

PH 205 PROBLEM SET II

DUE: TUES OCT. 4, 1988.

MAXIMUM RECORDED SCORE = 80 POINTS

① FALLING CHIMNEY IF A CHIMNEY IS UNDERMINED ON ONE SIDE

SO THAT IT FALLS, ROTATING ABOUT THE BASE,

IT USUALLY SNAPS BEFORE HITTING THE GROUND.

WE CAN ESTIMATE THE MOST LIKELY POSITION

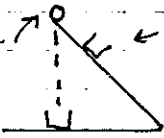
OF THE BREAK BY AN EXTENSION OF THE PRINCIPLES

OF STATICS TO A DYNAMICAL SITUATION. THIS IS THE

SPIRIT OF D'ALEMBERT.

YOU MIGHT WISH TO CONVINCE YOURSELF THAT THE ABOVE PICTURE

SHOWS THE BEHAVIOR AFTER A BREAK BY PERFORMING A HOME EXPERIMENT.

BALL  A BALL RESTS ON THE END OF A STICK HELD INITIALLY

AT SOME ANGLE ($\sim 45^\circ$) TO THE HORIZONTAL. LET THE SYSTEM

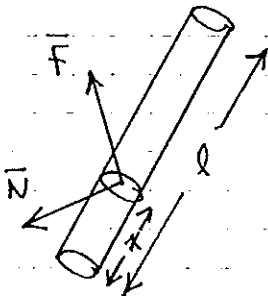
DROP. THE STICK WILL APPEAR TO FALL FASTER THAN THE

BALL. IF A CUP WAS PLACED ON THE STICK AT THE RIGHT PLACE,

THE BALL CAN BE CAUGHT BY THE CUP WHEN THE STICK HITS THE GROUND.

HENCE THE END OF THE STICK FALLS AT GREATER THAN $1g$, AND IF

THE STICK IS WEAK, IT WILL SNAP IN THE SENSE SHOWN AT TOP.



CONSIDER A SLICE THRU THE CHIMNEY A DISTANCE x

ABOVE THE BASE. THE INTERNAL FORCES ACROSS THIS

SLICE CAN BE COMBINED INTO A NET FORCE \vec{F}

APPLIED AT THE CENTER OF THE SLICE, AND A TORQUE

COUPLE \vec{N} ACTING ABOUT THE CENTER OF THE SLICE. — A

PRINCIPLE OF STATICS. "CLEARLY" \vec{N} IS \perp TO THE

PLANE OF THE FALLING CHIMNEY.

THE CHIMNEY MIGHT BREAK AT x FOR ANY OF 3 REASONS:

1. THE TENSION ALONG THE CHIMNEY, ($F_{||}$) IS TOO GREAT FOR THE MORTAR BETWEEN THE BRICKS TO SUSTAIN;
2. THE SHEAR ACROSS THE SLICE (F_{\perp}) IS TOO GREAT;
3. THE TORQUE N IS TOO GREAT AND THE CHIMNEY BENDS AND SNAPS.

EMPIRICALLY WE WILL CONCLUDE THAT THE CHIMNEY BREAKS DUE TO CAUSE 3 — THE TORQUE EFFECT.

CALCULATE $N(x)$ AND $F_{\perp}(x)$ AND FIND x SUCH THAT THESE ARE MAXIMUM.

YOU MAY WISH TO NOTE SOME TRICKS:

- a) AT $x=l$ = TOP OF CHIMNEY \vec{F} AND \vec{N} VANISH;
- b) AT THE BOTTOM, $x=0$, WE SUPPOSE \vec{N} VANISHES, TWO \vec{F} IS NON-ZERO;
- c) CONSIDER THE ANGULAR MOTION OF THE PIECE OF THE CHIMNEY BETWEEN 0 AND x , AS WELL AS THE PIECE $[l, x]$...

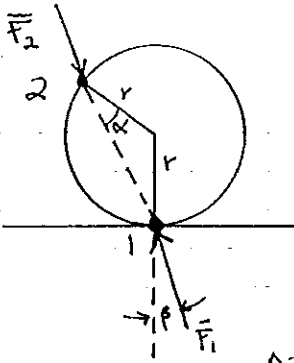
SHOW $N(x) = \frac{Mg \sin \theta \cdot x(l-x)^2}{4l^2}$ AND $F_L = \frac{Mg \sin \theta (l-x)(l-3x)}{4l^2}$

AND HENCE THE CHIMNEY IS MOST LIKELY TO SNAP AT $x = l/3$ IF TORQUE MATTERS, BUT AT $x = 2l/3$ IF SHEAR MATTERS.

SEE ALSO AMATEUR SCIENTIST, SCIENTIFIC AMERICAN, FEB 1979.

A TOWER OF BOTTLE CAPS SEEN ON T.V. BROKE AT $x \approx 2/3 l$. NO GLUE \Rightarrow SHEAR EFFECTS DOMINATE.

② THE NAPKIN RING AN EVERYDAY PROBLEM REQUIRING A VERY CAREFUL ANALYSIS.



A NAPKIN RING IN THE FORM OF A CYLINDRICAL SHELL OF MASS M , RADIUS r RESTS ON A HORIZONTAL TABLE. YOU PRESS ON IT WITH FORCE \vec{F}_2 AT SOME POINT.

WHAT HAPPENS?

WE OUTLINE A POSSIBLE FORMAT FOR THE DISCUSSION.

LET μ_1 AND μ_2 BE THE COEFFICIENTS OF FRICTION AT POINTS 1 AND 2.

FIRST, WHAT IS THE DIRECTION OF \vec{F}_2 FOR STATIC EQUILIBRIUM?

HOW BIG MUST μ_2 BE FOR STATIC EQUILIBRIUM TO HOLD?

IF $\mu_2 < \mu_{2 \text{ MIN}}$, THE RING WILL SLIP AT POINT 2 NO MATTER

HOW SMALL \vec{F}_2 IS. IF WE START WITH F_2 SMALL AND INCREASE IT, THE RING WILL BEGIN TO ROLL WITHOUT SLIPPING AT POINT 1 ($\mu_2 < \mu_{2 \text{ MIN}}$)

INSTEAD, SUPPOSE $\mu_2 > \mu_{2 \text{ MIN}}$. THEN THE RING WILL NEVER SLIP AT POINT 2. WHAT ABOUT SLIPPING AT POINT 1?

LET $\beta =$ ANGLE OF \vec{F}_1 TO THE VERTICAL. SHOW

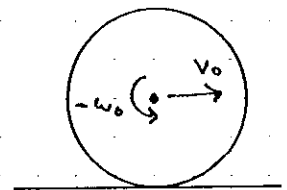
$$\cot \beta = \cot \alpha + \frac{Mg}{F_2 \sin \alpha}$$

WHAT IS $\mu_{1 \text{ MIN}}$ SUCH THAT THERE WILL NEVER BE ANY SLIPPING AT POINT 1?

THE RING JUST SITS THERE IF $\mu_1 > \mu_{1 \text{ MIN}}$ AND $\mu_2 > \mu_{2 \text{ MIN}}$.

BUT IF $\mu_1 < \mu_{1 \text{ MIN}}$ WHILE $\mu_2 > \mu_{2 \text{ MIN}}$, THE RING WILL SLIP AT POINT 1 AND SHOOT OUT WITH BACKSPIN!

SUPPOSE IT SHOOTS OUT WITH VELOCITY v_0 AND ANGULAR VELOCITY $-\omega_0$. LET $\mu =$ COEFFICIENT OF SLIDING FRICTION.

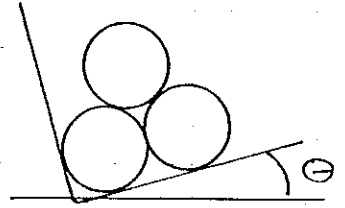


HOW LONG DOES IT TAKE UNTIL THE RING ROLLS WITHOUT SLIPPING?

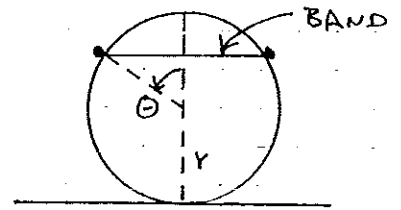
FOR WHAT v_0 & ω_0 WILL THE RING ROLL BACK TO YOU? ANS: $\omega_0 > \frac{v_0}{r}$

③ CONSIDER AN ARBITRARY MOTION OF A RIGID BODY (CHASLES' THEOREM). SHOW THAT FOR ANY PAIR OF PARTICLES WITHIN THE BODY THAT $\delta W_{ij} = \vec{f}_{ij} \cdot \delta \vec{r}_i + \vec{f}_{ji} \cdot \delta \vec{r}_j = 0$
 HENCE $\delta W_{INTERNAL} = \sum_{i,j} \delta W_{ij} = 0$ AS CLAIMED IN THE NOTES.

④ THREE CYLINDRICAL LOGS ARE RESTING IN THE TILTED BED OF A LUMBER TRUCK. WHAT IS THE MINIMUM ANGLE θ SUCH THAT ALL 3 LOGS REMAIN TOUCHING? ASSUME NO FRICTION.



⑤ A RUBBER BAND OF MASS M , REST LENGTH l_0 , SPRING CONSTANT K RESTS ON A BILLIARD BALL OF RADIUS r . ASSUMING THE BAND LIES IN A HORIZONTAL PLANE, WHAT IS THE POLAR ANGLE θ TO THE BAND?

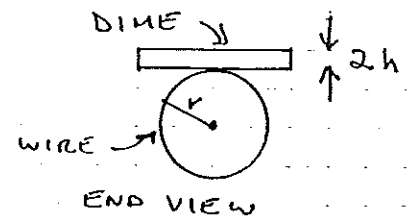


⑥ AN EQUILIBRIUM IS SAID TO BE STABLE IF $\delta W < 0$ IN ANY FINITE DISPLACEMENT FROM EQUILIBRIUM. (IF $\delta W < 0$ THE FINAL KINETIC ENERGY WOULD BE < 0)

a) A DIME OF THICKNESS $2h$ IS BALANCED ON A COAT HANGER WITH WIRE RADIUS r . WHAT IS THE CONDITION FOR STABILITY?



TRY IT! IF DONE WELL, YOU CAN TWIRL THE COAT HANGER (ABOUT A HORIZONTAL AXIS) AND THE DIME WILL STAY ON.

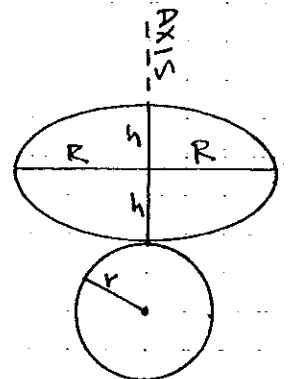


b) AN ELLIPSOID OF REVOLUTION HAS HEIGHT $2h$ ALONG ITS AXIS, AND RADIUS R . IT IS BALANCED ON A SPHERE OF RADIUS r .

SHOW THE EQUILIBRIUM IS STABLE IF

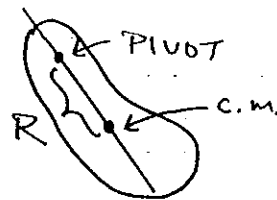
$$\frac{1}{h} > \frac{1}{r} + \frac{1}{p}$$

WHERE p = RADIUS OF CURVATURE OF THE ELLIPSOID AT THE POINT OF CONTACT. WHAT IS $p(h, R)$?



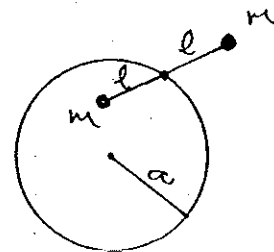
IF THE EQUILIBRIUM IS STABLE, THESE ARE CALLED ROCKING STONES; IF NOT, ROLLING STONES.

- 7) A PHYSICAL PENDULUM OF MASS M HAS MOMENT OF INERTIA I ABOUT A PIVOT POINT. THE C.M. IS DISTANCE R FROM THE PIVOT. WHERE IS THE CENTER OF OSCILLATION?

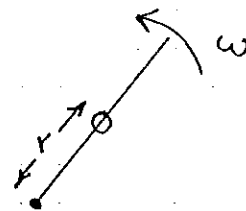


NOW SUPPOSE THE PENDULUM IS HUNG FROM A PIVOT POINT AT THE CENTER OF OSCILLATION FOUND ABOVE. SHOW THAT THE OLD PIVOT POINT IS THE NEW CENTER OF OSCILLATION, AND HENCE THE PERIOD OF OSCILLATION IS UNCHANGED.

- 8) TWO POINTS OF MASS M ARE JOINED BY A MASSLESS ROD OF LENGTH 2ℓ ; THE CENTER OF WHICH IS CONSTRAINED TO MOVE IN A CIRCLE OF RADIUS a . THE MOTION IS ENTIRELY IN THE PLANE OF THE CIRCLE. THERE ARE NO EXTERNAL FORCES, NO GRAVITY. DESCRIBE THE POSSIBLE MOTION USING THE TECHNIQUE OF SEPARATION INTO C.M. MOTION AND MOTION RELATIVE TO THE C.M. VERIFY YOUR ANSWER VIA LAGRANGE'S METHOD. IDENTIFY THE GENERALISED MOMENTA.



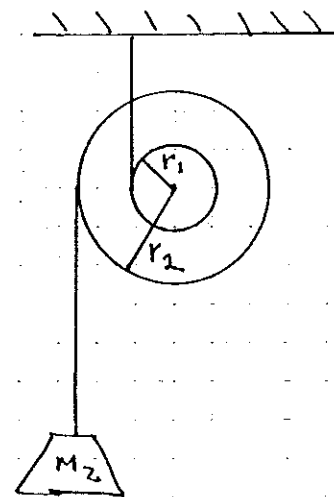
- 9) A MASS M SLIDES WITHOUT FRICTION ON A STRAIGHT WIRE WHICH IS CONSTRAINED TO ROTATE IN A PLANE WITH CONSTANT ANGULAR VELOCITY ω . THERE ARE NO EXTERNAL FORCES. USE LAGRANGE'S METHOD TO FIND $\psi(t)$ IF $\psi = \psi_0$ AND $\dot{\psi} = \dot{\psi}_0$ AT $t = 0$.



- 10) A DOUBLE CYLINDER OF MASS M_1 , MOMENT OF INERTIA I HANGS FROM A STRING WRAPPED AROUND THE CYLINDER OF RADIUS r_1 . A MASS M_2 IS SUSPENDED FROM A STRING WRAPPED AROUND THE CYLINDER OF RADIUS r_2 .

a) WHAT IS THE GENERAL CONDITION OF STATIC EQUILIBRIUM? DISCUSS THE SPECIAL CASE OF $r_1 = r_2$

b) USE ELEMENTARY METHODS TO FIND THE ACCELERATION OF THE C.M. OF THE CYLINDER, IF THE SYSTEM IS NOT IN EQUILIBRIUM.



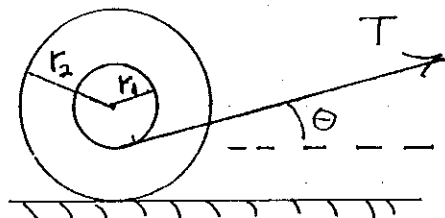
(10) c) USE LAGRANGE'S METHOD TO FIND THE ACCELERATION.

NOTE THAT THE ELEMENTARY METHOD REQUIRES ONE TO DERIVE EQUATIONS FOR THE TENSIONS IN THE STRINGS, WHILE LAGRANGE'S METHOD AVOIDS THIS; THE TENSIONS ARE CONSTRAINT FORCES.

(11) THE DOUBLE CYLINDER OF THE PREVIOUS PROBLEM RESTS ON A HORIZONTAL

SURFACE WITH FRICTION. A STRING WRAPPED AROUND THE CYLINDER OF RADIUS r_1 MAKES

ANGLE θ TO THE HORIZONTAL, AND IS PULLED SO AS TO MAINTAIN A CONSTANT TENSION T .



a) WHAT IS THE CONDITION OF STATIC EQUILIBRIUM? WHAT IS THE MINIMUM COEFFICIENT OF FRICTION REQUIRED?

b) SUPPOSE THE SYSTEM IS NOT IN EQUILIBRIUM, BUT THE FRICTION IS GREAT ENOUGH THAT THE CYLINDER ALWAYS ROLLS WITHOUT SLIPPING. FIND THE ACCELERATION OF THE C.M. BY ELEMENTARY METHODS. FOR WHAT ANGLE θ WOULD THE MOTION BE THE SAME EVEN IF FRICTION VANISHED?

c) FIND THE ACCELERATION BY LAGRANGE'S METHOD. USE THE LAGRANGE EQUATIONS WITH THE GENERALISED FORCE APPEARING EXPLICITLY. BE VERY CAREFUL IN CONSTRUCTING THE GENERALISED FORCE CORRESPONDING TO THE TENSION.

THIS PROBLEM IS CALLED 'GRANDMA & THE CAT.' GRANDMA DROPS HER SPOOL OF THREAD AND THE CAT PAWS IT OUT OF REACH. CAN GRANDMA RETRIEVE THE SPOOL SIMPLY BY PULLING ON THE THREAD? DOES SHE NEED LAGRANGE TO FIGURE IT OUT?