The Brown-Twiss Radio-Wave Interferometer

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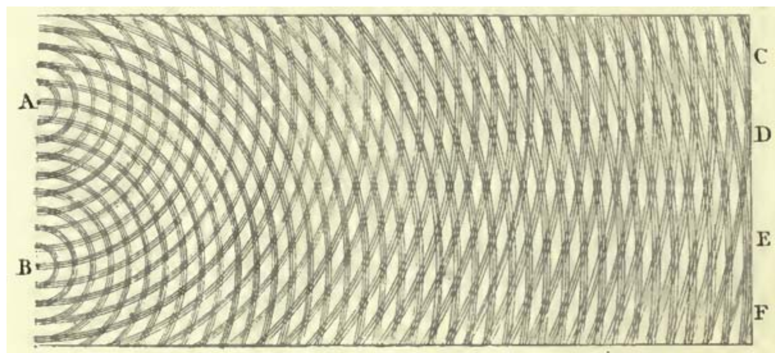
1 Problem

1.1 Young's Double Slit Experiment

Huygens argued in 1678 that light is a wave phenomenon which obeys a superposition principle, illustrated in the left figure below (p. 21 of [1]).



Young noted (p. 464 of [2], with a figure appearing before p. 777) that if light from a "point" source passes through two slits (A and B in the right figure above), then a series of bright and dark "fringes" can be observed on a distant screen (CDEF). The angular separation of the bright (or dark) fringes is λ/d, where λ is the wavelength of the light and d is the spacing between the slits.[1]

[1](Oct. 27, 2020) Young reported his double-slit experiment in [3] (1802), but with no figures. He was studying single-slit diffraction due to candlelight passing through a small, square hole when a hair accidentally fell across it, creating a double slit and altering the diffraction pattern. From the size of the hole and the diameter of the hair, Young inferred that the wavelength of the (yellow) candlelight was $1/43636''$, i.e., 582 nm. *For some details, with a drawing, see [4]; for historical commentary see, for example, [5].*

This was the first explicit measurement of the wavelength of light. However, Young noted that wavelength *deduced from Newton's experiments is 1/39200″, i.e.*, 648 nm, with no reference or explanation.

Newton's only published work from which a wavelength of light can be inferred is in Obs. 8, p. 99, Book 2 (.pdf page 254) of [6], where he deduced from observation of Newton's rings that the air gap which led to a yellow ring was $1/89000''$, corresponding to a wavelength of 571 nm. *See also [7].*

However, an extensive record of Newton's unpublished communications with the Royal Society of London on optics in 1675-76 is available in [8] (1757), pp. 247-305, which was likely known to Young. This is interspersed with controversy involving Hooke, which apparently delayed publication of [6] until after Hooke's death in 1703.

On p. 250 of [8] Newton first mentioned a version of the phenomenon now called Newton's rings. A figure on p. 264 shows that the center of the pattern is dark. The related discussion by Newton considered the possible wave nature of light, and argued that this would imply the center of the pattern to be bright. Newton did not appreciate that there is a 180° phase change for reflection of light at an air-glass interface but not at a glass-air interface. *Young showed (p. 393 of [3]) that if a liquid of high enough index of refraction is placed between the glass lenses, the central spot is bright: see also Fig. 450 of [2], before p. 787, and pp. 7-8 of [9].*

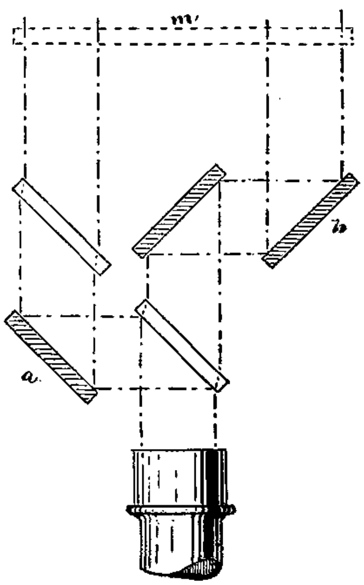
On p. 275, Newton stated that the air gap associated with a yellow ring is $1/80000''$ (*i.e.*, a wavelength of

If the light source has diameter D and is distance R from the screen, then the finite size of the source can be inferred from the pattern on the screen only if the angular extent D/R of the source is greater than the angular spacing λ/d of the fringes. That is, the smallest source diameter that could be inferred by observations of it through a double slit is $D_{\rm min} \approx \lambda R/d$.

This argument still holds approximately if the two slits are merged into a single aperture of width d, as in the case of the lens/mirror of a telescope. The lesson is that to resolve the size of distance objects, one needs a light-collecting system with a large width/diameter d.

Young's double-slit interference pattern can be considered as a quantum effect, in that the pattern can be built up slowly from the interference of single photons. First evidence of this was given in 1909 [11]. Later, Dirac remarked [12], "Each photon then interferes only with itself. Interference between two different photons never occurs." Indeed, a practical definition is that "classical" optics consists of phenomena due to the interference of photons only with themselves.[2]

1.2 Michelson's Stellar Interferometer



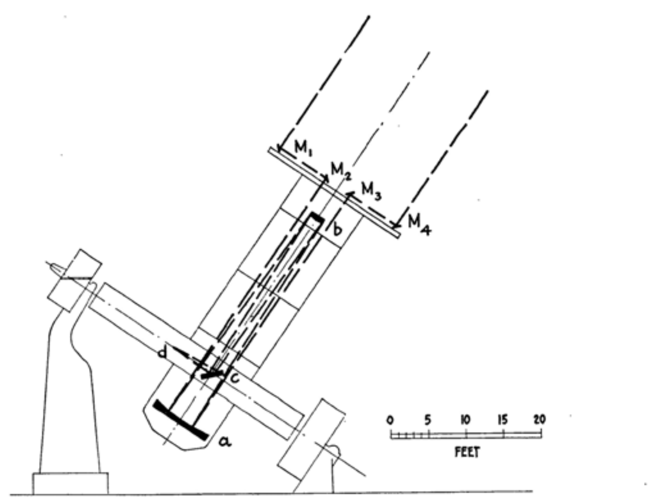


Diagram of optical path of interferometer pencils. M_1, M_2, M_3, M_4, mirrors; *a*, 100-inch paraboloid; *b*, convex mirror; *c*, coudé flat; *d*, focus.

Fizeau (1868) [14] made the first suggestion that a telescope with an effectively large aperture could be built using two small mirrors separated by distance d and arranged with a common focus. Apparently independently, Michelson made a similar suggestion in 1890 [15], illustrated in the left figure above, and built a version in 1891 sufficient to measure the diameters of Jupiter's moons [16]. In 1921 he used a larger version, with $d = 6$ m as shown on

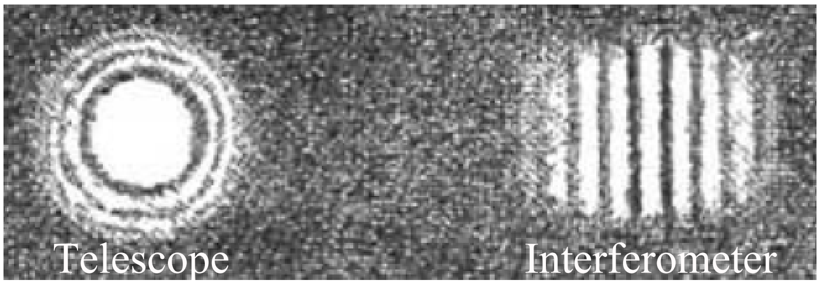
635 nm), while on p. 288 he associated this with orange-red (as is more consistent). This is likely the result of Newton mentioned by Young in [3].

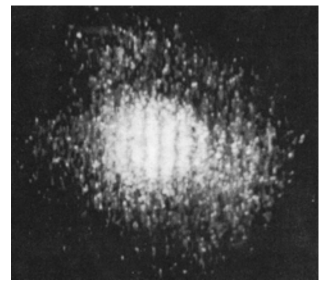
Young also described his double slit experiment in [10], again without any figures. Sunlight passed though *a piece of thick paper...perforated with a fine needle*. The circular aperture was made into a double slit with *a slip of card, about one-thirtieth of an inch in breadth*. Young compared his calculations of the wavelength of length to those of Newton in Obs. 8 of [6].

[2]For an example in which the quantum results of a simple experiment with two photons are dramatically different than that expected from a classical analysis, see [13].

the right on the previous page, to measure the diameter of the star Betelgeuse [17], which led this configuration to be called Michelson's stellar interferometer.[3]

Michelson noted [16] that the image of a spherical object in a telescope includes circular interference fringes (often not noticed), while the fringes in his stellar interferometer are parallel lines (perpendicular to the line between the two mirrors), as shown in the left figure below.

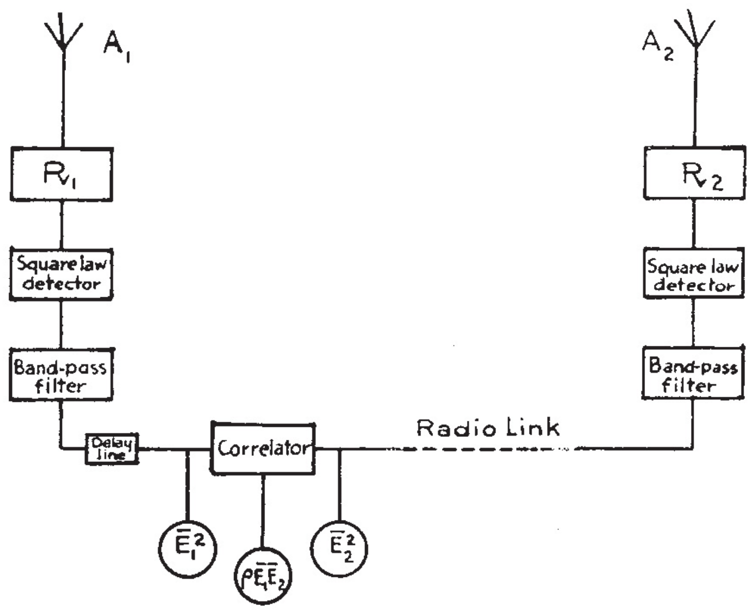




The image on the right above is from a stellar interferometer equipped with a so-called image intensifier [19]; each white dot is due to a single photon, and a fringe pattern is still observed.

1.3 The Brown-Twiss Interferometer

Interstellar radio waves were first detected in 1933 [20] as a background to communication between pairs of antennas on Earth. The first radio-frequency interferometer was built in 1946 [21], following the principle of Michelson's stellar interferometer by adding the signals (keeping both amplitude and phase information) from two radio telescopes.

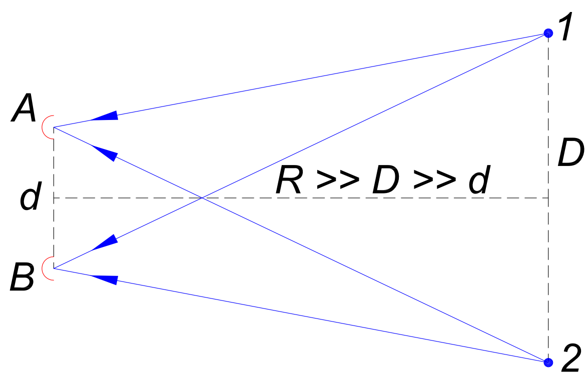


The longer wavelength of radio waves requires larger separation d between the two radio telescopes to maintain good angular resolution, such that it becomes more difficult to trans-

[3] For additional historical comments, see [18].

mit accurately both the amplitude and phase information over the larger distance.[4] This perceived difficulty led Hanbury Brown in 1952 [22] to develop a new type of interferometer in which the signal power $P \propto \langle E \rangle^2$ of each telescope, where $\mathbf{E}$ is the electric field of the incident radio wave, was measured at its site via so-called square-law detectors (which average the square of the input signal amplitude over each cycle), and then transmitted to a common point where their product was measured.

Subsequently, Brown and Twiss [23] gave a classical explanation of the operation of the new interferometer that was reasonably convincing. As for Michelson's interferometer, we expect the Brown-Twiss interferometer (as it came to be called) to perform the same in the limit that only one photon at a time is observed in each telescope. However, their quantum explanations [24, 25, 26, 27] seemed less satisfactory, and led to possibly confusing commentary by others [28, 29].[5]



Discuss the signal $\langle P_A P_B \rangle - \langle P_A \rangle \langle P_B \rangle$ in the simplified configuration of a Brown-Twiss interferometer sketched above, in which "point" sources 1 and 2 of equal strength are distance D apart, with detectors A and B separated by distance $d \ll D$. The line between the midpoints of the lines centers of the source pair and the detector pair has length $R \gg D$, and the two lines of centers are parallel. Sources 1 and 2 emit waves/photons of wavelengths λ_1 and λ_2, which are both very close (but not necessarily equal) to λ.

Give a classical argument first as to how the quantity $\langle P_A P_B \rangle - \langle P_A \rangle \langle P_B \rangle$ is related to the separation D between the sources. Then, consider the quantum case that the detectors each observe only a single photon at a time, with occasional observation of a photon by each detector at the same time.

2 Solution

The path lengths $1A$ and $2B$ are equal, as are the longer paths $1B$ and $2A$. The difference Δ in the lengths of the longer and shorter paths is,

$$\Delta = 1B - 1A = 2A - 2B = \sqrt{R^2 + \left(\frac{D+d}{2}\right)^2} - \sqrt{R^2 + \left(\frac{D-d}{2}\right)^2} \approx \frac{dD}{R}\,. \tag{1}$$

[4]Improvements in signal-transmission technology now permit good transmission of the radio-telescope signals over larger distances, such that most present radio interferometers are Michelson interferometers.

[5]The story of Feynman's initial skepticism as to the Brown-Twiss effect is recounted in [30].

Hence, the phase difference $\delta\phi$ for waves/photons on the longer paths compared to that on the shorter paths is approximated by,

$$\delta\phi_1 = 2\pi\frac{1B - 1A}{\lambda_1} \approx \delta\phi_2 = 2\pi\frac{2A - 2B}{\lambda_2} \approx \delta_\phi = 2\pi\frac{\Delta}{\lambda} \approx 2\pi\frac{dD}{\lambda R}\,, \tag{2}$$

noting that $\lambda_1 = 2\pi c/\omega_1 \approx \lambda_2 = 2\pi c/\omega_2 \approx \lambda = 2\pi c/\omega$, where the angular frequencies are related by $\omega = (\omega_1 + \omega_2)/2$, and c is the speed of light in vacuum (which is assumed to be the medium between the sources and the detectors).

The goal of the interferometry is to measure the phase $\delta\phi$, and hence the separation D of the two sources, assuming that distances d and R are known.

2.1 Classical Analysis

The waves from sources 1 and 2 are, in general, emitted with different phases ϕ_1 and ϕ_2, but we are free to define one of these, say ϕ_1, to be zero. Then, the electric field received at detector A has the form,

$$E_A = E_1\cos(\omega_1 t) + E_2\cos(\omega_2 t + \delta\phi_2 + \phi_2) \approx E[\cos(\omega t) + \cos(\omega t + \delta\phi + \phi_2)], \tag{3}$$

while that at detector B is,

$$E_B = E_1\cos(\omega_1 t + \delta\phi_1) + E_2\cos(\omega_2 t + \phi_2) \approx E[\cos(\omega t + \delta\phi) + \cos(\omega t + \phi_2)], \tag{4}$$

where to a very good approximation $E_1 = E_2 \equiv E$ for sources of equal strength. The instantaneous powers in the two detectors are then,

$$P_A \propto E_A^2 \approx E^2\left[\cos^2(\omega t) + \cos^2(\omega t + \delta\phi + \phi_2) + \cos(\delta\phi + \phi_2) + \cos(2\omega t + \delta\phi + \phi_2)\right], \tag{5}$$
$$P_B \propto E_B^2 \approx E^2\left[\cos^2(\omega t + \delta\phi) + \cos^2(\omega t + \phi_2) + \cos(\delta\phi - \phi_2) + \cos(2\omega t + \delta\phi - \phi_2)\right], \tag{6}$$

recalling that $2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$. The time-averaged powers are,

$$\langle P_A\rangle \propto \left\langle E_A^2\right\rangle \approx E^2[1 + \cos(\delta\phi + \phi_2)], \qquad \langle P_B\rangle \propto \left\langle E_B^2\right\rangle \approx E^2[1 + \cos(\delta\phi - \phi_2)], \tag{7}$$

and hence,

$$\langle P_A\rangle\langle P_B\rangle \propto E^4[1 + \cos(\delta\phi + \phi_2) + \cos(\delta\phi - \phi_2) + \cos(\delta\phi + \phi_2)\cos(\delta\phi - \phi_2)]. \tag{8}$$

To evaluate the time-averaged product $\langle P_A P_B\rangle$ of the powers in the two detectors, we note that,

$$\begin{aligned}
&\left\langle\cos^2(\omega t + \alpha)\cos^2(\omega t + \beta)\right\rangle = \frac{1}{4}\left\langle[\cos(\alpha - \beta) + \cos(2\omega t + \alpha + \beta)]^2\right\rangle \\
&= \frac{1}{4}\left\langle\cos^2(\alpha - \beta) + 2\cos(\alpha - \beta)\cos(2\omega t + \alpha + \beta) + \cos^2(2\omega t + \alpha + \beta)\right\rangle \\
&= \frac{1}{4}\left(\frac{1 + \cos 2(\alpha - \beta)}{2} + \frac{1}{2}\right) = \frac{1}{4} + \frac{\cos 2(\alpha - \beta)}{8}\,.
\end{aligned} \tag{9}$$

Then, we find from eqs. (5)-(6) and (9),

$$
\begin{aligned}
\frac{\langle P_A P_B \rangle}{E^4} &\propto \left\langle \cos^2(\omega t)\cos^2(\omega t + \delta\phi)\right\rangle + \left\langle \cos^2(\omega t)\cos^2(\omega t + \phi_2)\right\rangle \\
&\quad + \left\langle \cos^2(\omega t + \delta\phi + \phi_2)\cos^2(\omega t + \delta\phi)\right\rangle + \left\langle \cos^2(\omega t + \delta\phi + \phi_2)\cos^2(\omega t + \phi_2)\right\rangle \\
&\quad + \cos(\delta\phi + \phi_2) + \cos(\delta\phi - \phi_2) + \cos(\delta\phi + \phi_2)\cos(\delta\phi - \phi_2) \qquad (10) \\
&= 1 + \frac{\cos 2\delta\phi}{4} + \frac{\cos 2\phi_2}{4} + \cos(\delta\phi + \phi_2) + \cos(\delta\phi - \phi_2) + \cos(\delta\phi + \phi_2)\cos(\delta\phi - \phi_2),
\end{aligned}
$$

and, recalling eq. (8),

$$
\langle P_A P_B \rangle - \langle P_A \rangle \langle P_B \rangle \propto E^4 \left(\frac{\cos 2\delta\phi}{4} + \frac{\cos 2\phi_2}{4} \right). \qquad (11)
$$

Now, stars do not emit steady electromagnetic waves, but rather they are thermal sources of radiation, meaning that the waves consist of a series of short pulses, each with a random phase (and frequency that varies from pulse to pulse).[6] If a measurement were made only for one pulse from source 1 and one pulse from source 2 (that arrive at the two detectors at the same time), the second term on the right of eq. (11) would be nonzero, and the measurement could not determine the phase $\delta\phi$ (or the source separation D) without knowledge of the random phase ϕ_2. However, the actual measurement averages over many pairs of source pulses, with the result that the second term in eq. (11) averages to zero (since the difference ϕ_2 between the phases of waves from sources 1 and 2 has a random distribution). Further, on averaging over the random phase ϕ_2, $\langle P_A \rangle = \langle P_B \rangle \propto E^2$. That is,

$$
\frac{\langle P_A P_B \rangle}{\langle P_A \rangle \langle P_B \rangle} - 1 = \frac{\cos 2\delta\phi}{4}\,, \qquad (12)
$$

and the Brown-Twiss interferometer does measure the phase $\delta\phi = 2\pi dD/\lambda R$, and hence the source separation D.

2.2 Quantum Analysis

There is essentially no difference between a quantum analysis and the classical analysis if we consider that the star emits thermal photons, each of which corresponds to a wavefunction that is a short pulse which is phase coherent over the duration of the pulse, but which does not have phase coherence with other photons. Then, the quantum wave function of a photon from source 1 as observed at detector A at time t is proportional to $E_1\cos(\omega_1 t) \approx E\cos(\omega t)$, while that of a photon from source 2 (as observed at A) is proportional to $E_1\cos(\omega_2 t + \delta\phi_2 + \phi_2) \approx E\cos(\omega t + \delta\phi + \phi_2)$, *etc.*

[6] Stars emit radio waves, but a star is not like a broadcast antenna that emits a coherent electromagnetic wave whose origin cannot be localized to some part of the antenna. Broadcast antennas can be regarded as quantum devices that generate the quantum coherent states described by Glauber [31]. These quantum states differ in important ways from the thermal radiation of stars and light bulbs, which latter consists of incoherent photons emitted by individual, thermally excited atoms whose decays are not coherent with one another.

In the quantum view, P_A (P_B) can be interpreted as the probability of detection of a photon at detector A (B), and the rest of the argument of sec. 2.1 follows as before, but with a quantum interpretation.

It is important to note that photons from sources 1 and 2 are not coherent with one another (and in general have slightly different frequencies). The "interference" in the Brown-Twiss interferometer ultimately does not involve the phase difference ϕ_2 between the two photons (although it does involve the phase difference $\delta\phi$ in the two possible paths of each photon, which leads to an effect of each photon interfering with itself, as argued by Dirac).

There are additional possible "interference" effects in the quantum realm, that do not play a role in the Brown-Twiss interferometer. Namely, photons are bosons, with the implication that two "identical" photons obey Bose statistics. Roughly, this means that identical photons like to "clump" or "bunch" together.[7]

If the photons from sources 1 and 2 were "identical" in the quantum sense, which requires phase coherence between these two sources with, say, $\phi_2 = 0$, eq. (11) still holds, while eq. (12) would become $(1 + \cos 2\delta\phi)/4 = \frac{1}{2}\cos^2\delta\phi$, even in the limit that the detectors receive only one photon from each source at a time.[8] The interferometer would still determine $\delta\phi$, and hence the separation of the sources (provided we know to set ϕ_2 to zero rather than averaging $\cos\phi_2$ to zero as in going from eq. (11) to (12)).

Discussions of the Brown-Twiss interferometer such as [29] led to an awareness that other types of experiments with two or more photons would exhibit nonclassical phenomena, such that Brown and Twiss are sometimes generously considered to be the founders of now-very-active field of quantum optics, even though their interferometer uses pairs of photons in a manner that can be well described by classical optics.[9]

A review of Brown-Twiss interferometry is given in [34].

References

[1] C. Huygens, *Treatise on Light* (1678, 1690; English translation 1912),
http://kirkmcd.princeton.edu/examples/optics/huygens_treatise_on_light.pdf

[2] T. Young, *A Course of Lectures on Natural Philosophy and the Mechanical Arts*, Vol. 1 (London, 1807), http://kirkmcd.princeton.edu/examples/optics/young_lectures_07.pdf

[3] T. Young, *An Account of some Cases of the Production of Colours, not hitherto described*, Phil. Trans. Roy. Soc. London **92**, 387 (1802),
http://kirkmcd.princeton.edu/examples/optics/young_ptrsl_24_387_02.pdf

[7] See, for example, Chap. 4, Vol. III of the Feynman Lectures on Physics [32].

[8] This is the version posed in prob. 3-9 of [33]. In principle, it also could be arranged that the two sources have phase coherence, but with $\phi_2 = \pi/2$, in which case eq. (12) as written above would again describe the result of the interferometry.

[9] For an example in which the quantum results of a simple experiment with two photons are dramatically different than that expected from a classical analysis, see [13].

[4] Darthmouth Physics 1 Lab, *Thomas Young and the Wave Nature of Light* (Apr. 19, 2006), http://kirkmcd.princeton.edu/examples/optics/dsa_060419.pdf

[5] E.S. Barr, *Men and Milestones in Optics II. Thomas Young*, Appl. Opt. **2**, 639 (1963), http://kirkmcd.princeton.edu/examples/optics/barr_ao_2_639_63.pdf

[6] I. Newton, *Opticks*, Roy. Soc. London (1704), http://kirkmcd.princeton.edu/examples/optics/newton_opticks_04.pdf

[7] A. Macleod, *Newton's Rings*, Bull. Soc. Vac. Coat. 20 (Spring, 209), http://kirkmcd.princeton.edu/examples/optics/macleod_svc_20_spring_09.pdf https://www.flipsnack.com/svcdigitalpublications/2009-spring-bulletin.html

[8] T. Birch, *The History of the Royal Society of London*, Vol. 3 (Millar, 1757), http://kirkmcd.princeton.edu/examples/optics/birch_v3_57.pdf

[9] H. Crew, *Thomas Young's Place in the History of the Wave Theory of Light*, J. Opt. Soc. Am. **20**, 3 (1930), http://kirkmcd.princeton.edu/examples/optics/crew_josa_20_3_30.pdf

[10] T. Young, *The Bakerian Lecture. Experiments and Calculations relative to physical Optics*, Phil. Trans. Roy. Soc. London **94**, 1 (1804), http://kirkmcd.princeton.edu/examples/optics/young_ptrsl_94_1_04.pdf

[11] G.I. Taylor, *Interference fringes with feeble light*, Proc. Camb. Phil. Soc. **15**, 114 (1909), http://kirkmcd.princeton.edu/examples/QM/taylor_pcps_15_114_09.pdf

[12] P.A.M. Dirac, *The Principles of Quantum Mechanics*, 4$^{\rm th}$ ed. (Clarendon Press, London, 1958), p. 9, http://kirkmcd.princeton.edu/examples/QM/dirac_qm_58.pdf

[13] K.T. McDonald and L.J. Wang, *Bunching of Photons When Two Beams Pass Through a Beam Splitter* (Aug. 17, 2003), http://kirkmcd.princeton.edu/examples/bunching.pdf

[14] H. Fizeau, *Prix Bordin: Rapport sur le concours de l'année 1867*, Comp. Rend. Acad. Sci. **66**, 932 (1868), http://kirkmcd.princeton.edu/examples/astro/fizeau_cras_66_932_68.pdf

[15] A.A. Michelson, *On the Application of Interference Methods to Astronomical Measurements*, Phil. Mag. **30**, 1 (1890), http://kirkmcd.princeton.edu/examples/astro/michelson_pm_30_1_90.pdf

[16] A.A. Michelson, *Measurement of Jupiter's Satellites by Interference*, Nature **45**, 160 (1891), http://kirkmcd.princeton.edu/examples/astro/michelson_nature_45_160_91.pdf

[17] A.A. Michelson and F.G. Pease, *Measurement of the diameter of α Orionis with the interferometer*, Ap. J. **53**, 249 (1921); Proc. Natl. Acad. Sci. **7**, 143 (1921), http://kirkmcd.princeton.edu/examples/astro/michelson_apj_53_249_21.pdf

[18] P.G. Lawson, *Notes on the History of Stellar Interferometry*, p. 325 of *Principles of Long Baseline Stellar Interferometry*, JPL-00-009 (2000), http://kirkmcd.princeton.edu/examples/astro/lawson_jpl-00-009.pdf

[19] A. Labeyrie, *Interference Fringes Obtained on Vega with Two Optical Telescopes*, Ap. J. **196**, L71 (1975), http://kirkmcd.princeton.edu/examples/astro/labeyrie_apj_196_L71_75.pdf

[20] K.G. Jansky, *Radio Waves from Outside the Solar System*, Nature **132**, 66 (1933), http://kirkmcd.princeton.edu/examples/astro/jansky_nature_132_66_33.pdf

[21] M. Ryle and D. Vonberg, *Solar radiation on 175 Mc/s*, Nature **158**, 339 (1946), http://kirkmcd.princeton.edu/examples/astro/ryle_nature_158_339_46.pdf

[22] R. Hanbury Brown, R.C. Jennison and M.K. Das Gupta, *Apparent Angular Sizes of Discrete Radio Sources*, Nature **170**, 1061 (1952), http://kirkmcd.princeton.edu/examples/QED/brown_nature_170_1061_52.pdf

[23] R. Hanbury Brown and R.Q. Twiss, *A New Type of Interferometer for Use in Radio Astronomy*, Phil. Mag. **46**, 663 (1954), http://kirkmcd.princeton.edu/examples/QED/brown_pm_46_663_54.pdf

[24] R. Hanbury Brown and R.Q. Twiss, *Correlation between Photons in Two Coherent Beams of Light*, Nature **177**, 27 (1956), http://kirkmcd.princeton.edu/examples/QED/brown_nature_177_27_56.pdf

[25] R. Hanbury Brown and R.Q. Twiss, *The Question of Correlation between Photons in Coherent Light Rays*, Nature **178**, 1447 (1956), http://kirkmcd.princeton.edu/examples/QED/brown_nature_178_1447_56.pdf

[26] R. Hanbury Brown and R.Q. Twiss, *Interferometry of the intensity fluctuations in light I. Basic theory: the correlation between photons in coherent beams of radiation*, Proc. Roy. Soc. London A **242**, 300 (1957), http://kirkmcd.princeton.edu/examples/QED/brown_prsla_242_300_57.pdf

[27] R. Hanbury Brown and R.Q. Twiss, *Interferometry of the intensity fluctuations in light III. Applications to astronomy*, Proc. Roy. Soc. London A **248**, 199 (1958), http://kirkmcd.princeton.edu/examples/QED/brown_prsla_248_199_58.pdf

[28] E. Brannen and H.I.S. Ferguson, *The Question of Correlation between Photons in Coherent Light Rays*, Nature **178**, 481 (1956), http://kirkmcd.princeton.edu/examples/QED/brannen_nature_178_481_56.pdf

[29] E.M. Purcell, *The Question of Correlation between Photons in Coherent Light Rays*, Nature **178**, 1449 (1956), http://kirkmcd.princeton.edu/examples/QED/purcell_nature_178_1449_56.pdf

[30] V. Radhakrishnan, *Robert Hanbury Brown*, Phys. Today **55**(7), 75 (2002), http://kirkmcd.princeton.edu/examples/astro/radhakrishnan_pt_55-7_75_02.pdf

[31] R.J. Glauber, *Coherent and Incoherent States of the Radiation Field*, Phys. Rev. **131**, 2766 (1963), http://kirkmcd.princeton.edu/examples/QM/glauber_pr_131_2766_63.pdf

[32] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, 1965), http://feynmanlectures.caltech.edu/III_04.html

[33] G. Neugebauer, *The Feynman Lectures on Physics, Vol. 3, Exercises* (Addison-Wesley, 1965), http://kirkmcd.princeton.edu/examples/QM/feynman_v3_prob3-9.pdf

[34] G. Baym, *The physics of Hanbury BrownTwiss intensity interferometry: from stars to nuclear collisions* (Zakopane, 1997), http://kirkmcd.princeton.edu/examples/QM/bayn_zakopane_97.pdf