Ph 205 Final Exam

January 15, 1981, 8:30 A.M.

Time limit: 3 hours

- Do not begin until told to do so.
- The exam consists of 5 problems each worth 20 points.
- Please do all work you wish graded in the exam booklets provided
- You must write and sign the pledge for this exam.
- The exam is closed book, closed notes.

① BIO-MECHANICS (YOU MUST SHOW THE LOGIC OF YOUR SOLUTION, AS WELL AS GIVING THE ANSWER)

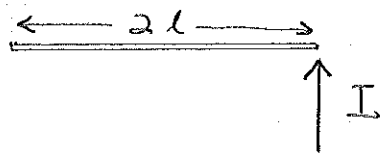
a) HOW DOES THE LENGTH OF TIME AN ANIMAL CAN SURVIVE ON A HOT, DRY DESERT DEPEND ON ITS LENGTH (OR HEIGHT), L?

b) ANIMALS ARE HEAT ENGINES. HOW DOES 'HORSEPOWER' DEPEND ON THE SIZE, L, OF THE ANIMAL?

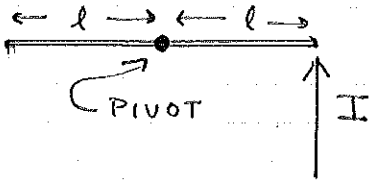
c) HOW DOES THE MAXIMUM RUNNING SPEED OF AN ANIMAL DEPEND ON ITS SIZE, L, IF AIR RESISTANCE IS THE LIMIT? , — IF GRAVITY IS THE LIMIT, AS IN RUNNING UPHILL?

d) HOW DOES THE HEIGHT AN ANIMAL CAN JUMP DEPEND ON ITS SIZE L? ASSUME THE RADII OF BONES ARE PROPORTIONAL TO L.

② a) A THIN ROD OF LENGTH 2ℓ, AND MASS $2M$ IS STRUCK BY AN IMPULSE, I, AT ONE END. THE IMPULSE IS PERPENDICULAR TO THE ROD. WHAT IS THE VELOCITY OF THE C.M., AND THE ANGULAR VELOCITY OF THE MOTION JUST AFTER THE IMPULSE?

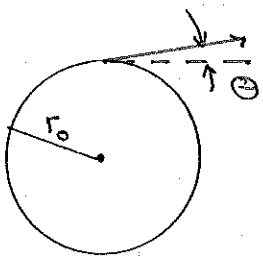


b) TWO THIN RODS EACH OF LENGTH ℓ, MASS, M, ARE JOINED BY A FRICTIONLESS PIVOT AS SHOWN. INITIALLY THE RODS LIE ALONG A STRAIGHT LINE. AN IMPULSE, I, IS APPLIED PERPENDICULAR TO THE END OF ONE ROD.



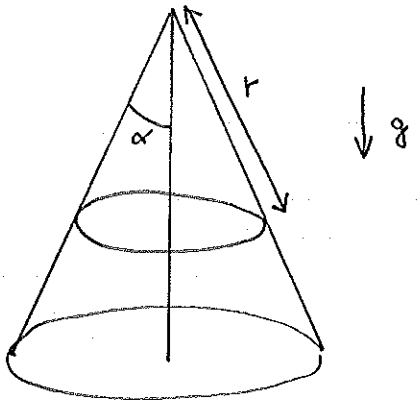
WHAT IS THE VELOCITY OF THE PIVOT, AND THE ANGULAR VELOCITIES OF THE TWO RODS JUST AFTER THE IMPULSE? (AS A FINAL CHECK, YOU MAY WISH TO CALCULATE THE VELOCITY OF THE C.M.)

(3) a) A SATELLITE IS TO BE PUT IN A CIRCULAR ORBIT OF RADIUS r_0 ABOUT THE EARTH. WHEN IT REACHES RADIUS r_0, CORRECTION ROCKETS FIRE TO GIVE THE SATELLITE THE PROPER VELOCITY AND DIRECTION. SUPPOSE THE VELOCITY IS CORRECT, BUT THE DIRECTION IS WRONG BY A SMALL ANGLE Θ AS SHOWN. TO FIRST ORDER IN Θ, WHAT IS THE APOGEE OF THE ORBIT? (APOGEE = MAXIMUM RADIUS)



b) SUPPOSE WHEN $r = r_0$, THE CORRECTION ROCKETS GIVE THE SATELLITE THE PROPER DIRECTION FOR A CIRCULAR ORBIT, BUT VELOCITY $V = V_0 + \epsilon$, WHERE V_0 IS THE PROPER VELOCITY. WHAT IS THE APOGEE OF THE ORBIT, TO FIRST ORDER IN ϵ?

(4) AN ELASTIC STRING OF MASS M, REST LENGTH l_0 AND SPRING CONSTANT K, LIES ON A CONE OF HALF-ANGLE α. THE AXIS OF THE CONE IS VERTICAL



a) WHAT IS THE EQUILIBRIUM DISTANCE, r_0, FROM THE APEX OF THE CONE TO THE STRING (MEASURED ALONG THE CONE)?

b) WHAT IS THE FREQUENCY OF THE LOWEST OSCILLATORY MODE ABOUT THE EQUILIBRIUM OF PART a)?

(5) A HOLLOW CYLINDER OF RADIUS a, MASS M, ROLLS WITHOUT SLIPPING ON A HORIZONTAL PLANE. ANOTHER HOLLOW CYLINDER, RADIUS $b < a$, MASS M, ROLLS WITHOUT SLIPPING ON THE INNER SURFACE OF THE FIRST CYLINDER. DESCRIBE THE NORMAL MODES AS COMPLETELY AS POSSIBLE. (BUT WORDS ALONE WILL NOT SUFFICE!)