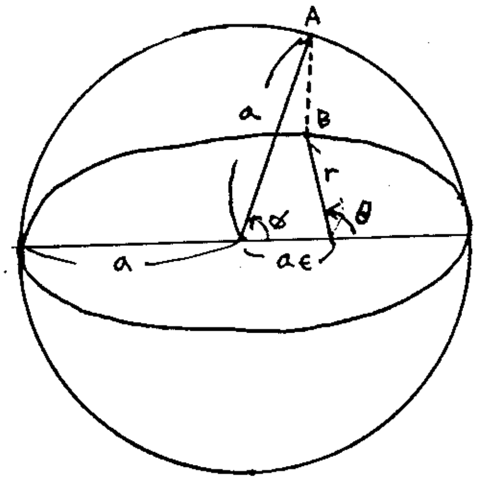


. a) Kepler's Equation of Time:

From the right figure we get the relation,

$$a\cos\phi + r\cos(180-\theta) = a\epsilon$$

Putting the orbit equation

$$\frac{1}{r} = \frac{1+\epsilon\cos\theta}{a(1-\epsilon^2)}$$

into the above equation yields

$$\cos\phi = \frac{(1-\epsilon^2)\cos\theta}{1+\epsilon\cos\theta} + \epsilon = \frac{\cos\theta+\epsilon}{1+\epsilon\cos\theta}$$

Inverting above equation, we get

$$\cos\theta = \frac{\cos\phi-\epsilon}{1-\epsilon\cos\phi} \qquad (1)$$

We differentiate above equation to get

$$-\sin\theta\,\dot\theta = -\frac{(1-\epsilon^2)}{(1-\epsilon\cos\phi)^2}\dot\phi \;\Rightarrow\; \dot\theta = \frac{\sqrt{1-\epsilon^2}}{1-\epsilon\cos\phi}$$

where we used

$$\sin\theta = \frac{\sqrt{1-\epsilon^2}\,\sin\phi}{1-\epsilon\cos\phi}$$ (notice that the sign of $\sin\theta$ & $\sin\varphi$ are the same. $1-\epsilon\cos\phi>0$)

which can be obtained from (1). Now, combing orbit equation and (1), we solve r^2 to get,

$$r^2 = \frac{a^2(1-\epsilon^2)^2}{(1+\epsilon\cos\theta)^2} = a^2(1-\epsilon\cos\phi)^2$$

Consequently,

$$L = mr^2\dot\theta = ma^2\sqrt{1-\epsilon^2}\,(1-\epsilon\cos\phi)\frac{d\phi}{dt}$$

$$\Rightarrow K\,dt = (1-\epsilon\cos\phi)\,d\phi \quad\Rightarrow\quad Kt = \phi - \epsilon\sin\phi$$

where we define,

$$K = \frac{L}{ma^2}\cdot\frac{1}{\sqrt{1-\epsilon^2}}$$

From the properties of an ellipse, we know,

$$b^2 = (1-\epsilon^2)a^2$$

From the constancy of the angular momentum, we get

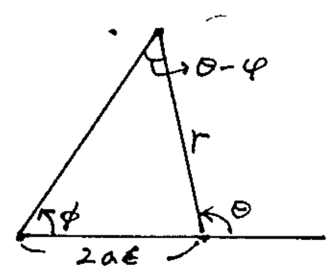
$$\int_{1\text{ period}} \frac{L}{2}\,dt = \frac{LT}{2} = m\int \frac{1}{2}r^2\,d\theta = m\cdot\text{Area} = m\pi ab$$

where we used the geometrical fact that the area of an ellipse is πab. From the note we get

$$L^2 = GMm^2a(1-\epsilon^2)$$

Combining these results, we get

$$K = \frac{L}{mab} = \frac{2\pi}{T} = \Omega = \sqrt{\frac{GM}{a^3}}$$



b) From the law of sine applied to the right figure, we get,

$$\frac{2a\epsilon}{\sin(\theta-\phi)} = \frac{r}{\sin\phi}$$

Using $\sin(\theta-\phi) = \sin\theta\cos\phi - \sin\phi\cos\theta$, the above equation immediately becomes,

$$\tan\phi = \frac{\sin\theta}{\cos\theta + \frac{2a\epsilon}{r}}$$

We differentiate both sides with respect to time and get,

$$(1+\tan^2\phi)\dot{\phi} = \left(\dot{\theta}\cos\theta\left(\cos\theta+\frac{2a\epsilon}{r}\right)-\sin\theta\left(-\sin\theta\,\dot{\theta}-\frac{2a\epsilon}{r^2}\dot{r}\right)\right)\Big/\left(\cos\theta+\frac{2a\epsilon}{r}\right)^2$$

$$\Rightarrow \left(1+\frac{4a\epsilon}{r}\cos\theta+\frac{4a^2\epsilon^2}{r^2}\right)\dot{\phi} = \dot{\theta}\left(1+\frac{2a\epsilon}{r}\cos\theta+\frac{2a\epsilon}{r^2}\cdot\frac{\dot{r}}{\dot{\theta}}\sin\theta\right) \quad (2)$$

From the equation,

$$r = \frac{a(1-\epsilon^2)}{1+\epsilon\cos\theta} \Rightarrow \dot{r} = \frac{-\epsilon a(1-\epsilon^2)}{(1+\epsilon\cos\theta)^2}\sin\theta\,\dot{\theta}$$

we find that (2) can be written in the form

$$\dot{\phi} = \dot{\theta}\,\frac{1+2a\epsilon/r\cos\theta+O(\epsilon^2)+\cdots}{1+4a\epsilon/r\cos\theta+O(\epsilon^2)+\cdots} = \dot{\theta}\left(1-\frac{2a}{r}\epsilon\cos\theta\right)+O(\epsilon^2)+\cdots \quad (3)$$

where $O(\epsilon^2)$ represents the second order in ϵ^2 which is assumed to be small Also from the orbit equation, we find that

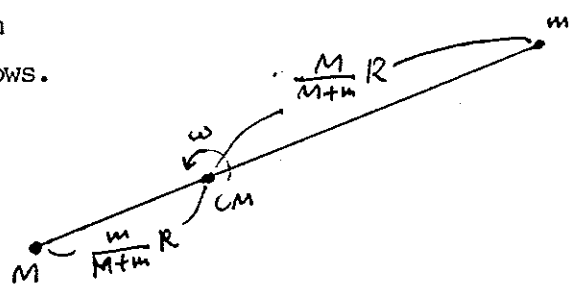
$$r \simeq a(1-\epsilon\cos\phi)+O(\epsilon^2)+\cdots$$

Thus (3) can be rewritten as,

$$\dot{\phi} = \frac{mr^2\dot{\theta}}{ma^2}+O(\epsilon^2)+\cdots = \frac{L}{ma^2}+O(\epsilon^2)+\cdots$$

The above equation shows that $\dot{\phi}$ is a constant upto the first order in ϵ.

2. Spin-Orbit Coupling



Referring to the right figure which shows the configuration about the center of mass, we can write down the total angular momentum as follows.

$$L = I\Omega + M\cdot\left(\frac{m}{M+m}R\right)^2\omega + m\left(\frac{M}{M+m}R\right)^2\omega$$

$$= I\Omega + \frac{Mm}{M+m}R^2\omega \quad (1)$$

From the Newton's law,

$$\frac{GMm}{R^2} = m\cdot\frac{M}{M+m}R\cdot\omega^2$$

we can rewrite (1) in the form

$$L = I\Omega + C\omega^{-1/3}$$

where

$$C \equiv G^{2/3}Mm/(M+m)^{1/3}$$

The total energy consists of the kinetic energy and the gravitational potential energy. That is,

$$E = \frac{1}{2}I\Omega^2+\frac{1}{2}M\left(\frac{m}{M+m}R\omega\right)^2+\frac{1}{2}m\left(\frac{M}{M+m}R\omega\right)^2-\underbrace{\frac{GMm}{R}}_{\text{via Newton's law}}$$

$$= \frac{1}{2}I\Omega^2-\frac{1}{2}\frac{Mm}{M+m}R^2\omega^2$$

where we used Newton's law once. Comparing the above equation with (1), we conclude

$$E = \frac{1}{2}I\Omega^2-\frac{1}{2}C\omega^{2/3}$$

As suggested in the problem, we introduce a variable x,

$$x \equiv C\omega^{-1/3}$$

In this variable E and L can be written as

$$L = I\Omega + x$$

$$E = \frac{1}{2} I \Omega^2 - \frac{c^3}{2x^2}$$

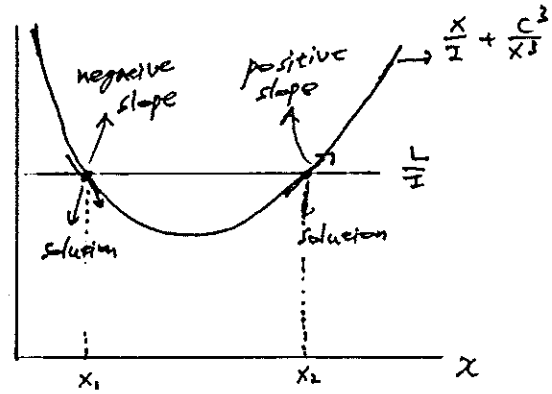
During the tidal friction, we can assume that L is a constant. (It is a direct consequence of the fact that gravitational interaction is purely a central one.) Thus, we can express E in terms of the constant L as follows.

$$E = \frac{1}{2I}(L-x)^2 - \frac{c^3}{2x^2}$$

This looks equivalent to the one-dimensional potential problem. The equilibrium state would be such that

$$\frac{dE}{dx} = 0 \Rightarrow -\frac{L-x}{I} + \frac{c^3}{x^3} = -\Omega + \omega = -\frac{L}{I} + \left(\frac{x}{I} + \frac{c^3}{x^3}\right) = 0$$

For simplicity we assume $L > 0$, $\Omega > 0$, and $\omega > 0$. From the above equation, we find that once equilibrium exists, it will be the case when $\omega = \Omega$. From the figure below, we see that the solution of the above equation actually exists only if the minimum value of $\frac{x}{I} + \frac{c^3}{x^3}$ is less than $\frac{L}{I}$. Since the minimum value of former can be obtained as follows,



$$\frac{d}{dx}\left(\frac{x}{I} + \frac{c^3}{x^3}\right) = \frac{1}{I} - \frac{3c^3}{x^4} = 0$$

$$\therefore \left.\frac{x}{I} + \frac{c^3}{x^3}\right|_{min} = \frac{4}{3^{3/4}}(c^3 I)^{\frac{3}{4}} \cdot \frac{1}{I}$$

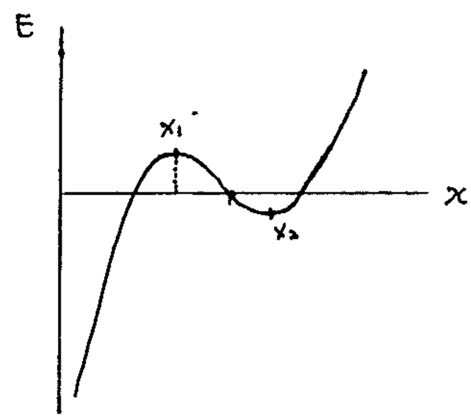
one condition we should require for the initial condition is,

$$L \geq \frac{4}{3^{3/4}}(c^3 I)^{3/4}$$

Clearly, if this condition does not hold, the total energy is monotonic increasing function with no equilibrium at all. Under the condition, the graph of the energy becomes as shown below. (Notice that this form is nearly determined by the previous graph.) Since the energy should decrease all the time, if we additionally require that

$$x_1 \leq x_{initial}$$

where x_1 is the smaller root of the $\frac{d}{dx}E = 0$ equation, existence of which is guaranteed from the first condition.



Thus, under these two conditions in initial values, we can be sure that there would be a stable equilibrium with In earth-moon system, $L > 0$, $\Omega > 0$, and $\omega > 0$ do hold. And R can be written as,

$$R = (G(M+m))^{1/3} \frac{1}{c^2}(L - I\Omega)^2$$

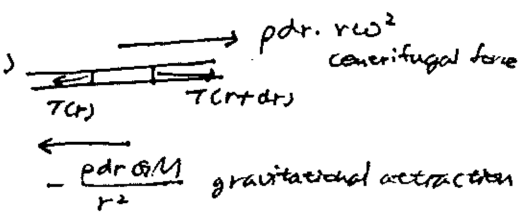
where we used Newton's law. By the direct differentiation, we find that,

$$\frac{dR}{d\Omega} = -I\,(G(M+m))^{1/3}\,\frac{1}{c^2}\,(L - I\Omega) < 0 \quad (\text{Notice that } \omega > 0)$$

which tells us that R increases as Ω decreases.

3. Skyhook

We apply the force balancing to the infinitesmal element of rope shown right. Then, we get, (ρ: mass density)



$$T(r+dr) - T(r) - \frac{\rho GM\,dr}{r^2} + \rho\,dr\,r\omega^2 = 0$$

which yields,

$$\frac{dT}{dr} = \frac{\rho GM}{r^2} - \rho r\omega^2$$

Using the boundary condition, $T(r_{min})=0$ and $T(r_{max})=0$, we can integrate above equation and get, (We use only $T(r_{min})=0$)

$$T(r) = -\frac{\rho GM}{r} - \frac{\rho r^2\omega^2}{2} + \frac{\rho GM}{r_e} + \frac{\rho r_e^2\omega^2}{2}$$

The condition $T(r_{max})=0$, now, gives,

$$\frac{r_{max}^3}{2}\omega^2 + \left(\frac{GM}{r_e} + \frac{r_e^2\omega^2}{2}\right) r_{max} - GM = 0$$

$$\Rightarrow (r_{max} - r_e)\left(\frac{\omega^2}{2} r_{max}^2 + \frac{\omega^2 r_e}{2} r_{max} - \frac{GM}{r_e}\right) = 0 \qquad (1)$$

Above quadratic equation is easily solved to give

$$r_{max} = \frac{r_e}{2}\left(\sqrt{1 + \frac{8GM}{\omega^2 r_e^3}} - 1\right) \approx \underline{1.5 \times 10^8\,m}$$

where we choose only positive solution. The effective potential of this problem can be written as,

$$V_{eff} = \int_r^{r+\Delta} \left(-\frac{GM\rho}{r}\,dr - \frac{1}{2}\rho r^2\omega^2\right) dr \quad (\text{assuming no coiling!})$$

where Δ is a constant denoting the length of the rope. (Notice the minus sign in centrifugal term! One should construct effective potential from L = T − V rather than H = T + V) We differentiate twice the above expression to get,

$$\frac{d^2}{dr^2} V_{eff} = -\rho\left(\omega^2\Delta + \left(\frac{1}{r^2} - \frac{1}{(r+\Delta)^2}\right)\right) \quad \left(\frac{d}{dr} V_{eff} = 0 \text{ gives (1) again, in fact.}\right)$$

which is clearly negative. Thus, the rope is unstable.

comment. We could have used virial theorem. In this case, we first evaluate the total energy and find that it is positive. According to the virial theorem the value should be negative. Thus, this contradiction leads us to the conclusion that the motion is unbounded. This is exactly the same as the assertion that the rope is unstable.

4. a) Modelling

Since we use the same material, the density should not be changed. Thus,

$$\rho' = \rho = \frac{m'}{\ell'^3} = \frac{m'}{\alpha^3 \ell^3} = \frac{m}{\ell^3}$$

from which we conclude that $m' = \alpha^3 m$. From the invariance of Newton's law, we get

$$F' = m' \frac{d^2}{dt'^2} x' = \alpha^3 \cdot \alpha \cdot \frac{1}{\beta^2} m \frac{d^2}{dt^2} x = \alpha^4 \beta^{-2} F$$

$$(cf.\ v' = \frac{\alpha}{\beta} v)$$

If the gravitation is involved in our consideration, the gravitational force will be rescaled as,

$$F_g' = m' g = \alpha^3 mg = \alpha^3 F_g$$

(cf. Notice that one should not use $F = \frac{G m_1 m_2}{r^2}$. In modelling we are dealing with the phenomena occuring near the surface of the earth. If we rescale using the latter, we are performing astronomical simulation!) Since all other forces should be rescaled the same way, we get,

$$\alpha^3 = \alpha^4 \beta^{-2} \quad \Rightarrow \quad \beta = \sqrt{\alpha} \qquad (1)$$

For a given v', the corresponding v is $v = \frac{\cancel{\alpha}\,\beta}{\cancel{\beta}\,\alpha} v'$. If gravity is involved, we get $v = \frac{1}{\sqrt{\alpha}} v' = \sqrt{n}\, v'$ due to (1). Given the form

$$F = k v^p A^q$$

, the force should transform like

$$F' = k v'^p A'^q = \alpha^{p+2q} \beta^{-p} F$$

which should be equated with $\alpha^4 \beta^{-2} F$. Thus, we get $p=2, q=1$ from $p+2q=4$ & $p=2$ ✓. If we assume that gravity should be included all the time (which is not necessary in some effectively 2-dimensional case with no vertical motion), we get one relation,

$$\frac{p}{2} + 2q = 3$$

(b) From the force law,

$$\vec{F} = -A\vec{r} - B\vec{v}$$

we find the dimension of A and B.

$$\dim A = [\text{mass}][\text{time}]^{-2}, \qquad \dim B = [\text{mass}][\text{time}]^{-1}$$

Since the particle was initially at rest, the resulting motion is one dimensional. Thus, the time to reach the origin is

$$t = t(m, A, B, \vec{r})$$

Since the motion is one dimensional, there can be no dimensionless directional dependence. Furthermore, since the only parameter which has the dimension of length is the initial position, there can not be any dimensionless parameter containing it. Since the dimension of function is time, there is no way of appearing of the initial position since there is no counter term having inverse length dimension. Thus, the time is independent of initial position.

(c) From the force law,

$$\vec{F} = -\frac{A}{r^n}\hat{r}$$

we find the dimension of the constant A is, $[m][t]^{-2}[\ell]^{n+1}$. Since the initial speed is zero, the time can be a function of only A, m, and R. Thus, we set,

$$T \sim A^{\alpha} m^{\beta} R^{\gamma}$$

The dimensionality consideration gives us,

$$\alpha + \beta = 0, \quad -2\alpha = 1, \quad \gamma + (n+1)\alpha = 0 \quad \Rightarrow \quad \gamma = \frac{n+1}{2}.$$

$$\therefore \quad T \sim R^{(n+1)/2}$$

5. Bio-Mechanics

(a) We can assume that the animal dies if it loses some portion of its water. The initial amount of water contained in the animal is proportional to L^3. Since it loses water through sweating, the loss of water per unit time should proportional to L^2. Thus, the living time should proportional to L.

(b) Their production of power accompanies the sweating which is proportiona to surface area, i.e., L^2. Thus, the power should be proportional to L^2.

(c) If the air resistance is the limit, the power should be balanced with the resistance force times speed. Since the power is proportional to L^2 and the air resistance is also proportional to L^2, two factor cancel out. Thus, the speed is independent of L.
If the gravity is the limit, the power should be balanced with the gravitational force time speed, where gravitational force is proportional to L^3 if we assume the density of animals are approximately the same. Thus, the speed is proportional to L^{-1}.

(d) Since The radius of bone is L, the area of bone which is proportional to maximum force which should be produced in the process of jumping is proportional to L^2. Thus, the amount of the work done by the force is proportional to force times body length, i.e., L^3. Since the gravitational potential energy is mgh where m is proportional to L^3, the factor L^3 in both sides cancels. Thus, the height is independent of L.

8. (a) We consider the scattering of BBs from a steel ball in their center of mass frame. As is well-known, in the center of mass frame, the steel ball moves in one direction before the collision and moves into the opposite direction with the same speed. Thus, during the collision process it momentarily stops. Thus, the scattering of BBs satisfies the property that the incident angle is same as the reflecting angle . Thus, from the figure in the next page, we get,

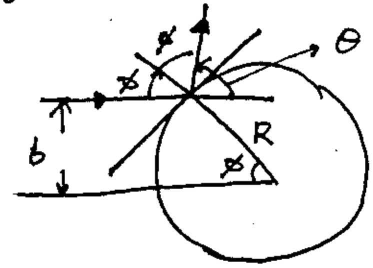
$$b = R\sin\phi, \quad \theta = \pi - 2\phi$$

Combining two eqautions,

$$b = R\cos\frac{\theta}{2}$$

Thus,

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b\, db}{2\pi \sin\theta\, d\theta} = \frac{R^2}{2}\frac{\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{\sin\theta} = \frac{R^2}{4}, \quad \frac{d\sigma}{d\cos\theta} = \frac{\pi}{2}R^2.$$



(b) Let us consider the elastic scattering process. The number of particles which is deflected into solid angle $d\Omega$ is given by

$$n\frac{d\sigma}{d\Omega}\,d\Omega$$

where n is the number of bombarding BBs per unit area. As shown in the figure they trnasfer momentum along the direction of incidence

$$mv(1-\cos\theta)$$

From the spherical symmetry, other components of momentum transfer vanish. Thus, the total momentum transfer is,

$$\int n\frac{d\sigma}{d\Omega}\,mv(1-\cos\theta)\,d\Omega = n\cdot mv\cdot\frac{R^2}{4}\cdot 2\pi\underbrace{\int_0^\pi (1-\cos\theta)\sin\theta\, d\theta}_{=\int_{-1}^{1}(1-x)dx\,=\,2 \text{ with } x=\cos\theta}$$

$$= n(\pi R^2)\,mv.$$

which is clearly same as the momentum transfer by the complete inelastic process. Since the incident direction can vary randomly, this calculation imples that the average drag force caused by elastic scattering is same as the average drag force caused by inelastic scattering.

(c) This problem is beautifully explained in section 17 of Landau and Lifshitz. According to their equation (17.5) of page 46,

$$v_2' = \frac{2mv}{(M+m)}\sin\frac{\theta}{2}$$

where v_2' represents the speed of the steel ball after collision.From the energy conservation,

$$dT_{lost} = dT_{gain}$$

and, since the steel ball was initially at rest, its final kinetic energy is,

$$T_{gain} = \frac{2Mm^2v^2}{(M+m)^2}\sin^2\frac{\theta}{2}$$

Thus, by the direct differentiation, we get,

$$\frac{dT_{gain}}{d\Omega} = \frac{T_{max}}{2\pi\sin\theta}\frac{d}{d\theta}\sin^2\frac{\theta}{2} = \frac{1}{4\pi}T_{max}$$

where $T_{MAX} = \frac{2Mm^2v^2}{(M+m)^2}$ which is easily seen from (1) since the angle can be any angle between 0 and 180. Consequently,

$$\frac{d\sigma}{dT_{lost}} = \frac{d\sigma}{dT_{gain}} = \frac{d\sigma}{d\Omega}\left(\frac{dT_{gain}}{d\Omega}\right)^{-1} = \frac{R^2}{4}\cdot\frac{4\pi}{T_{max}} = \frac{\pi R^2}{T_{max}}$$

Clearly, T_{min} = 0 when $\theta = 0$.

comment. Especially when you solve homework problems, it is not advisable to quote the results as I did. My intention was to give you more clear explanations which can be found in Landau's book.

(see comment of problem 11, for sign convention of $\frac{d\sigma}{d\Omega}$.)

9. (a) These two problems can be solved easily using elementary method. First assume that moon is a point particle with the same mass. Then the distance of closest approach can be calculated from conservations laws. From the angular momentum conservation, we have (b: impact parameter, v_0: speed at infinity)

$$L = m v_0 b = m r_{min} v(r_{min})$$

where we used the fact that there is no radial motion at the moment when the closest approach is achieved. Energy conservation gives,

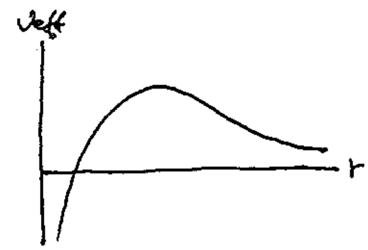
$$\frac{1}{2} m v_0^2 = \frac{\sigma G M m}{r_{min}} + \frac{1}{2} m v^2(r_{min})$$

where $\sigma = 1$ represents repulsive interaction and $\sigma = -1$ represents attractive interaction. Combining these two equation gives,

$$b^2 = r_{min}^2 - \sigma\, v_{L,esc}^2 / v_0^2\ r_L \cdot r_{min}$$

where we defind escape velocity $\frac{1}{2} v_{L,esc}^2 = \frac{GM}{r_L}$. Clearly, the maximum impact parameter is obtained when we set $r_{min} = r_L$, the radius of the moon. Thus, the total cross-section for collision is, (Of course, for smaller impact parameter, the particle will hit the moon.)

$$\sigma = \pi b_{max}^2 = \pi r_L^2 \left(1 - \sigma \frac{v_{L,esc}^2}{v_0^2}\right)$$



(b) Using $L = m v_0 b$, the effective potential can be written as,

$$V_{eff} = -\frac{C}{r^4} + \frac{L^2}{2mr^2} = -\frac{C}{r^4} + \frac{b^2}{r^2} \underbrace{E_0}_{\text{initial energy}} \quad \left(E_0 = \frac{1}{2} m v_0^2\right)$$

where the graph of which is shown right. If particle's initial energy E_0 is larger than the maximum height of the potential then the particle will be captured. The maximum height of potential is obtained by requiring,

$$\frac{d}{dr} V_{eff} = 0 = 4\frac{C}{r^5} - 2\frac{b^2}{r^3} E_0 \quad \Rightarrow \quad r^2 = \frac{2C}{b^2 E_0}, \quad \therefore V_{eff}\Big|_{max} = -\frac{E_0^2 b^4}{4C^2} C + b^2 E_0 \frac{b^2 E_0}{2C} = \frac{b^4 E_0^2}{4C}$$

Thus,

$$V_{eff}\big|_{max} < E_0$$

gives maximum impact parameter $b_{max} = \left(\frac{4C}{E_0}\right)^{1/4}$. For a smaller impact paramer, the particle will be captured. Thus, the total cross section is,

$$\sigma = \pi b_{max}^2 = \pi \left(\frac{4C}{E_0}\right)^{1/2}$$

11. From the equation of motion, we have, (radial component)

$$m\ddot{r} = \frac{L}{mr^3} - \frac{\partial}{\partial r}V(r)$$

from which, we get,

$$\frac{1}{2}m\dot{r}^2 - \frac{1}{2}mv_0^2 = -\frac{L^2}{2mr^2} - U(r) \Rightarrow \dot{r} = \sqrt{v_0^2 - \frac{L^2}{m^2r^2} - \frac{2}{m}V(r)}$$

Using the angular momentum conservation,

$$L = mv_0 b = mr^2\dot{\phi} \quad \Rightarrow \quad d\phi = \frac{v_0 b}{r^2}\frac{dt}{dr}dr$$

we get,

$$\phi = \int_{r_{min}}^{\infty} \frac{b}{r^2}\frac{dr}{\sqrt{1 - b^2/r^2 - 2/mv_0^2\, V(r)}}$$

In this case, we set $V(r) = \frac{\lambda^2}{r^2}$. Then,

$$\phi = \int_{r_{min}}^{\infty} \frac{b}{r^2}\frac{1}{\sqrt{(1 - (b^2 + 2\lambda^2/mv_0^2)\,1/r^2}}\,dr \qquad (x = r/\sqrt{b^2 + 2\lambda^2/mv_0^2}\;)$$

$$= \frac{b}{\sqrt{b^2 + 2\lambda^2/mv_0^2}}\int_1^{\infty}\frac{1}{x^2}\frac{dx}{\sqrt{1 - \frac{1}{x^2}}} \qquad (x \to \tfrac{1}{x})$$

$$= \frac{b}{\sqrt{b^2 + 2\lambda^2/mv_0^2}}\int_0^1 \frac{dt}{\sqrt{1 - t^2}} \Longrightarrow = \int_0^{\pi/2}\frac{\cos t\, dt}{\cos t} = \frac{\pi}{2}.$$

Consequently, we have,

$$\theta = \pi - 2\phi = \pi\left(1 - b/\sqrt{b^2 + 2\lambda^2/mv_0^2}\,\right) \quad \Rightarrow \quad b^2 = \frac{2\lambda^2}{mv_0^2}\frac{(\pi - \theta)^2}{\theta(2\pi - \theta)}$$

Thus, the differential cross section is given by,

$$\frac{d\sigma}{d\Omega} = \left|\frac{2\pi b\, db}{2\pi \sin\theta\, d\theta}\right| = \left|\frac{1}{2\sin\theta}\frac{d}{d\theta}b^2\right| = \frac{2\lambda^2}{mv_0^2}\frac{\pi^2(\pi - \theta)}{\sin\theta\cdot\theta^2(2\pi - \theta)^2}$$

The corresponding graph is,

Notice that it is a monotonic decreasing function of θ and diverges at $\theta \to 0$.

comment. Although it is not clear from the notes, we define the differential cross section as a positive number such that it can be proportional to the number of scattered particles with, of course, positive proportionality coefficient. Thus, all you have to do is to make the final answer positive by putting arbitrary sign, brutally.