The Antenna Paradox

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(December 17, 2004)

Abstract

The antenna paradox (enunciated in 1941 by Schelkunoff as the "radiation paradox") is that a good/perfect conductor can support little/no tangential electric field at its surface, and hence the Poynting vector can have little/no normal component at the surface. Thus, good/perfect conductors can emit little/no energy, and hence the conductors of an antenna cannot be the source of the energy radiated by the antenna. The resolution of this paradox favored by Schelkunoff is to regard an antenna as a waveguide, or as the last element in a transmission line, that directs energy from an oscillatory source into the far zone. This view remains counterintuitive to many who wish to view antennas as having a more active role in the radiation process. Here we offer an alternative view in which energy is first delivered from the source to points on the antenna, at which points the energy is (re)radiated) into the far zone. Our prescription is readily illustrated with Numerical Electromagnetic Codes (NECs) that have become important tools for the calculation of radiation by antennas with realistic geometries.

1 Introduction

A method of accounting for the flow of energy in electromagnetic waves was introduced by Poynting in 1884 [1] via the cross product of the electric field vector **E** and the magnetic field vector **H**,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}. \tag{1}$$

The Poynting vector **S** has dimensions of energy per area per time, and also velocity times energy per volume, and so is consistent with its interpretation as a quantitative measure of energy flow.

Poynting realized that the use of his vector **S** would require some revision in thinking about the role of electrical currents. hus, on p. 192 of his paper he states:

> *If we accept Maxwell's theory of energy residing in the medium, we must no longer consider a current as something conveying energy along the conductor. A current is rather to be regarded as consisting essentially of a convergence of electric and magnetic energy from the medium upon the conductor and its transformation there into other forms.*

And on p. 193 Poynting notes that there can be situations in which

> *the energy merely streams round the outside of the conductor.*

The essence of the antenna paradox is contained in these remarks of Poynting, although the development of antennas by Hertz was still three years away.

The antennas of Hertz that emitted radiation did so using spark discharges. Hertz gave an extremely appealing analysis of the radiation process in terms of electromagnetic waves created by an oscillating electric dipole [2]. In this type of antenna metallic conductors play only the supporting role of electrodes and the radiation was understood to be emitted by free, accelerating charges in the gap between the electrodes.

The radio transmission industry that emerged in the following years sought greater reliability than spark discharges by separating the radiation process into parts involving a high-frequency oscillator that was connected via a transmission line to a metallic device that has come to be called the antenna. The spirit of Poynting is that the flow of high-frequency electromagnetic energy begins in the oscillator, from which energy passes down the transmission line and into the far zone in a manner that is influence by the configuration of the antenna. However, analyses of this process tended to be made in three separate steps, and the overall view of Poynting was perhaps obscured thereby.

An important analytical advance was made by Pocklington in 1897 [3] who noted on one hand that standing-wave currents in thin wires are to a good approximation sinusoidal functions of the wavelength $\lambda = c/f$ (assuming that the waves have a pure frequency f and that c is the speed of light in the surrounding medium), and on the other hand these currents and their associated electromagnetic fields satisfy an integral equation from which they could all, in principle, be calculated given knowledge of relevant drive voltages. Pocklington's integral equation, as solved by the so-called method of moments [4], is the basis of the Numerical Electromagnetic Codes (NECs) used nowadays for modeling of antennas. In this paper we will use the NEC4 code [5].

Considerable time elapsed between Pocklington's statement of his integral equation and practical solution of it for antenna problems [6]. In the mean time, useful progress was made by assuming sinusoidal current distributions in simple antennas, beginning with the work of Abraham in 1901 [7]. This approach was first summarized in English by Pierce [8], and is now presented in many contemporary textbooks [9]. In particular, a center-fed linear antenna of length $d = 2a$ situated along the z axis is assumed to have a current distribution of the form

$$I(z,t) = I_0 \sin\left(ka - k\left|z\right|\right) \cos\omega t, \tag{2}$$

where $k = 2\pi/\lambda = \omega/c$ is the wave number of the radiation whose angular frequency is ω. The far-zone radiation pattern calculated from eq. (2) is found to be in good agreement with experiment,

$$\frac{dP}{d\Omega} = 377 \text{ Ohms} \frac{I_0^2}{8\pi^2} \left| \frac{\cos\left(ka\cos\theta\right) - \cos\left(ka\right)}{\sin\theta} \right|^2, \tag{3}$$

where P is radiated power, $d\Omega$ is an element of solid angle, and θ is the polar angle with respect to the z axis.

In 1927 Kliatzkin noted (in a Russian publication) that the electric and magnetic fields associated with the current distribution (2) can be calculated analytically in the near zone as well as the far zone [10]. This result was communicated to the English-speaking community by Pistolkors [11] and by Bechmann [12], and is reviewed in the treatises of Stratton [13] and Smythe [14]. For example, the z component of the electric field can be expressed in the cylindrical coordinates shown in Fig. 1 as

$$E_z(\mathbf{r},t) = 377 \text{ Ohms}\frac{I_0}{4\pi}\left[2\cos ka\frac{\sin(kr_0-\omega t)}{r_0} - \frac{\sin(kr_1-\omega t)}{r_1} - \frac{\sin(kr_2-\omega t)}{r_2}\right]. \quad (4)$$

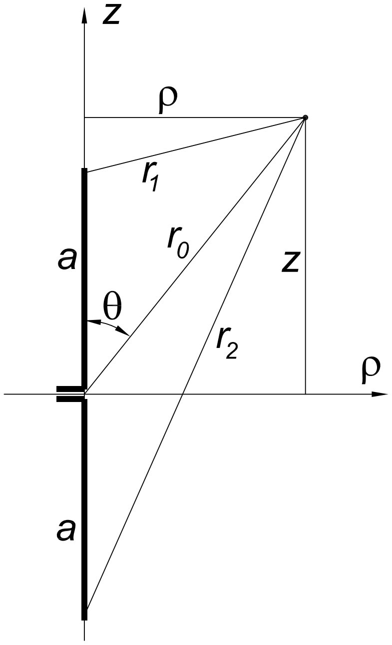

Figure 1: Geometry of a center-fed linear antenna of length $2a$ in a cylindrical coordinate system (ρ, ϕ, z).

The near-zone Poynting flux associated with the fields calculated from the current distribution (2) is illustrated in Fig. 2 for a half-wave dipole antenna ($d = 2a = \lambda/2$). This result is appealing in that the lines of Poynting flux emanate from points distributed along the antenna, which is consistent with an interpretation that the radiation is created by the distributed current (2).

The difficulty with this interpretation is that the electric field (4) has a non-zero tangential component in the region ($\rho = 0, |z| < a$), and so cannot be an accurate representation of the electric field at the surface of a real antenna built from good metallic conductors. Thus, we arrive at one form of the antenna paradox, which is that the current distribution (2) provides a good description of the far-zone radiation pattern of dipole antennas, while being incompatible with the metallic boundary condition for electric fields.

Indeed, the metallic boundary condition is that the electric field $\mathbf{E}$ can have no tangential component at the surface of a conductor in the limit of infinite conductivity. Hence, in this limit, the Poynting vector, $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, has no component perpendicular to the surface of the conductor. No (net) energy can flow outwards from the surface of a perfect conductor, and so such conductors cannot be sources of radiation.

Such concerns no doubt influenced Schelkunoff in his efforts to model antennas as structures of built from good/perfect conductors of finite thickness [15, 16, 17]. He argued that

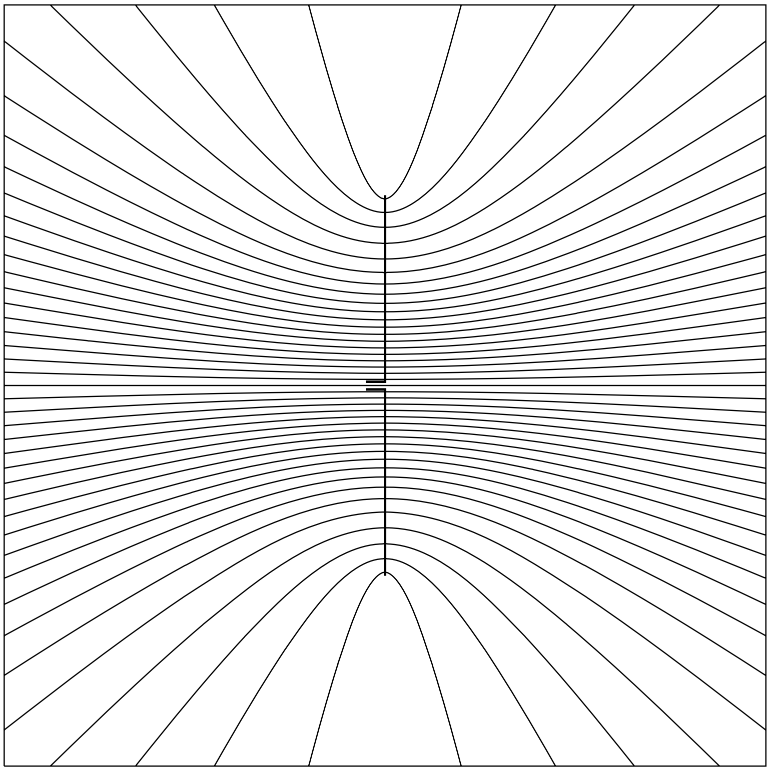

Figure 2: Lines of Poynting flux for the fields of a half-wave, center-fed dipole antenna whose current distribution is assumed to be given by eq. (2).

antennas should not be considered to radiate by themselves, but rather should be regarded as the last element in the transmission line that guides power into the far zone from the high-frequency oscillator that is more properly taken to be the source of the radiation.

The viewpoint of an antenna as a waveguide is supported by numerical calculations using the NEC4 program of the Poynting flux from a resonant, half-wave dipole antenna, as shown in Fig. 3. In detail, the antenna was $d = 10$ m long, with (perfect) conductors of 2.5 cm radius. The excitation frequency was 14.18 MHz, which is not quite $c/2d = 15$ MHz but rather is the frequency at which the terminal impedance of the antenna is purely real (meaning that the current at the feed point is in phase with the feed voltage).

In contrast to Fig. 2, the Poynting flux from an antenna that obeys the metallic boundary condition is parallel rather than perpendicular to the surface of the conductors. The Poynting flux in Fig. 3 emanates from the feed point of the antenna, at the center of the plot, after arriving there from the high-frequency source via a transmission line that is not shown.

Plots such as Fig. 3, which require considerable computational resources, do not appear in the works of Schelkunoff and his contemporaries [18, 19, 20]. The earliest versions of such plots of which we are aware are due to the Landstorfer group [21].

Thus, numerical computations (based on solution to Pocklington's integral equation) confirm the view of Schelkunoff, as anticipated by Poynting, that conductors of an antenna play a somewhat passive role in guiding energy from the active source into the far zone.

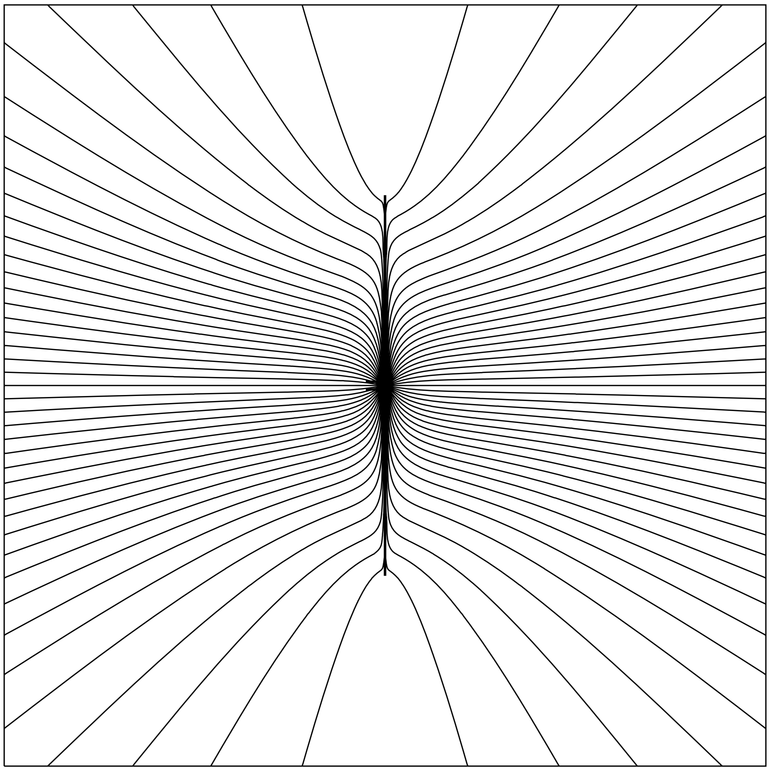

Figure 3: Lines of Poynting flux for the fields of a resonant, half-wave, center-fed dipole antenna whose arms are perfect conductors, calculated using the NEC4 program [5].

Indeed, since lines of Poynting flux cannot intersect a good conductor, the role of the antenna conductors can be considered as "telling the radiation where not to go".

Hence, a direct interpretation of Maxwell's equations, as elaborated upon by Poynting, appears to contradict the appealing view of Abraham *et al.* that the conductors of antennas are sources of radiation. This is the antenna paradox.

2 A Multistep View of Radiation by Antennas

A further statement of the antenna paradox is that the conductors contain oscillating currents, which are caused by accelerating charges. Shouldn't we be able to consider these accelerating charges as the sources of the radiation from the antenna?

Examples from optics may be of relevance here. A mirror reflects light, and a plate of glass refracts light. In both cases the incident radiation causes acceleration of electrons in the mirror or plate, and the radiation of these electrons interferes with the incident radiation to produce the output radiation pattern [22]. However, we do not typically speak of mirrors or window glass as radiators. We are content to regard mirrors and windows as passive objects rather than sources of radiation.

This note expands a bit on the Comments in sec. 2.8 of [23].

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