Spin of the Λ Hyperon via the Adair Method

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(October 13, 2013)

1 Problem

Λ^0 hyperons are produced by a pion beam in the reaction $\pi^- p \to K^0\Lambda^0$, and observed by the decay $\Lambda^0 \to p\pi^-$ (which is a weak interaction that does not conserve parity). Let J denote the spin of the Λ (considered to be unknown in this problem, while the spins of the π^-, p and K^0 are known), and θ be the angle of a decay product in the Λ rest frame, relative to the direction of the Λ in the lab frame. In the case where the Λ is produced exactly along the beam direction, what are the possible values of J_z?

Show that for unpolarized beam protons, and for Λ's produced along the beam direction, the Λ-decay angular distribution depends on J according to

$$\begin{aligned} J &= 1/2, \quad \text{isotropic}, \\ J &= 3/2, \quad 3\cos^2\theta + 1, \\ J &= 5/2, \quad 5\cos^4\theta - 2\cos^2\theta + 1. \end{aligned} \tag{1}$$

Hints in Sakurai, Invariance Principles and Elementary Particles *(1964), p. 17.*

This problem is based on R.K. Adair, *Angular Distribution of Λ^0 and θ^0 Decays*, Phys. Rev. **100**, 1540 (1955), http://puhep1.princeton.edu/~mcdonald/examples/EP/adair_pr_100_1540_55.pdf.
The principle of this problem was used to determine that the Λ^0 has spin-1/2 by F. Eisler *et al.*, *Experimental Determinations of the Λ^0 and Σ^- Spins*, Nuovo Cim. **7** 222 (1958), http://puhep1.princeton.edu/~mcdonald/examples/EP/eisler_nc_7_222_58.pdf.

2 Solution

A two-particle state can only have orbital-angular-momentum component $L_z = 0$ along a z-axis.

If the Λ^0 moves along the beam axis, taken to be the z-axis, then so also does the K^0, and no matter what is their orbital angular momentum L, $L_z = 0$. Of course, the initial $\pi^- p$ state has $L_z = 0$, and $J_z = \pm 1/2$, since the pion is spinless and the proton has spin-1/2. Conservation of angular momentum then implies that $J_z = \pm 1/2$ for the final state; these two states are distinguishable, so it suffices to consider only one, say $J_z = 1/2$.

Similarly, since the initial state can only have $J = n/2$ for odd n this also holds for the final state, which in turn implies that the spin of the Λ^0 is $m/2$ for odd m, since the K^0 is spinless.

1. $J_\Lambda = 1/2$.

   In general, the decay final state $\pi^- p$ could have $L = 0$ or 1 such that $J = 1/2$. If the Λ has $J_z = \pm 1/2$ in its rest frame, then this couples to the $L = 0$ $\pi^- p$ state according to

$$|1/2, 1/2\rangle = |0, 0\rangle |1/2, \pm 1/2\rangle, \tag{2}$$

   and couples to the $\pi^- p$ states with orbital angular momentum $L = 1$ and (proton) spin $S = \pm 1/2$ according to

$$|1/2, 1/2\rangle = \sqrt{\frac{2}{3}} |1, 1\rangle |1/2, -1/2\rangle - \sqrt{\frac{1}{3}} |1, 0\rangle |1/2, 1/2\rangle, \tag{3}$$

$$|1/2, -1/2\rangle = -\sqrt{\frac{2}{3}} |1, -1\rangle |1/2, 1/2\rangle + \sqrt{\frac{1}{3}} |1, 0\rangle |1/2, -1/2\rangle, \tag{4}$$

   using the Clebsch-Gordon coefficients from
   http://pdg.lbl.gov/2013/reviews/rpp2012-rev-clebsch-gordan-coefs.pdf.

   The initial $J_z = \pm 1/2$ states, and the decay final states are all distinguishable by the proton spin component, so we have four amplitudes to consider,

$$\alpha |0, 0\rangle |1/2, 1/2\rangle - \beta \sqrt{\frac{1}{3}} |1, 0\rangle |1/2, 1/2\rangle, \tag{5}$$

$$\beta \sqrt{\frac{2}{3}} |1, 1\rangle |1/2, -1/2\rangle, \tag{6}$$

$$\alpha |0, 0\rangle |1/2, -1/2\rangle + \beta \sqrt{\frac{1}{3}} |1, 0\rangle |1/2, -1/2\rangle, \tag{7}$$

$$-\beta \sqrt{\frac{2}{3}} |1, -1\rangle |1/2, 1/2\rangle, \tag{8}$$

   where α is the strength of the interaction with the $L = 0$ state, and β is the strength of the interaction with the $L = 1$ state. We square amplitudes (5)-(8) and add to find the angular distribution, noting that the orbital angular momentum states $|L, L_z\rangle$ correspond to spherical harmonics $Y_L^{L_z}(\theta, \phi)$, where θ is the angle of, say, the decay pion with respect to the z-axis in the Λ rest frame.

$$Y_0^0 = \sqrt{\frac{1}{4\pi}}, \quad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{\pm i\phi}, \quad Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta. \tag{9}$$

The four amplitudes (5)-(8) are then (after multiplying by $\sqrt{4\pi}$),

$$\alpha - \beta\sqrt{\frac{1}{3}} \cos\theta, \quad -\beta\sqrt{\frac{1}{3}} \sin\theta \, e^{i\phi}, \quad \alpha + \beta\sqrt{\frac{1}{3}} \cos\theta, \quad -\beta\sqrt{\frac{1}{3}} \sin\theta \, e^{-i\phi}. \tag{10}$$

Squaring, and adding, leads to the angular distribution

$$2\left|\alpha\right|^2 + \frac{2\left|\beta\right|^2}{3}(\sin^2\theta + \cos^2\theta) = 2\left|\alpha\right|^2 + \frac{2\left|\beta\right|^2}{3} = \text{isotropic}. \tag{11}$$

We note that the target protons needed to be unpolarized so that the cases of $J_z = \pm 1/2$ for the initial state are equally likely, and the cross terms between different L in the final $\pi^- p$ state cancel out. We assume this holds for the cases of higher possible Λ spin, and consider than contributions to the angular distribution from different L separately.

2. $J_\Lambda = 3/2$.

In this case the orbital angular momentum of the $\pi^- p$ final state can be $L = 1$ or 2 such that $J = 3/2$. If the Λ has $J_z = 1/2$ in its rest frame, then this couples to the $\pi^- p$ final states with orbital angular momentum $L = 1$ and (proton) spin $S = 1/2$ according to

$$|3/2, 1/2\rangle = \sqrt{\frac{1}{3}}|1,1\rangle|1/2,-1/2\rangle + \sqrt{\frac{2}{3}}|1,0\rangle|1/2,1/2\rangle, \tag{12}$$

which implies an angular distribution proportional to

$$\left|Y_1^1\right|^2 + 2\left|Y_1^0\right|^2 \propto \frac{\sin^2\theta}{2} + 2\cos^2\theta \propto 3\cos^2\theta + 1. \tag{13}$$

Similarly, the coupling to the $\pi^- p$ final states with orbital angular momentum $L = 2$ is

$$|3/2, 1/2\rangle = \sqrt{\frac{3}{5}}|2,1\rangle|1/2,-1/2\rangle - \sqrt{\frac{2}{5}}|2,0\rangle|1/2,1/2\rangle, \tag{14}$$

which implies an angular distribution of

$$3\left|Y_2^1\right|^2 + 2\left|Y_2^0\right|^2 \propto 3\frac{15}{2}\sin^2\theta\cos^2\theta + 2\frac{5}{4}(3\cos^2\theta - 1)^2 \propto 3\cos^2\theta + 1, \tag{15}$$

noting that

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta \, e^{i\phi}, \qquad Y_2^0 = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1). \tag{16}$$

Thus, either value of L for the $\pi^- p$ final states leads to the same angular distribution, namely $3\cos^2\theta + 1$.

3. $J_\Lambda = 5/2$.

   In this case the possible orbital angular momenta of the final $\pi^- p$ states are $L = 2$ and 3.

   We content ourselves with calculating only $L = 2$.

$$|5/2, 1/2\rangle = \sqrt{\frac{2}{5}}|2, 1\rangle|1/2, -1/2\rangle + \sqrt{\frac{3}{5}}|2, 0\rangle|1/2, 1/2\rangle, \tag{17}$$

   which implies an angular distribution of

$$2\left|Y_2^1\right|^2 + 3\left|Y_2^0\right|^2 \propto 2\frac{15}{2}\sin^2\theta\cos^2\theta + 3\frac{5}{4}(3\cos^2\theta - 1)^2 \propto 5\cos^4\theta - 2\cos^2\theta + 1. \tag{18}$$