

ACCELERATED COORDINATE SYSTEMS

[THIS MATERIAL IS WELL TREATED IN BÉ0 SECS 6-1, 2, 3, 4, L&L SEC 39]

UP TO NOW WE HAVE BASED OUR ELEMENTARY DESCRIPTIONS OF MOTION ON INERTIAL FRAMES - ONES IN WHICH NEWTON'S 1ST LAW HOLDS.

OFTEN IN USING LAGRANGE'S METHOD IN PROBLEMS WITH MOVING CONSTRAINTS, WE HAVE INTRODUCED TIME-DEPENDENT GENERALISED COORDINATES, AND HAVE SEEN 'EXTRA' TERMS APPEAR IN THE EQUATIONS OF MOTION. WE HAVE SOMETIMES CALLED THESE EXTRA TERMS 'FICTITIOUS FORCES' OR 'COORDINATE FORCES.' WE NOW GO BACK TO $\vec{F} = m\vec{a}$ TO SEE HOW THESE TERMS ARISE WHEN WE USE ACCELERATED COORDINATE SYSTEMS.

THERE ARE 2 KINDS OF ACCELERATED COORDINATE SYSTEMS:

- THE ORIGIN OF THE COORDINATE SYSTEM IS ACCELERATED (WITH RESPECT TO SOME INERTIAL FRAME), BUT THE AXES REMAIN PARALLEL TO THOSE OF THE INERTIAL FRAME.
- THE ORIGIN REMAINS FIXED, BUT THE AXES ARE ROTATING ABOUT SOME INSTANTANEOUS AXIS AT ANGULAR VELOCITY $\vec{\Omega}(t)$ (WITH RESPECT TO AN INERTIAL FRAME).

OF COURSE, A COMBINATION OF a) & b) IS THE GENERAL CASE (BY CHASLES' THEOREM).

TRANSLATING ORIGIN

LET $\vec{R}(t)$ = VECTOR TO ORIGIN OF THE ACCELERATED FRAME W.R.T. OUR INERTIAL FRAME.

$\vec{Y}(t)$ = VECTOR TO A POINT MASS m , AS OBSERVED IN THE ACCELERATED FRAME.

THEN $\vec{Y}_I = \vec{R} + \vec{Y}$ = VECTOR TO MASS m IN OUR INERTIAL FRAME.

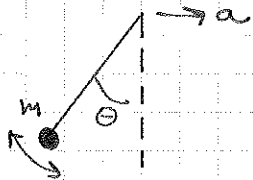
NOW $m \ddot{\vec{Y}}_I = \vec{F}$ HOLDS IN THE INERTIAL FRAME.

$\therefore m \ddot{\vec{Y}} = \vec{F} - m \ddot{\vec{R}} = \vec{F}_{\text{EFF}}$ IN THE ACCELERATED FRAME

IF WE WANT TO KEEP THE FORM OF NEWTON'S LAW AS $\vec{F}_{\text{EFF}} = m \ddot{\vec{Y}}$ EVEN IN THE ACCELERATED FRAME, THEN WE MUST INTRODUCE THE 'FICTITIOUS FORCE' $-m \ddot{\vec{R}}$.

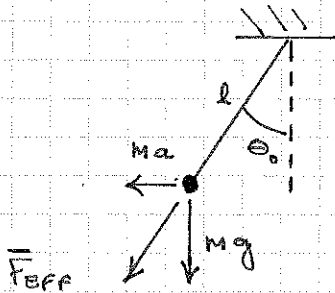
NOTE THAT THIS PROCEDURE HAS MUCH THE FLAVOR OF D'ALEMBERT'S METHOD.

EXAMPLE. THE POINT OF SUPPORT OF A PENDULUM MOVES HORIZONTALLY WITH CONSTANT ACCELERATION a . WHAT IS THE OSCILLATION OF THE PENDULUM?



DIRECT USE OF $\vec{F} = m\vec{a}$ IS NOT EASY. WE NEED THE FORCE ON THE BOB, BUT KNOW ONLY THE MOTION OF THE SUPPORT. OF COURSE, LAGRANGE'S METHOD WOULD HAVE NO DIFFICULTY WITH THIS PROBLEM!

HOWEVER, LET'S USE AN ACCELERATED COORDINATE SYSTEM WITH ITS ORIGIN AT THE SUPPORT. IN THIS FRAME THE 'FORCES' ARE AS SHOWN.



$$\tan \theta_0 = a/g$$

$$|\vec{F}_{eff}| = m \sqrt{a^2 + g^2} = m g_{eff}$$

'CLEARLY' THE PENDULUM OSCILLATES ABOUT THE EQUILIBRIUM ANGLE θ_0 WITH FREQUENCY

$$\omega = \sqrt{\frac{g_{eff}}{l}}$$

THIS WOULD APPEAR TO GIVE AN 'ACCELEROMETER' - A MEANS OF DETERMINING WHETHER YOU ARE IN AN ACCELERATED FRAME. BUT EINSTEIN NOTED THAT YOU CANNOT (LOCALLY) DISTINGUISH THE CASE OF AN ACCELERATED FRAME FROM THAT OF A UNIFORM GRAVITATIONAL FIELD OF STRENGTH g_{eff} . THIS IS THE BASIC PRINCIPLE OF GENERAL RELATIVITY.

OF COURSE, IF YOU ARE NEAR A BLACK HOLE, THE GRAVITATIONAL FIELD IS SO NON-UNIFORM THAT YOUR TOES FEEL MORE TUG THAN YOUR HEAD. THIS DIFFERENTIAL STRETCHING (TIDAL EFFECT) CANNOT BE DUPLICATED BY AN ACCELERATED FRAME. THUS WE SAY THAT THE 'PRINCIPLE OF EQUIVALENCE' IS ONLY A LOCAL RATHER THAN A GLOBAL PRINCIPLE.

ROTATING AXES (COMMON ORIGIN) GOAL: $\vec{F} = m \frac{d^2 \vec{r}}{dt^2} \Big|_{inertial} = m \frac{d^2 \vec{r}}{dt^2} \Big|_{accel} + \dots$

LET $\vec{\omega}(t)$ = ANGULAR VELOCITY OF THE (AXES OF) THE ACCELERATED FRAME W.R.T. THE INERTIAL FRAME.

WE ALREADY KNOW THAT A VECTOR \vec{r} WHICH IS CONSTANT IN THE ROTATING FRAME OBEYS

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r} \quad \text{IN THE INERTIAL FRAME}$$

- IT JUST ROTATES ABOUT THE AXIS $\vec{\omega}$!

IF THE VECTOR IS MOVING IN THE ROTATING FRAME WITH VELOCITY $\frac{\delta \vec{r}}{\delta t}$ IN THAT FRAME, WE CLAIM

$$\left. \frac{d\vec{r}}{dt} \right|_{\text{INERTIAL}} = \left. \frac{\delta \vec{r}}{\delta t} \right|_{\text{ROTATING}} + \vec{\Omega} \times \vec{r}$$

THIS IS JUST CHASLES' THEOREM IN A NEW GUISE.

ONE WAY TO DEMONSTRATE THE RESULT IS TO WRITE

$$\vec{r} = r_1 \hat{e}_1 + r_2 \hat{e}_2 + r_3 \hat{e}_3 \quad \text{WHERE } \hat{e}_1, \hat{e}_2, \hat{e}_3 \text{ ARE}$$

THE UNIT VECTORS OF THE COORD. AXES IN THE ROTATING FRAME

$$\begin{aligned} \text{THEN } \frac{d\vec{r}}{dt} &= \sum_i \frac{dr_i}{dt} \hat{e}_i + \sum_i r_i \frac{d\hat{e}_i}{dt} \\ &= \frac{\delta \vec{r}}{\delta t} + \sum_i r_i (\vec{\Omega} \times \hat{e}_i) = \frac{\delta \vec{r}}{\delta t} + \vec{\Omega} \times \vec{r} \end{aligned}$$

WE MAKE A DIGRESSION ABOUT COMPOSITION OF ROTATIONS

SUPPOSE THE MOTION OF \vec{r} IN THE ROTATING FRAME IS ALSO A ROTATION. $\frac{\delta \vec{r}}{\delta t} = \vec{\omega} \times \vec{r}$ AS VIEWED IN THAT FRAME,

IN GENERAL, $\vec{\omega}$ IS NOT PARALLEL TO $\vec{\Omega}$.

THEN IN OUR INERTIAL FRAME,

$$\frac{d\vec{r}}{dt} = \frac{\delta \vec{r}}{\delta t} + \vec{\Omega} \times \vec{r} = \vec{\omega} \times \vec{r} + \vec{\Omega} \times \vec{r} = (\vec{\omega} + \vec{\Omega}) \times \vec{r}$$

WHICH IS A ROTATION ABOUT THE VECTOR SUM $\vec{\omega} + \vec{\Omega}$.

THUS, THE SUM OF TWO ANGULAR VELOCITIES IS A 3RD ANGULAR VELOCITY.

EXAMPLE THE EARTH ROTATES ABOUT ITS AXIS WITH $\omega \Rightarrow T = \frac{2\pi}{\omega} = 24$ HOURS

AS MEASURED WITH RESPECT TO THE SUN. SINCE THE EARTH IS ROTATING ABOUT THE SUN WITH Ω GIVEN BY $T = \frac{2\pi}{\Omega} = 365$ DAYS, WE SEE

THAT THE 24 HOUR DAY IS MEASURED IN AN ACCELERATED FRAME.

IN AN INERTIAL FRAME (= FRAME OF THE 'FIXED STARS') THE ROTATION FREQUENCY IS $\omega + \Omega$ (THE SENSE OF THE 2 ROTATIONS IS

THE SAME). $\therefore T_{\text{INERTIAL}} = \frac{2\pi}{\omega + \Omega} = \frac{2\pi}{\omega} \frac{\omega}{\omega + \Omega} = 24 \text{ HOURS} \cdot \frac{365}{366}$, SINCE

$\omega = 365\Omega$. THUS $T_{\text{INERTIAL}} < 24$ HOURS. IT IS CALLED THE SIDEREAL DAY.

NOW LET'S CONSIDER THE ADDITION OF ANGULAR DISPLACEMENTS, NOT JUST ANGULAR VELOCITIES.

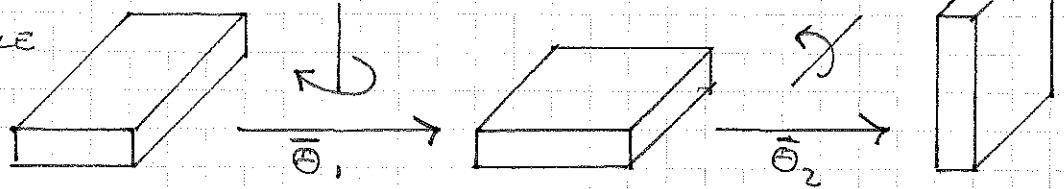
IF $\bar{\omega} = \bar{\omega}_1 + \bar{\omega}_2$ AND $\bar{\omega} = d\bar{\theta}/dt$

THEN $d\bar{\theta} = d\bar{\theta}_1 + d\bar{\theta}_2$ HAS A VECTOR MEANING

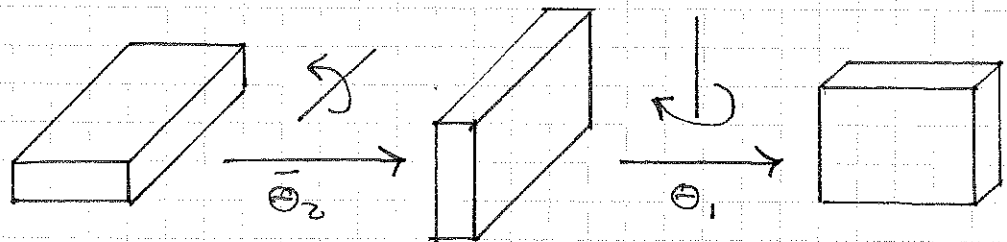
SUPPOSE WE INTEGRATE OVER A FINITE TIME. CAN WE WRITE

$\bar{\theta} = \bar{\theta}_1 + \bar{\theta}_2$? No!

FOR EXAMPLE



WHILE



THAT IS, $\bar{\theta}_1 + \bar{\theta}_2 \neq \bar{\theta}_2 + \bar{\theta}_1$

SO WE CANNOT WRITE $\bar{\theta}_1 + \bar{\theta}_2$ WITH A UNIQUE VECTOR MEANING. THE MATHEMATICAL MESSAGE IS THAT FINITE ANGULAR DISPLACEMENTS CANNOT BE DESCRIBED BY VECTORS CONSISTENTLY - BUT INFINITESIMAL ANGULAR DISPLACEMENTS CAN!

WE RETURN TO OUR RELATION $\frac{d\bar{r}}{dt} = \frac{\delta\bar{r}}{\delta t} + \bar{\omega} \times \bar{r}$

OUR GOAL IS TO TRANSFORM $\bar{F} = m \frac{d^2\bar{r}}{dt^2}$ INTO $m \frac{\delta^2\bar{r}}{\delta t^2} = \bar{F}_{eff}$.

$$\begin{aligned} \frac{d^2\bar{r}}{dt^2} &= \frac{\delta}{\delta t} \left(\frac{d\bar{r}}{dt} \right) + \bar{\omega} \times \frac{d\bar{r}}{dt} = \frac{\delta}{\delta t} \left(\frac{\delta\bar{r}}{\delta t} + \bar{\omega} \times \bar{r} \right) + \bar{\omega} \times \frac{\delta\bar{r}}{\delta t} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) \\ &= \frac{\delta^2\bar{r}}{\delta t^2} + \frac{\delta\bar{\omega}}{\delta t} \times \bar{r} + 2\bar{\omega} \times \frac{\delta\bar{r}}{\delta t} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) \end{aligned}$$

WE WILL WRITE $\frac{\delta\bar{r}}{\delta t} = \bar{v}$ = VELOCITY IN THE ROTATING FRAME.

ALSO, $\frac{d\bar{\omega}}{dt} = \frac{\delta\bar{\omega}}{\delta t} + \bar{\omega} \times \bar{\omega} = \frac{\delta\bar{\omega}}{\delta t} \equiv \dot{\bar{\omega}}$ UNIQUELY

EXERCISE: VERIFY OUR RESULT FOR $\frac{d^2\bar{r}}{dt^2}$ BY DIFFERENTIATING $\frac{d}{dt} \left(\frac{d\bar{r}}{dt} \right) = \frac{d}{dt} \left[\frac{\delta\bar{r}}{\delta t} + \bar{\omega} \times \bar{r} \right]$

HENCE $m \frac{d^2 \bar{r}}{dt^2} = \bar{F}$ BECOMES

$$m \frac{d^2 \bar{r}}{dt^2} = \bar{F} - m \left\{ 2 \bar{\omega} \times \bar{v} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) + \dot{\bar{\omega}} \times \bar{r} \right\}$$

BEFORE DISCUSSING THIS WE CONSIDER THE GENERAL CASE OF ROTATING AXES AND TRANSLATING ORIGIN.

$$\bar{r}_I = \bar{R} + \bar{r}$$

\bar{R} = VECTOR TO ORIGIN OF THE ROTATING COORD SYSTEM

\bar{r} = POSITION IN THE ROTATING COORD SYSTEM

$$\text{so } \bar{F} = m \frac{d^2 \bar{r}_I}{dt^2} = m \frac{d^2 \bar{R}}{dt^2} + m \frac{d^2 \bar{r}}{dt^2}$$

$$\text{AND } m \frac{d^2 \bar{r}}{dt^2} = \bar{F} - m \left\{ 2 \bar{\omega} \times \bar{v} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) + \dot{\bar{\omega}} \times \bar{r} + \frac{d^2 \bar{R}}{dt^2} \right\}$$

WE HAVE 4 FICTITIOUS FORCES IN ALL:

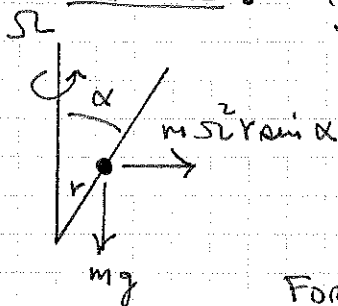
- 1) $-2m \bar{\omega} \times \bar{v}$ = CORIOLIS FORCE
- 2) $-m \bar{\omega} \times (\bar{\omega} \times \bar{r})$ = CENTRIFUGAL FORCE
- 3) $-m \dot{\bar{\omega}} \times \bar{r}$ = AZIMUTHAL FORCE
- 4) $-m \frac{d^2 \bar{R}}{dt^2}$ = TRANSLATIONAL FORCE

IN MOST APPLICATIONS, $\dot{\bar{\omega}}$ IS SMALL, AND WE IGNORE IT HERE.

IF $\bar{\omega}$ IS BIG THE CENTRIFUGAL FORCE IS PROMINENT. BUT IF $\bar{\omega}$ IS SMALL, AS IN THE ROTATION OF THE EARTH, THE CORIOLIS TERM IS MORE IMPORTANT.

CENTRIFUGAL FORCE : $-m \bar{\omega} \times (\bar{\omega} \times \bar{r})$

EXAMPLE : A BEAD SLIDES ON A WIRE MAKING ANGLE α TO THE VERTICAL. THE WIRE ROTATES WITH ω ABOUT THE VERTICAL.

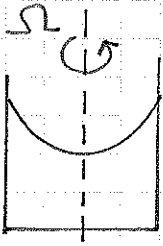


IN THE ROTATING FRAME THE CENTRIFUGAL FORCE IS $-m \bar{\omega} \times (\bar{\omega} \times \bar{r}) = m \omega^2 r \sin^2 \alpha$ OUTWARD FROM THE AXIS.

FOR EQUILIBRIUM, THE 'FORCES' ALONG THE WIRE CANCEL:
 $mg \cos \alpha = m \omega^2 r \sin^2 \alpha \Rightarrow r = \frac{g \cos \alpha}{\omega^2 \sin^2 \alpha}$ AT EQUILIBRIUM.

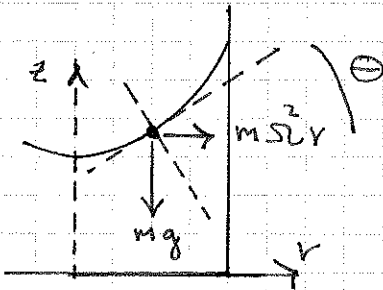
EXAMPLE

A BUCKET OF WATER ROTATES WITH ANGULAR VELOCITY Ω ABOUT THE VERTICAL AXIS OF THE BUCKET.



WHAT IS THE SHAPE OF THE SURFACE OF THE WATER?

THE SURFACE WILL BE \perp TO THE FORCE ON WATER AT THE SURFACE.



$$\frac{dz}{dr} = \tan \theta = \frac{\Omega^2 r}{g}$$

$$\Rightarrow z = z_0 + \frac{\Omega^2 r^2}{2g} \quad \text{PARABOLIC}$$

ANOTHER METHOD IS TO NOTE THAT THE SURFACE WILL BE \perp TO THE FORCE IF THE SURFACE IS AN EQUIPOTENTIAL.

[SINCE $\vec{F} = -\frac{\partial V}{\partial \vec{r}}$ IS \perp TO SURFACES $V = \text{CONSTANT}$.]

OF COURSE, IN THE ROTATING FRAME THE POTENTIAL MUST INCLUDE A TERM TO DESCRIBE THE CENTRIFUGAL FORCE.

$$\vec{F}_{\text{EFF}} = -mg \hat{z} + m\Omega^2 r \hat{r} = -\frac{\partial V}{\partial z} \hat{z} - \frac{\partial V}{\partial r} \hat{r}$$

$$\Rightarrow V = mgz - \frac{m\Omega^2 r^2}{2}$$

THE TERM $-\frac{m\Omega^2 r^2}{2}$ IS THE SO-CALLED CENTRIFUGAL POTENTIAL

$$V = \text{CONSTANT} \Rightarrow z = \frac{\Omega^2 r^2}{2g} + z_0$$

THE DISTORTED SURFACE OF THE WATER SUGGESTED TO NEWTON A METHOD OF DETECTING THAT YOU ARE IN A ROTATING FRAME.

(I.E., ABSOLUTE ROTATIONS ARE DETECTABLE WITHOUT REFERENCE TO ANY DISTANT INERTIAL FRAME.

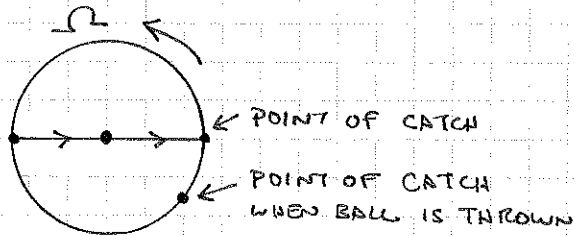
[IS THIS IN DISAGREEMENT WITH EINSTEIN'S PRINCIPLE OF EQUIVALENCE?]

CORIOUS FORCE : $-2m(\vec{\Omega} \times \vec{v})$

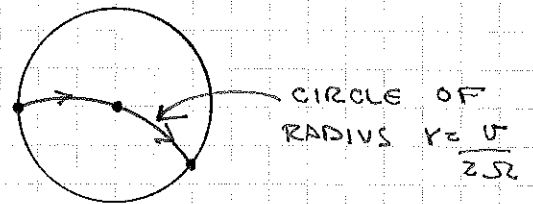
WE ARE LESS FAMILIAR WITH THIS TERM, WHICH HOWEVER PLAYS AN IMPORTANT ROLE IN LARGE-SCALE PHENOMENA ON THE EARTH.

EXAMPLE: A BALL IS THROWN WITH VELOCITY v FROM A POINT ON A SLOWLY ROTATING MERRY-GO-ROUND.

a) BALL THROWN INWARDS SO AS TO PASS THRU THE AXIS OF ROTATION. (WE NEGLECT CENTRIFUGAL FORCE, WHICH GOES LIKE Ω^2)



VIEW FROM INERTIAL FRAME



VIEW FROM ROTATING FRAME

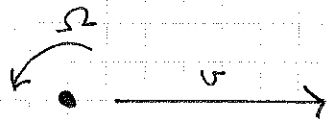
IN THE ROTATING FRAME $\vec{F} = -2m\vec{\Omega} \times \vec{v}$ IS \perp TO \vec{v} .

IT'S MAGNITUDE IS $2m\Omega v = \text{CONSTANT} \Rightarrow$ MOTION IN A CIRCLE

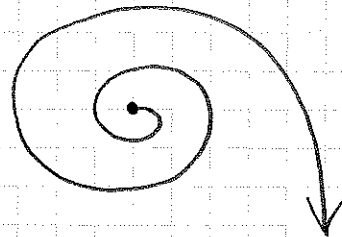
OF RADIUS $\frac{mv^2}{F} = 2m\Omega v$ OR $r = \frac{v}{2\Omega}$

b) THE BALL IS THROWN RADIIALLY OUTWARD.

IF Ω IS LARGE, OR r IS LARGE, WE MUST COMBINE THE CENTRIFUGAL FORCE ($m\Omega^2 r$) WITH THE CORIOUS FORCE ($2m\Omega v$) TO UNDERSTAND THE MOTION.



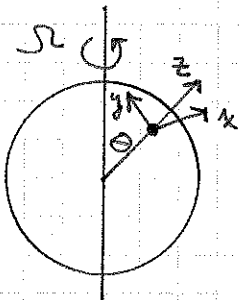
INERTIAL OBSERVER



ROTATING OBSERVER

THE CORIOUS FORCE WOULD IMPLY CIRCULAR MOTION, BUT CENTRIFUGAL FORCE OUTWARDS DISTORTS THE MOTION INTO A SPIRAL.

EXAMPLE THE EARTH AS A ROTATING FRAME.



WE SET UP A LOCAL COORDINATE SYSTEM WITH ITS ORIGIN AT A POINT ON THE SURFACE. LET \hat{z} POINT UPWARDS, \hat{x} EAST AND \hat{y} NORTH.

A MASS m AT POSITION \vec{r} IN THIS FRAME OBEYS THE EQUATION OF MOTION

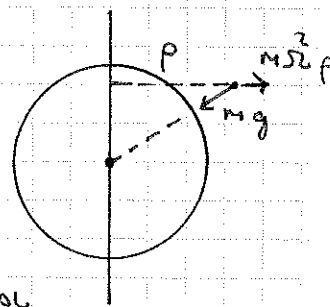
$$m \frac{\delta^2 \vec{r}}{\delta t^2} = \vec{F} - 2m \vec{\Omega} \times \vec{v} - m \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) - m \frac{d^2 \vec{R}}{dt^2}$$

Now $\frac{d\vec{R}}{dt} = \vec{\Omega} \times \vec{R}$ so $\frac{d^2 \vec{R}}{dt^2} = \vec{\Omega} \times (\vec{\Omega} \times \vec{R})$ [\vec{R} = POSITION OF ORIGIN RELATIVE TO EARTH'S CENTER]

AND $m \frac{\delta^2 \vec{r}}{\delta t^2} = \vec{F} - 2m \vec{\Omega} \times \vec{v} - m \vec{\Omega} \times (\vec{\Omega} \times (\vec{r} + \vec{R})) = \vec{F}_{\text{eff}}$

SUPPOSE $\vec{v} = 0$, AND $\vec{F} = -mg\hat{z}$ DUE TO GRAVITY.

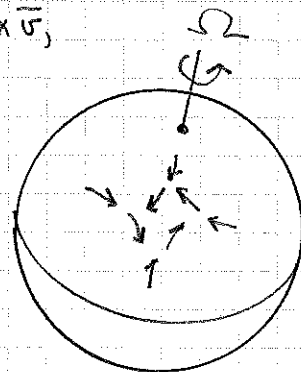
THEN \vec{F}_{eff} IS NOT ALONG \hat{z} , AND THE 'VERTICAL' AS DEFINED BY A PLUMB LINE DOES NOT POINT TOWARDS THE CENTER OF THE EARTH! THE CENTRIFUGAL FORCE TILTS THE 'VERTICAL' A SMALL AMOUNT - SINCE Ω^2 IS TINY.



OF COURSE WE CAN'T DISTINGUISH THE CENTRIFUGAL FORCE FROM GRAVITY IF WE ARE STANDING ON THE EARTH (AND HAVE ONLY A 'LOCAL' VIEWPOINT. COMPARE NEWTON'S CLAIM ON P173.) WHEN WE SET UP OUR LOCAL COORD. SYSTEM WE JUST CHOOSE THE z AXIS ALONG THE APPARENT VERTICAL - AND WE WILL NOT NOTICE ANY FURTHER EFFECT OF CENTRIFUGAL FORCE - SO LONG AS WE STAY NEAR THE ORIGIN!

WE NOW CONSIDER THE CORIOLIS FORCE $-2m \vec{\Omega} \times \vec{v}$, FOR PARTICLES WITH \vec{v} IN THE x - y PLANE $\Rightarrow \vec{v}$ ALONG THE EARTH'S SURFACE.

FOR EXAMPLE: IN A STORM WITH A LOW PRESSURE CENTER THE WINDS BLOW TO THE CENTER AND GET DEFLECTED AS SHOWN TO PRODUCE A COUNTER-CLOCKWISE CIRCULATION \Rightarrow HURRICANE.



IN THE SOUTHERN HEMISPHERE HURRICANES SWIRL CLOCKWISE, AGAIN DUE TO THE CORIOLIS EFFECT!

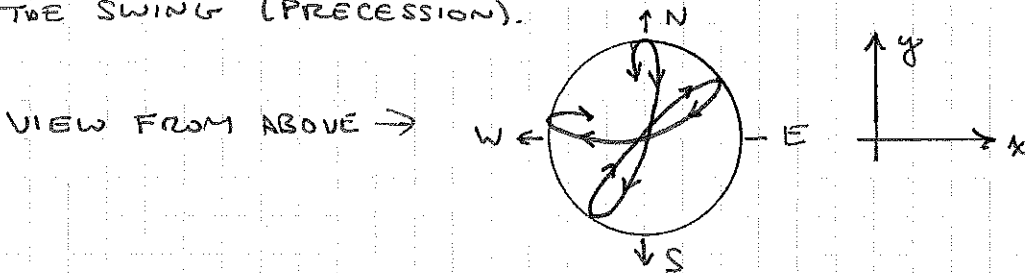
(SEE SCI. AM. MAY 1952)

FOLK LORE: WHEN YOU PULL THE PLUG IN A SINK THE WATER FLOWS RADIALLY INWARDS & OFTEN SPIRAL FLOW DEVELOPS. IF THIS IS AFFECTED BY THE CORIOLIS FORCE THE SPIRAL SHOULD BE COUNTER-CLOCKWISE IN THE NORTHERN HEMISPHERE, CLOCKWISE IN THE SOUTHERN. (BUT, SUPPOSE THE WATER HAS SOME ANGULAR MOMENTUM ABOUT THE DRAIN....)

EXAMPLE: BEER'S LAW EROSION IN RIVERS (AND ON RAILROAD TRACKS) IN THE NORTHERN HEMISPHERE IS GREATER ON THE RIGHT-HAND SIDE - DUE TO THE CORIOLIS EFFECT. EINSTEIN DISCUSSES THIS, RIVER MEANDERS, AND THE STIRRED-TEA EFFECT IN 'IDEAS & OPINIONS' P 249.

EXAMPLE: FOUCAULT PENDULUM

THE CORIOLIS FORCE WILL DEFLECT THE BOB OF A SWINGING PENDULUM, CAUSING A SLOW ROTATION OF THE PLANE OF THE SWING (PRECESSION).



FROM P 174 WE EXPECT CURVATURE $\approx \frac{v(t)}{2\Omega \cos \theta}$, LEADING TO THE ROSETTE PATTERN SKETCHED ABOVE.

TO ANALYZE THE MOTION, WE SUPPOSE THE LENGTH OF THE PENDULUM IS VERY LONG, AND THE SWING SMALL, SO WE MAY NEGLECT THE VERTICAL MOTION OF THE BOB. IF THERE WERE NO CORIOLIS FORCE, THE EQUATIONS OF MOTION WOULD BE

$$\begin{aligned} \ddot{x} + \omega_0^2 x &= 0 \\ \ddot{y} + \omega_0^2 y &= 0 \end{aligned} \quad \omega_0^2 = g/l$$

WE MUST ADD THE CORIOLIS TERM $\vec{F}/m = -2\vec{\Omega} \times \vec{v}$

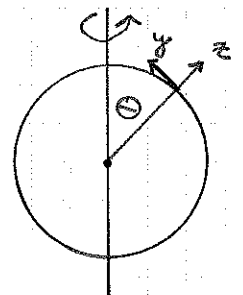
$$\vec{v} = (\dot{x}, \dot{y}, 0) \quad ; \quad \vec{\Omega} = (0, \Omega \sin \theta, \Omega \cos \theta)$$

$$\vec{\Omega} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \Omega \sin \theta & \Omega \cos \theta \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = -\dot{y} \Omega \cos \theta \hat{x} + \dot{x} \Omega \cos \theta \hat{y} - \dot{x} \Omega \sin \theta \hat{z}$$

$$\begin{aligned} \text{so } \ddot{x} + \omega_0^2 x &= 2\dot{y} \Omega \cos \theta \\ \ddot{y} + \omega_0^2 y &= -2\dot{x} \Omega \cos \theta \end{aligned}$$

FOR PROBLEMS LIKE THIS WITH COUPLED x & y OSCILLATIONS WE CAN EXTEND OUR TRICK OF USING COMPLEX NOTATION BY THINKING OF x & y AS THE REAL AND IMAGINARY PARTS OF A COMPLEX NUMBER

$$C = x + iy$$



WE COMBINE OUR 2 DIFFERENTIAL EQUATIONS TO GET A SINGLE DIFFERENTIAL EQUATION FOR C

$$(\ddot{x} + i\dot{y}) + \omega_0^2 (x + iy) = 2\Omega \omega_0 \theta (\dot{y} - i\dot{x}) = -2i\Omega \omega_0 \theta (\dot{x} + i\dot{y})$$

$$\text{OR } \ddot{C} + \omega_0^2 C = -2i\Omega \omega_0 \theta \dot{C}$$

LET'S TRY AN OSCILLATORY SOLUTION: $C = y_0 e^{i\omega t}$ (y_0 REAL)

$$\text{SO } -\omega^2 + \omega_0^2 = 2\omega\Omega \omega_0 \theta$$

$$\Rightarrow \omega = -\Omega \omega_0 \theta \pm \sqrt{\omega_0^2 + \Omega^2 \omega_0^2 \theta^2}$$

IF WE NEGLECT TERMS IN Ω^2 AS SMALL,

$$\omega = -\Omega \omega_0 \theta \pm \omega_0$$

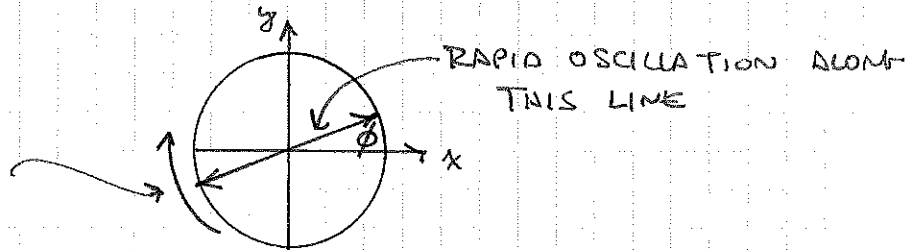
$$\text{SO } C = x + iy = y_0 e^{i\omega_0 t} e^{-i\Omega \omega_0 \theta t} \quad (\text{FOR THE } + \text{ SOLUTION})$$

RAPID OSCILLATION
SLOW PRECESSION

IT MAY HELP TO THINK OF THIS AS $x + iy = y_0 e^{i(\omega t + \phi)}$

WITH $\phi = -\Omega \omega_0 \theta t$

ϕ SLOWLY VARIES IN A CLOCKWISE SENSE



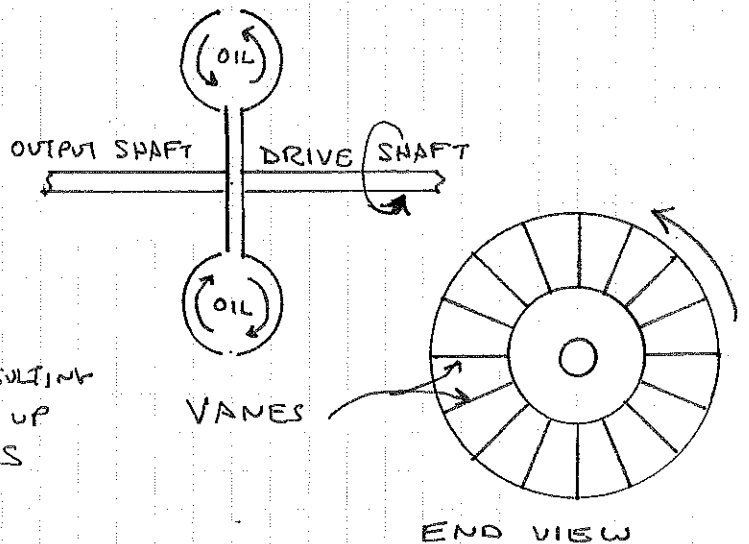
THE PERIOD OF PRECESSION IS $T = \frac{2\pi}{\Omega \omega_0 \theta} = \frac{24 \text{ HOURS}}{\omega_0 \theta}$

$T = 24 \text{ HOURS}$ AT THE POLE ; $T \rightarrow \infty$ AT THE EQUATOR.

EXAMPLE: FLUID CLUTCH

THE FLUID CLUTCH OF AN AUTOMOBILE WORKS BY A COMBINATION OF CENTRIFUGAL & CORIOLIS FORCE - IF ANALYSED IN THE ROTATING FRAME.

THE DRIVE SHAFT SPINS & THE RESULTING CENTRIFUGAL FORCE ON THE OIL SETS UP A CIRCULAR MOTION OF THE OIL AS SHOWN.



THE CORIOLIS FORCE ON THE OUTWARD MOVING OIL FORCES IT TO MOVE OPPOSITE TO THE SENSE OF ROTATION OF THE DRIVE SHAFT. THE OIL THEN PUSHES ON THE VANES AND CAUSES A TORQUE AGAINST THE MOTOR - TRYING TO SLOW IT DOWN.

ON THE OUTPUT SHAFT SIDE THE OIL FLOWS INWARDS. THE CORIOLIS FORCE NOW CAUSES A TORQUE IN THE SAME SENSE AS $\bar{\omega}$ DRIVE. THUS THE OUTPUT SHAFT WILL TRY TO ROTATE JUST LIKE THE DRIVE SHAFT.

IN STEADY MOTION THE CLUTCH CAN TRANSFER ABOUT 98% OF THE MECHANICAL ENERGY OF THE MOTOR TO THE OUTPUT SHAFT.

CULTURAL SUPPLEMENT:

UMBERTO ECO FOUCAULT'S PENDULUM

Translated from the Italian by

WILLIAM WEAVER

A Helen and Kurt Wolff Book
Harcourt Brace Jovanovich, Publishers

SAN DIEGO NEW YORK LONDON

I

(ב) והנה כהיות אור האיס נמשך,
בכחית (ה) קו ישר תוך החלל
הנל, לא נמשך ונתפשט (ו) תיכף
עד למטה, אמנם היה מתפשט לאט
לאט, רצוני לומר, כי בתחילה התי-
חיל קו האור להתפשט, ושם תיכף
(ז) בתחילת התפשטותו בסוד קו,
נתפשט ונמשך ונעשה, כעין (ח)
גלגל אחד עגול מסביב.

That was when I saw the Pendulum.

The sphere, hanging from a long wire set into the ceiling of the choir, swayed back and forth with isochronal majesty.

I knew—but anyone could have sensed it in the magic of that serene breathing—that the period was governed by the square root of the length of the wire and by π , that number which, however irrational to sublunar minds, through a higher rationality binds the circumference and diameter of all possible circles. The time it took the sphere to swing from end to end was determined by an arcane conspiracy between the most timeless of measures: the singularity of the point of suspension, the duality of the plane's dimensions, the triadic beginning of π , the secret quadratic nature of the root, and the unnumbered perfection of the circle itself.

I also knew that a magnetic device centered in the floor beneath issued its command to a cylinder hidden in the heart of the sphere, thus assuring continual motion. This device, far from interfering with the law of the Pendulum, in fact permitted its manifestation, for in a vacuum any object hanging from a weightless and unstretchable wire free of air resistance and friction will oscillate for eternity.

The copper sphere gave off pale, shifting glints as it was struck by the last rays of the sun that came through the great stained-glass

windows. Were its tip to graze, as it had in the past, a layer of damp sand spread on the floor of the choir, each swing would make a light furrow, and the furrows, changing direction imperceptibly, would widen to form a breach, a groove with radial symmetry—like the outline of a mandala or pentaculum, a star, a mystic rose. No, more a tale recorded on an expanse of desert, in tracks left by countless caravans of nomads, a story of slow, millennial migrations, like those of the people of Atlantis when they left the continent of Mu and roamed, stubbornly, compactly, from Tasmania to Greenland, from Capricorn to Cancer, from Prince Edward Island to the Svalbards. The tip retraced, narrated anew in compressed time what they had done between one ice age and another, and perhaps were doing still, those couriers of the Masters. Perhaps the tip grazed Agarththa, the center of the world, as it journeyed from Samoa to Novaya Zemlya. And I sensed that a single pattern united Avalon, beyond the north wind, to the southern desert where lies the enigma of Ayers Rock.

At that moment of four in the afternoon of June 23, the Pendulum was slowing at one end of its swing, then falling back lazily toward the center, regaining speed along the way, slashing confidently through the hidden parallelogram of forces that were its destiny.

Had I remained there despite the passage of the hours, to stare at that bird's head, that spear's tip, that obverse helmet, as it traced its diagonals in the void, grazing the opposing points of its astigmatic circumference, I would have fallen victim to an illusion: that the Pendulum's plane of oscillation had gone full circle, had returned to its starting point in thirty-two hours, describing an ellipse that rotated around its center at a speed proportional to the sine of its latitude. What would its rotation have been had it hung instead from the dome of Solomon's Temple? Perhaps the Knights had tried it there, too. Perhaps the solution, the final meaning, would have been no different. Perhaps the abbey church of Saint-Martin-des-Champs was the true Temple. In any case, the experiment would work perfectly only at the Pole, the one place where the Pendulum, on the earth's extended axis, would complete its cycle in twenty-four hours.

But this deviation from the Law, which the Law took into ac-

count, this violation of the rule did not make the marvel any less marvelous. I knew the earth was rotating, and I with it, and Saint-Martin-des-Champs and all Paris with me, and that together we were rotating beneath the Pendulum, whose own plane never changed direction, because up there, along the infinite extrapolation of its wire beyond the choir ceiling, up toward the most distant galaxies, lay the Only Fixed Point in the universe, eternally unmoving.

So it was not so much the earth to which I addressed my gaze but the heavens, where the mystery of absolute immobility was celebrated. The Pendulum told me that, as everything moved—earth, solar system, nebulae and black holes, all the children of the great cosmic expansion—one single point stood still: a pivot, bolt, or hook around which the universe could move. And I was now taking part in that supreme experience. I, too, moved with the all, but I could see the One, the Rock, the Guarantee, the luminous mist that is not body, that has no shape, weight, quantity, or quality, that does not see or hear, that cannot be sensed, that is in no place, in no time, and is not soul, intelligence, imagination, opinion, number, order, or measure. Neither darkness nor light, neither error nor truth.

I was roused by a listless exchange between a boy who wore glasses and a girl who unfortunately did not.

"It's Foucault's Pendulum," he was saying. "First tried out in a cellar in 1851, then shown at the Observatoire, and later under the dome of the Panthéon with a wire sixty-seven meters long and a sphere weighing twenty-eight kilos. Since 1855 it's been here, in a smaller version, hanging from that hole in the middle of the rib."

"What does it do? Just hang there?"

"It proves the rotation of the earth. Since the point of suspension doesn't move . . ."

"Why doesn't it move?"

"Well, because a point . . . the central point, I mean, the one right in the middle of all the points you see . . . it's a geometric point; you can't see it because it has no dimension, and if something has no dimension, it can't move, not right or left, not up or down. So it doesn't rotate with the earth. You understand? It can't even rotate around itself. There is no 'itself.'"

"But the earth turns."