}. \quad \text{(30)}
$$

$$
\tilde{P} = -\int_0^\infty E \, dx = -\frac{C_1}{jk} (e^{i\omega} - 1) + \frac{C_2}{jk} (e^{-i\omega} - 1) - \frac{2\alpha - j\omega}{k^2} J x, \quad \text{(31)}
$$

$$
\tilde{P}_n = \tilde{P}(1) = Z (J_{ns} + J_{np}) + Z J, \quad \text{(32)}
$$

where

$$
Z = \frac{1}{k^2} (Z J_{np} - 2\alpha + j\omega). \quad \text{(34)}
$$

Note that currents are positive in the plus $x$ direction while the potential is measured from the $x = 0$ end. The impedance in the normal sense is obtained by changing the sign of $Z$. We see that $E$, $J_n$, $J_p$, and $\tilde{P}_n$ are composed of three components proportional to $J_{ns}$, $J_{np}$, and $J$.

The time average of power dissipation is given by

$$
-\left< \frac{d\omega}{dt} \right> = \frac{1}{2} \text{Re} \left\{ (J_n + J_p) \tilde{E}^* \right\}. \quad \text{(35)}
$$

The energy is stored as the field energy:

$$
\omega = \frac{1}{2} \text{Re} \left< |\tilde{E}|^2 \right>. \quad \text{(36)}
$$

The "$Q$" is given by

$$
Q = \frac{\omega}{\int_0^1 \left< \frac{d\omega}{dt} \right> \, dx}. \quad \text{(37)}
$$

In the numerical example $\alpha = 1$ corresponding to avalanche breakdown and $|\alpha'| = 1.672$. 
ACKNOWLEDGMENT

The author would like to thank B. C. De Loach, Jr., and R. M. Ryder for valuable discussions, comments, and suggestions. Thanks are also due G. A. Baraff, A. G. Chynoweth, H. K. Gummel, and C. A. Lee for comments and suggestions on an early version of the manuscript.

REFERENCES

3. Ibid., p. 65.

Negative Resistance in p-n Junctions Under Avalanche Breakdown Conditions, Part II

T. MISAWA

Abstract—The small-signal impedance of the space-charge region of p-n junctions under avalanche breakdown conditions is calculated using reasonably realistic dependences of electron and hole ionization rates and drift velocities upon electric field. Two structures are analyzed: one is a p+—n+ structure which has a fairly uniform distribution of avalanche multiplication, and the other is a singly diffused junction which is a hybrid of an abrupt and a linear graded junction. Both structures show negative resistance when the transit time of carriers becomes appreciable.

A computer program was evolved which requires, as input, the impurity profile and field dependences of ionization rates and drift velocities. The program first calculates the dc field and electron and hole currents and then solves the ac small-signal problem. Both the ac small-signal impedance and the Q of the diode are calculated.

INTRODUCTION

A previous paper [1] showed that the inherent instability of an avalanching electron-hole plasma gives rise to a negative resistance in the space-charge region of a p-n junction with uniform distribution of avalanche. The analysis employed several unrealistic assumptions in order to simplify the mathematics. This paper presents results obtained from a computer program designed to include more realistic physical assumptions.

Also, cases intermediate to the above and that treated by Read [2] (in which the avalanche is restricted to one edge of the space-charge depletion region) are calculated. The latter cases are particularly interesting since they can be compared with recently reported experimental results [3].

The program requires as input the impurity profile and the dependence of electron and hole ionization rates and drift velocities on electric field. The program first determines the space-charge region, then calculates the spatial distribution of the dc electric field and electron and hole currents, and then the small-signal ac impedance at a given bias current and frequency. It also calculates the Q of the device which is useful in discussing threshold conditions for oscillation and, hopefully, gives information on large-signal operation.

In this report the results of calculations for two Si structures are reported. One of them, AVX-1, simulates the previously discussed idealized model of uniform avalanche [1]. The other structure, AVX-2, simulates the p-n junction which showed CW oscillations in the microwave region [3]. The latter is a typical example intermediate between Read’s type and the uniform distribution of avalanche.

It has been found that the first structure shows impedance characteristics quite similar to the previously reported idealized model [1], and the second structure has impedance characteristics which are hybrids of the first type and Read’s. It has also been found that the more uniform the avalanche is, the better the Q. This advantage is more pronounced when the carrier drift velocities are dependent on the electric field and the depletion layer capacitance is lossy.

STRUCTURES

The impurity profile, AVX-1, which is intended to be as close as possible to the idealized model previously reported [1], simulates the structure...