Measurement of $\sin^2 2\theta_{13}$ via Inverse β-Decay due to $\overline{\nu}_e$ from Multiple Nuclear Reactors

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Abstract

This note transcribes lore developed by members of the Daya Bay reactor neutrino experiment and others as to how, in a measurement of the neutrino-mixing parameter using multiple detectors and $\overline{\nu}_e$ from multiple nuclear reactors, it may be preferable to weight the data from different detectors in a manner than minimizes the effect of uncertainties in the reactor neutrino fluxes.

1 $\overline{\nu}_e$ Disappearance

Reactor neutrino experiments [1] can search for the neutrino-mixing parameter $\sin^2 2\theta_{13}$ and measure it if its value is large enough by observing the probability of disappearance of electron antineutrinos ($\overline{\nu}_e$) as a function of distance from their production in a nuclear reactor. For distances of less than hundreds of km so-called matter effects can be neglected, and the probability that an antineutrino that is created as a $\overline{\nu}_e$ of energy $E_\nu \approx$ 1-10 MeV remains a $\overline{\nu}_e$ after traveling a distance $L \lesssim 2$ km can be written as,

$$P_{\rm sur} = 1 - P_{\rm dis} \approx 1 - A(E_\nu, L)\sin^2 2\theta_{13}, \qquad (1)$$

where the "analyzing power" A has the form,

$$A(E_\nu, L) = \sin^2\Delta_{31} = \sin^2\left[1,267\Delta m_{31}^2({\rm eV}/c^2)^2 \frac{L({\rm km})}{E_\nu({\rm MeV})}\right] \approx \sin^2\left[2.8\frac{L({\rm km})}{E_\nu({\rm MeV})}\right], \qquad (2)$$

using the current best estimate [2] that the neutrino mass splitting parameter Δm_{31}^2 is $2.2 \pm 0.5 \times 10^{-3}$ $({\rm eV}/c^2)^2$.

The electron antineutrinos are detected via the inverse β-decay reaction,

$$\overline{\nu}_e + p \to n + e^+ \qquad (3)$$

on free protons in a suitable detector, typically Gd-loaded liquid scintillator to enhance the detection of the final-state neutron [1].

2 One Detector, One Reactor, One Distance

If a single neutrino detector is placed at a distance L from a single nuclear reactor, the number $N(E_\nu)$ of detected neutrino interactions in an interval dE_ν about energy E_ν during time T can be written as the product,

$$N(E_\nu) = \int_0^T \phi_r(E_\nu, dE_\nu, t)M_d(t)\epsilon_d(t)\,dt\frac{\sigma_{\overline{\nu}}}{4\pi L_{dr}^2}(1 - A\sin^2 2\theta_{13}) \approx \frac{\overline{\phi}_r KT}{L_{dr}^2}(1 - A\sin^2 2\theta_{13}) \quad (4)$$

where $\phi_r(E, dE, t)$ is the total flux per unit time at unit distance of antineutrinos from the reactor at time t in interval dE about E, L_{dr} is the distance from the detector to the reactor, $\sigma_{\overline{\nu}_e}$ is the cross section for the neutrino interaction (3), M_d is the (possibly time-dependent) detector mass, ϵ_d is the (generally time-dependent) detection efficiency, and,

$$\overline{\phi}_r = \frac{1}{T}\int_0^T \phi_r(E_\nu, dE_\nu, t)\, dt, \qquad \text{and} \qquad K = \frac{\sigma_{\overline{\nu}}}{4\pi T}\int_0^T M_d(t)\epsilon_d(t)\, dt \tag{5}$$

are the time-average antineutrino flux and time-average detector factor, respectively.

The spectrum of reactor antineutrinos falls with energy, while the cross section for rises with energy. As shown in Fig. 1 the most probable energy for observation of inverse β-decay is for an antineutrino energy of 4 MeV.

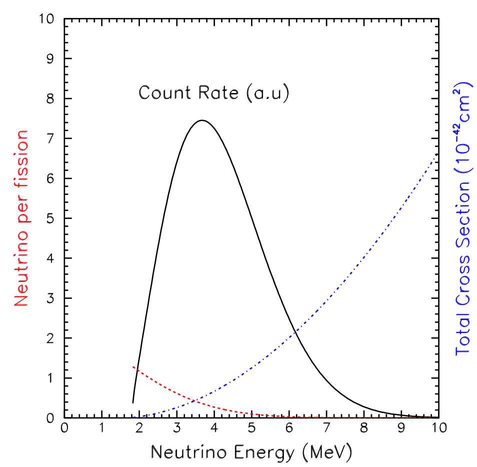

Figure 1: The $\overline{\nu}_e$ production rate in a nuclear reactor, the cross section for inverse β-decay, and the probability of observation of the inverse β-decay process in a reactor neutrino experiment, as a function of the energy of the antineutrino.

Clearly, the distance L_{dr} should be chosen such that the analyzing power A is maximal. Using $\Delta m^2_{31} = 2.2 \times 10^{-3}$ $(\mathrm{eV}/c^2)^2$ in eq. (2), the analyzing power would be 1 for $L = 2.25$ km and $E = 4$ MeV.

In principle, a determination of $\sin^2 2\theta_{13}$ can be made using eq.(4) using a single detector at a single distance from a single reactor. The distance L can be well measured, the neutrino flux $\overline{\phi}_r$ can be calculated reasonably well from measurement of the reactor power [3], but the detector factor K is not well known in practice. Hence, extensions of the measurement technique are desirable that reduce the dependence of the result for $\sin^2 2\theta_{13}$ on the uncertainties in the product $\overline{\phi}_r K$.

3 One Detector, One Reactor, Two Distances

If a single detector is operated at different times at two distances $L^{(1)}$ and $L^{(2)}$ from a single reactor, two sets of neutrino interactions $N^{(1)}$ and $N^{(2)}$ can be observed. The ratio of these observations is,

$$\frac{N^{(2)}}{N^{(1)}} = \frac{\overline{\phi}^{(2)} T^{(2)}}{\overline{\phi}^{(1)} T^{(1)}} \left(\frac{L^{(1)}}{L^{(2)}}\right)^2 \frac{1 - A(L^{(2)}) \sin^2 2\theta_{13}}{1 - A(L^{(1)}) \sin^2 2\theta_{13}}\,, \tag{6}$$

assuming that the detector factor K is independent of time.

The typical strategy is to choose distance $L^{(2)}$ to be at the first maximum of the oscillating analyzing power (2), and to use a distance $L^{(1)}$ that is much less than $L^{(2)}$. Then, the analyzing power $A(L^{(1)})$ very small, and we set it to zero in the following.

We call distance $L^{(2)}$ the Far distance and write $N^{(2)} = N_{\rm Far}$. Similarly, we say that $L^{(1)}$ the Near distance and write $N^{(1)} = N_{\rm Near}$.

Then eq.(6) can be written as,

$$\frac{N_{\rm Far}}{N_{\rm Near}} \approx \frac{\overline{\phi}_{\rm Far} T_{\rm Far}}{\overline{\phi}_{\rm Near} T_{\rm Near}} \frac{L^2_{\rm Near}}{L^2_{\rm Far}} (1 - \sin^2 2\theta_{13}), \approx \frac{T_{\rm Far}}{T_{\rm Near}} \frac{L^2_{\rm Near}}{L^2_{\rm Far}} (1 - \sin^2 2\theta_{13}), \tag{7}$$

where the second approximation is that the reactor flux is independent of time, $\overline{\phi}_{\rm Far} = \overline{\phi}_{\rm Near}$. Then, $\sin^2 2\theta_{13}$ is measured as,

$$\sin^2 2\theta_{13} \approx 1 - \frac{T_{\rm Near}}{T_{\rm Far}} \frac{L^2_{\rm Far}}{L^2_{\rm Near}} \frac{N_{\rm Far}}{N_{\rm Near}}\,. \tag{8}$$

Thus, the use of two distances eliminates the dependence of the measurement of $\sin^2 2\theta_{13}$ on the imperfectly known quantity K.

The one-standard-deviation statistical uncertainty on the observation of N events is $\delta_N = \sqrt{N}$. Hence, the statistical uncertainty on the measurement of $\sin^2 2\theta_{13}$ according to eq. (8) would be,

$$\delta_{\sin^2 2\theta_{13}} \approx \frac{T_{\rm Near}}{T_{\rm Far}} \frac{L^2_{\rm far}}{L^2_{\rm Near}} \frac{N_{\rm Far}}{N_{\rm Near}} \sqrt{\frac{1}{N_{\rm Far}} + \frac{1}{N_{\rm Near}}}. \tag{9}$$

If the number $N_{\rm Far}$ of events observed at the Far distance is small compared to the number $N_{\rm Near}$ observed at the Near distance, then,

$$\delta_{\sin^2 2\theta_{13}} \approx \frac{T_{\rm Near}}{T_{\rm Far}} \frac{L^2_{\rm Far}}{L^2_{\rm Near}} \frac{\sqrt{N_{\rm Far}}}{N_{\rm Near}} \qquad (N_{\rm Far} \ll N_{\rm Near}). \tag{10}$$

For small values of $\sin^2 2\theta_{13}$ the factor $L^2 N/T$ is essentially independent of distance, so that we can make the further approximation,

$$\delta_{\sin^2 2\theta_{13}} \approx \frac{1}{\sqrt{N_{\rm Far}}} \qquad (N_{\rm Far} \ll N_{\rm Near},\ \sin^2 2\theta_{13} \ll 1). \tag{11}$$

If we wish to determine $\sin^2 2\theta_{13}$ with a statistical uncertainty of, say, 0.001, then the number of antineutrino events observed at the far location should be $N_{\rm far} \approx 10^6$.

4 Two Detectors, One Reactor, Two Distances

One way to improve the statistical accuracy of the measurement of $\sin^2 2\theta_{13}$ in a fixed time is by use of multiple detectors.

For example, if $N_i^{(k)}$ is the number of antineutrino interactions observed in detector i when in configuration (k), for which its distance to the reactor is $L_i^{(k)}$, then,

$$N_i^{(k)} = \frac{\overline{\phi}_r K_i T^{(k)}}{\left(L_i^{(k)}\right)^2}(1 - A\left(L_i^{(k)}\right)\sin^2 2\theta_{13}). \tag{12}$$

The value of the constants K_i could well be different for different detectors. To minimize the effect of the possible variation of the constant K_i between detectors, they should not be operated at a single distance. A simple procedure is to operate each detector at both distances $L_1^{(1)} = L_{\rm Near}$ and $L_1^{(2)} = L_{\rm Far}$, which requires "swapping" the two detectors at the midpoint of the duration of the experiment. That is, $L_2^{(1)} = L_1^{(2)} = L_{\rm Far}$ and $L_2^{(2)} = L_1^{(1)} = L_{\rm Near}$. Then, if $A(L_{\rm Near}) \approx 0$ and $A(L_{\rm Far}) \approx 1$, we measure,

$$\begin{aligned} \sin^2 2\theta_{13} &\approx 1 - \frac{\overline{\phi}^{(1)} T^{(1)}}{\overline{\phi}^{(2)} T^{(2)}} \frac{L_{\rm Far}^2}{L_{\rm Near}^2} \frac{N_1^{(2)}}{N_1^{(1)}} \approx 1 - \frac{\overline{\phi}^{(2)} T^{(2)}}{\overline{\phi}^{(1)} T^{(1)}} \frac{L_{\rm Far}^2}{L_{\rm Near}^2} \frac{N_2^{(1)}}{N_2^{(2)}} \\ &\approx 1 - \frac{T^{(1)}}{T^{(2)}} \frac{L_{\rm Far}^2}{L_{\rm Near}^2} \frac{N_1^{(2)}}{N_1^{(1)}} \approx 1 - \frac{T^{(2)}}{T^{(1)}} \frac{L_{\rm Far}^2}{L_{\rm Near}^2} \frac{N_2^{(1)}}{N_2^{(2)}}, \end{aligned} \tag{13}$$

where the second line of eq. (13) assumes that $\overline{\phi}^{(1)} = \overline{\phi}^{(2)} \equiv \overline{\phi}_r$. If the two detectors are nearly identical, *i.e.,* $K_1 \approx K_2$, the accuracy of each of the two measurements of $\sin^2 2\theta_{13}$ will be the same, and the best estimate is the average of the two measurements,

$$\sin^2 2\theta_{13} = 1 - \frac{1}{2} \frac{L_{\rm Far}^2}{L_{\rm Near}^2} \left(\frac{T^{(1)}}{T^{(2)}} \frac{N_1^{(2)}}{N_1^{(1)}} + \frac{T^{(2)}}{T^{(1)}} \frac{N_2^{(1)}}{N_2^{(2)}} \right) \qquad (K_1 \approx K_2). \tag{14}$$

Further, if the duration of operation in the two configurations is nearly identical we can write $N_1^{(2)} \approx N_2^{(1)} \approx N_{\rm Far}/2$ and $N_1^{(1)} \approx N_2^{(2)} \approx N_{\rm Near}/2$, so that eq. (14) reduces to eq. (7),

$$\sin^2 2\theta_{13} \approx 1 - \frac{L_{\rm Far}^2}{L_{\rm Near}^2} \frac{N_{\rm Far}}{N_{\rm Near}} \qquad (K_1 \approx K_2,\ \text{equal times}). \tag{7}$$

The statistical uncertainty on the measurement (7) of $\sin^2 2\theta_{13}$ is again given by eq. (11).

In principle, the four measurements $N_i^{(k)}$ suffice to determine the four unknown quantities $\sin^2 2\theta_{13}$, $\overline{\phi}_r$, K_1 and K_2. However, only $\sin^2 2\theta_{13}$ and the products $\overline{\phi}_r K_1 = N_1^{(1)} L_{\rm Near}^2$ and $\overline{\phi}_r K_2 = N_2^{(2)} L_{\rm Near}^2$ are determined in a simple manner. Extraction of $\overline{\phi}_r$, K_1 and K_2 requires use of a χ^2-minimization procedure with,

$$\chi^2 = \sum_{i=1}^{2} \sum_{k=1}^{2} \frac{\left[N_i^{(k)} - \frac{\overline{\phi}_r K_i T^{(k)}}{(L_i^{(k)})^2} (1 - A(L_i^{(k)}) \sin^2 2\theta_{13}) \right]^2}{N_i^{(k)}}, \tag{15}$$

taking the uncertainty on the number $N_i^{(k)}$ of observed events to be, $\delta_{N_i^{(k)}} = \sqrt{N_i^{(k)}}$.

5 8 Detectors, 6 Reactors (the Daya Bay Reactor Neutrino Experiment)

Nuclear reactor complexes typically have more than one reactor. If we observe antineutrino interactions from m reactors with a set of n detectors, there are $m+n+1$ unknowns, $\sin^2 2\theta_{13}$, the reactor fluxes $\overline{\phi}_j$ for $j = 1$ to m, and the detector constants K_i for $i = 1$ to n (assuming that K_i and $\overline{\phi}_j$ are independent of time).

A single configuration of the detectors around the reactors leads to the observation of n numbers of events in the n detectors. If k different configurations are the detectors are used over the course of the experiment, a total of kn measurements are made. If $\sin^2 2\theta_{13}$ is to be determined with no dependence on the $m + n$ fluxes and detector constants, then kn must be greater than or equal to $m + n + 1$.

The Daya Bay reactor complex will have $m = 6$ reactors operational by the completion of the experiment. To determine $\sin^2 2\theta_{13}$ independent of the K_i and the $\overline{\phi}_j$ with only two detector configurations requires a minimum of $n = 7$ detectors.

The proposal is to use 8 detectors, arrayed in two "near" groups of two detectors, and one "far" group of 4 detectors, as shown in Fig. 2.

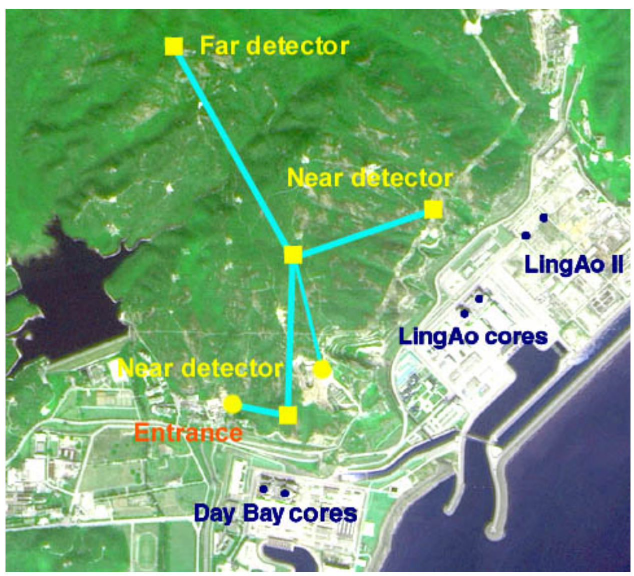

Figure 2: Layout of the Daya Bay reactor neutrino experiment.

To a first approximation, the analyzing powers (2) are unity for detectors and the far site, and zero for detectors at the near sites.

We consider two configurations, indicated by superscripts $^{(1)}$ and $^{(2)}$, such that in configuration 1 detectors 1-4 are at near sites while detectors 5-8 are at the far site. Then, in configuration 2 detectors 1-4 are at the far site and detectors 5-8 are at near far sites.

The observed rates $N_i^{(k)}$ of antineutrino events in the 8 detectors can be written as,

$$N_i^{(1)} = K_i \sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(1)}}{(L_{i,j}^{(1)})^2}, \qquad i = 1\text{-}4, \qquad (16)$$

$$
\begin{aligned}
N_i^{(1)} &= K_i \sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(1)}}{(L_{i,j}^{(1)})^2}(1-\sin^2 2\theta_{13}), \qquad i = \text{5-8}, && (17)\\
N_i^{(2)} &= K_i \sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(2)}}{(L_{i,j}^{(2)})^2}(1-\sin^2 2\theta_{13}), \qquad i = \text{1-4}, && (18)\\
N_i^{(2)} &= K_i \sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(2)}}{(L_{i,j}^{(2)})^2}, \qquad i = \text{5-8}, && (19)
\end{aligned}
$$

We form the 8 ratios,

$$
\begin{aligned}
\frac{N_i^{(2)}}{N_i^{(1)}} &= \frac{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(2)}}{(L_{i,j}^{(2)})^2}}{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(1)}}{(L_{i,j}^{(1)})^2}}(1-\sin^2 2\theta_{13}) = \rho_i(1-\sin^2 2\theta_{13}), \qquad i = \text{1-4}, && (20)\\
\frac{N_i^{(1)}}{N_i^{(2)}} &= \frac{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(1)}}{(L_{i,j}^{(1)})^2}}{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(2)}}{(L_{i,j}^{(2)})^2}}(1-\sin^2 2\theta_{13}) = \rho_i(1-\sin^2 2\theta_{13}), \qquad i = \text{5-8}, && (21)
\end{aligned}
$$

where,

$$
\rho_i = \frac{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(2)}}{(L_{i,j}^{(2)})^2}}{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(1)}}{(L_{i,j}^{(1)})^2}}, \qquad i = \text{1-4}, \qquad \text{and} \qquad \rho_i = \frac{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(1)}}{(L_{i,j}^{(1)})^2}}{\sum_{j=1}^{6} \frac{\overline{\phi}_j T^{(2)}}{(L_{i,j}^{(2)})^2}}, \qquad i = \text{5-8}, \tag{22}
$$

are ratios of sums of the reactor fluxes weighted by distances in the two configurations of the experiment. Using the notation that $L_{i,j,\mathrm{Far}}$ and $L_{i,j,\mathrm{Near}}$ represent the distances from detector i to reactor j when the detector is in its Far and Near location, respectively, the ratios ρ_i can be written as,

$$
\rho_i = \frac{\sum_{j=1}^{6} \frac{\overline{\phi}_j}{L_{i,j,\mathrm{Far}}^2}}{\sum_{j=1}^{6} \frac{\overline{\phi}_j}{L_{i,j,\mathrm{Near}}^2}}, \quad i = \text{i-8}, \;\; T^{(1)} = T^{(2)}, \tag{23}
$$

assuming also that the times of operation in the two configurations are equal.

We now have 8 measurements of $\sin^2 2\theta_{13}$,

$$
\sin^2 2\theta_{13} = 1 - \rho_i \frac{N_i^{\mathrm{Far}}}{N_i^{\mathrm{Near}}}, \tag{24}
$$

where $N_i^{\mathrm{Far}} = N_i^{(2)}$ for i = 1-4 and $N_i^{\mathrm{Far}} = N_i^{(1)}$ for i = 5-8, and similarly $N_i^{\mathrm{Near}} = N_i^{(1)}$ for i = 1-4 and $N_i^{\mathrm{Near}} = N_i^{(2)}$ for i = 5-8. All 8 detectors will be nearly identical, so if we operate for equal times in the two configurations, the numbers of events in the numerator of eq. (24) are all approximately equal to $N_{\mathrm{Far}}/8$, where N_{Far} is the total number of events observed at the Far site.

From the 8 measurements (23) of $\sin^2 2\theta_{13}$ we can report a single result based on the weighted average,

$$\sin^2 2\theta_{13} = 1 - \sum_{i=1}^{4} w_i \rho_i \frac{N_i^{(2)}}{N_i^{(1)}} - \sum_{i=5}^{8} w_i \rho_i \frac{N_i^{(1)}}{N_i^{(2)}} = 1 - \sum_{i=1}^{8} w_i \rho_i \frac{N_i^{\rm Far}}{N_i^{\rm Near}}, \qquad \text{where} \qquad \sum_{i=1}^{8} w_i = 1, \tag{25}$$

where to a first approximation the weights w_i are all equal to 1/8. Equation (25) is a generalization of eqs. (7) and (14), and likewise involves ratios of rates in Far detectors to those in Near detectors. The detector factors K_i do not appear in eq. (25), but ratios of the reactor fluxes $\overline{\phi}_j$ do appear via the ρ_i.

In the approximation that the statistical uncertainty in the measurement ofsin$^2 2\theta_{13}$ is due only to the numbers of events in Far detectors, that uncertainty is, recalling eq. (10),

$$\delta_{\sin^2 2\theta_{13}} \approx \sqrt{\sum_{i=1}^{8} \left(w_i \rho_i \frac{\sqrt{N_i^{\rm Far}}}{N_i^{\rm Near}} \right)^2}. \tag{26}$$

It is a good approximation that all the numbers of events in the numerators of eq. (26) are $N_i^{\rm Far} \approx N_{\rm Far}/8$. For small $\sin^2 2\theta_{13}$ it is also a reasonable approximation that the numbers of events in the denominators can be written as $N_i^{\rm Near} \approx N_{\rm Far}/(8\rho_i)$. Then, we obtain the form,

$$\delta_{\sin^2 2\theta_{13}} \approx \frac{\sqrt{8 \sum_{i=1}^{8} w_i^2}}{\sqrt{N_{\rm Far}}} \approx \frac{1}{\sqrt{N_{\rm Far}}}, \tag{11}$$

where the second approximation follows if the weights w_i are all 1/8.

The measurement (25) of $\sin^2 2\theta_{13}$ depends on our knowledge of the reactor fluxes $\overline{\phi}_j$ through the factors ρ_i of eq. (22). In principle these fluxes can be determined from the 16 measurements $N_i^{(k)}$, but this determination requires use of a χ^2-minimization procedure, with,

$$\chi^2 = \sum_{i=1}^{8} \sum_{j=1}^{6} \sum_{k=1}^{2} \frac{\left[N_i^{(k)} - \frac{\overline{\phi}_j K_i}{(L_{i,j}^{(k)})^2} (1 - A(L_{i,j}^{(k)}, \Delta m_{31}^2) \sin^2 2\theta_{13}) \right]^2}{N_i^{(k)}}. \tag{27}$$

This procedure can fit for 16 parameters, which can include the mass-splitting parameter Δm_{31}^2 via its role in the analyzing power (2) along with the 15 parameters $\sin^2 2\theta_{13}$, K_i and $\overline{\phi}_j$.

6 Detector "Swapping"

As discussed in sec. 5, the 8 detector constants K_i can in principle be determined along with $\sin^2 2\theta_{13}$ if each detector is operated at one of two Near sites and at the Far site for equal amounts of time. In practice it will not be convenient to "swap" all 8 detectors halfway through the experiment. Rather, we propose to make 3 "swaps" of 4 detectors each at the end of each quarter of the experiment, as indicated in Table 6.

Careful calibration of the detectors *before and after* each "swap" will be necessary to insure that each detector's performance does not change significantly due to relocation.

Table 1: Swapping scheme with 4 running periods. The detectors (labeled 1-8) are deployed at the Near(DB), Near(LA), and Far sites during each period as indicated in this table. Each detector is operated for 50% of the experiment at the Far site.

Run Period	Near(DB)	Near(LA)	Far
I	1,3	5,7	2,4,6,8
II	2,3	6,7	1,4,5,8
III	2,4	6,8	1,3,5,7
IV	1,4	5,8	2,3,6,7

7 Statistical Uncertainty on $\sin^2 2\theta_{13}$

As discussed in secs. 3-5, the statistical uncertainty on our measurement of $\sin^2 2\theta_{13}$ is approximately $1/\sqrt{N_{\rm Far}}$, where $N_{\rm Far}$ is the total number of antineutrino events observed in all detectors at the Far site.

Each detector with have a fiducial mass of about 20 tons of liquid scintillator, as constrained by requirements of operation in the underground site. The far site is approximately 2 km from the Daya Bay reactors, so that the analyzing power (2) is close to unity. Then, each of the 4 detectors at the far site at any moment will observe about 90 antineutrino events per day. In 1000 days of operation, a total of $N_{\rm far} \approx 3.6 \times 10^5$ events will be observed, and the corresponding statistical uncertainty on the measurement will be,

$$\delta_{\sin^2 2\theta_{13}} \approx 0.0017 \qquad \text{(statistical)}. \tag{28}$$

The statistical uncertainty (28) is well below the goal of an overall sensitivity of $\sin^2 2\theta_{13} \approx 0.01$. The contribution to the uncertainty on the measurement of $\sin^2 2\theta_{13}$ from other sources than the number of observed antineutrino events will be discussed in the remainder of this section. These systematic uncertainties are of 3 types: reactor, detector and background.

For a historical perspective, the best existing limit of $\sin^2 2\theta_{13} < 0.17$ at 90% confidence level was obtained by the CHOOZ experiment [9] (with an analyzing power of 1/2 at their far site) which quotes a statistical uncertainty of 2.8% and a systematic uncertainty of 2.7%.

8 Reactor-related uncertainties

For a single reactor (with only one core), the reactor flux $\overline{\phi}_r$ is well determined by use of one far detector and one near detector (assuming the average distances are precisely known), as discussed in sec. 3. *At present*, the Daya Bay Power Plant has four cores in two groups, the Daya Bay Plant and the Ling Ao Plant. Another two cores will be installed adjacent to Ling Ao, called Ling Ao II, which will start to generate electricity in 2010. Figure 2 shows the locations of the Daya Bay cores, Ling Ao cores, and the future Ling Ao II cores. Superimposed on the figure are the tunneling scheme and the proposed detector sites. The distance between the two cores at each reactor site, called a pair here, is about 88 m. The Daya Bay pair is 1100 m from the Ling Ao pair, and the maximum span of cores will reach 1600 m when Ling Ao II starts operation.

Uncertainties in the reactor fluxes are associated with the power levels of the different cores and the effective locations of the cores relative to the detectors.

8.1 Reactor-Power Uncertainties

Knowledge of the reactor power is needed to calculate the ratios ρ_i of sum of distance-weighted antineutrino fluxes given by eq. (23). To assess the effect on the uncertainties $\delta_{\overline{\phi}_j}$ on the reactor fluxes, we recall eq. (25) and take the derivative,

$$\frac{\partial \rho_i}{\partial \overline{\phi}_j} = \rho_i \left(\frac{\frac{1}{L^2_{i,j,\mathrm{Far}}}}{\sum_{j'=1}^6 \frac{\overline{\phi}_{j'}}{L^2_{i,j',\mathrm{Far}}}} - \frac{\frac{1}{L^2_{i,j,\mathrm{Near}}}}{\sum_{j'=1}^6 \frac{\overline{\phi}_{j'}}{L^2_{i,j',\mathrm{Near}}}} \right) \approx \frac{\rho_i}{\overline{\phi}} \left(\frac{\frac{1}{L^2_{i,j,\mathrm{Far}}}}{\sum_{j'=1}^6 \frac{1}{L^2_{i,j',\mathrm{Far}}}} - \frac{\frac{1}{L^2_{i,j,\mathrm{Near}}}}{\sum_{j'=1}^6 \frac{1}{L^2_{i,j',\mathrm{Near}}}} \right) \equiv \frac{\rho_i}{\overline{\phi}} A_{ij}, \tag{29}$$

where we suppose that all 6 reactor fluxes $\overline{\phi}_j$ are close to a common value $\overline{\phi}$.

The dependence of the measurement (25) of $\sin^2 2\theta_{13}$ on the ratios ρ_i is such that the contribution to its uncertainty due to the uncertainty in the reactor fluxes is,

$$\begin{aligned} \delta_{\sin^2 2\theta_{13}} &= \sqrt{\sum_{j=1}^6 \left(\delta_{\overline{\phi}_j} \sum_{i=1}^8 w_i \frac{\partial \rho_i}{\partial \overline{\phi}_j} \frac{N_i^{\mathrm{Far}}}{N_i^{\mathrm{Near}}} \right)^2} \approx \sqrt{\sum_{j=1}^6 \left(\frac{\delta_{\overline{\phi}_j}}{\overline{\phi}} \sum_{i=1}^8 w_i A_{ij} \rho_i \frac{N_i^{\mathrm{Far}}}{N_i^{\mathrm{Near}}} \right)^2} \\ &\approx \sqrt{\sum_{j=1}^6 \left(\frac{\delta_{\overline{\phi}_j}}{\overline{\phi}} \sum_{i=1}^8 w_i A_{ij} \right)^2}. \end{aligned} \tag{30}$$

The coefficients A_{ij} that appear in eq. (29) can be evaluated from Table 8.1 of the distance between detector sites and reactors. We adopt the convention that detectors i = 1-4 use the Daya Bay near site, detectors i = 5-8 use the Ling Ao near site, reactors j = 1-2 are the Daya Bay cores, reactors j = 3-4 are the Ling Ao cores, and reactors j = 5-6 are the Ling Ao II cores, then there are only 6 distinct coefficients, $A_{ij} = -0.547$, 0.230 and 0.316 for j = 1-2, 3-4 and 4-5, respectively, when i = 1-4, and 0.192, -0.158 and -0.034 for j = 1-2, 3-4 and 4-5, respectively, when i = 5-8. There are only 3 distinct averages, $\sum_{i=1}^8 A_{ij}/8 = -0.178$, 0.036 and 0.141 for j = 1-2, 3-4 and 4-5, respectively. We combine the 6 terms of eq. (30) in quadrature, assuming that all reactors have a similar uncertainty $\delta_{\overline{\phi}}$ in their flux and that all weights w_i are equal to 1/8, to find,

$$\delta_{\sin^2 2\theta_{13}} \approx \frac{\delta_{\overline{\phi}}}{\overline{\phi}} \sqrt{\sum_{j=1}^6 \left(\sum_{i=1}^8 w_i A_{ij} \right)^2} \approx 0.23 \frac{\delta_{\overline{\phi}}}{\overline{\phi}} \qquad (w_i = 1/8). \tag{31}$$

This result is only slightly better than that of the approximation that all of the coefficients A_{ij} are 1/6. The coefficients A_{ij} associated with the Daya Bay near site have opposite sign to those associated with the Ling Ao near site, so combining the data from these sites yield a lower uncertainty than that associated with either site alone.[1]

Table 2: Average distances of the detector sites to the two cores of each reactor pair, for the configuration shown in Fig. 2.

	Daya Bay cores	Ling Ao cores	Ling Ao II cores
Daya Bay near site (m)	360	850	1300
Ling Ao near site (m)	1330	440	500
Far site (m)	1980	1620	1610

If the Ling Ao II cores are not operational, the effect of the uncertainty in the other 4 reactor fluxes would be, $\delta_{\sin^2 2\theta_{13}} \approx 0.10\delta_{\overline{\phi}}/\overline{\phi}$, assuming that all the weights w_i are 1/8.

The uncertainty (31) is small, but not negligible. It can be reduced by use of an appropriate set of weights w_i for the various detectors. A simple choice is that $w_i = w_{\rm DB}$ for detectors 1 = 1-4 whose near location is the Daya Bay near site, and that $w_i = w_{\rm LA}$ for detectors 1 = 5-8 whose near location is the Ling Ao site. The weights that yield the minimum uncertainty in $\sin^2 2\theta_{13}$ due to the uncertainty in the reactor fluxes when 6 cores are operational are $w_{\rm DB} = 0.066$ and $w_{\rm LA} = 0.183$,

$$\delta_{\sin^2 2\theta_{13}} \approx \frac{\delta_{\overline{\phi}}}{\overline{\phi}} \sqrt{\sum_{j=1}^{6} \left(\sum_{i=1}^{8} w_i A_{ij} \right)^2} \approx 0.08 \frac{\delta_{\overline{\phi}}}{\overline{\phi}} \qquad (w_{\rm DB} = 0.066,\ w_{\rm LA} = 0.183). \tag{32}$$

The use of weights other than 1/8 increasing the statistical uncertainty in the measurement of $\sin^2 2\theta_{13}$ according to eq. (10). Using the weights that minimize the effect of uncertainty in the reactor flux, the statistical uncertainty increases by 10% to,

$$\delta_{\sin^2 2\theta_{13}} \approx 0.0019 \qquad (\text{statistical},\ w_{\rm DB} = 0.066,\ w_{\rm LA} = 0.183). \tag{33}$$

If the Ling Ao II cores are not operational, the use of weights $w_{\rm DB} = 0.101$ and $w_{\rm LA} = 0.149$ reduces the effect of the uncertainty in the reactor fluxes essentially to zero, with an accompanying increase in the statistical uncertainty in the measurement of $\sin^2 2\theta_{13}$ of only 2%

Studies of the measured neutrino spectra in the different near detectors during different parts of the reactor fuel cycle can help provide additional constraints on the fuel cycle effects on the spectrum.

The Bugey experiment [8] claimed an uncertainty of $\delta_{\overline{\phi}}/\overline{\phi} = 1.4\%$ in their knowledge of the reactor flux, and the CHOOZ experiment [9] claimed an uncertainty of only 0.6%. Here, we suppose that the typical uncertainty in the Daya Bay reactor flux will be 2%.

[1]The analysis up to this point closely parallels that by Suekane [6]. See also [7].

Table 3: The effect of reactor-related uncertainties on the measurement of $\sin^2 2\theta_{13}$ for different reactor configurations. The uncorrelated uncertainty in the antineutrino flux from a single core is assumed to be $\delta_{\overline{\phi}}/\overline{\phi} = 2\%$. The locations of the reactor cores are assumed to be known to 0.3 m.

Number of cores	$w_{\rm DB}$	$w_{\rm LA}$	$\delta_{\sin^2 2\theta_{13}}$(power)	$\delta_{\sin^2 2\theta_{13}}$(location)	$\delta_{\sin^2 2\theta_{13}}$(total)
4	0.101	0.149	0.0%	0.14%	0.14%
6	0.066	0.183	0.16%	0.11%	0.18%

8.2 Location Uncertainties

To assess the effect on the uncertainties $\delta_{L_{i,j}}$ on the reactor fluxes, we recall eq. (25) and take the derivative,

$$\frac{\partial \rho_i}{\partial L_{i,j,\rm Far}} = -\frac{2\rho_i \overline{\phi}_j}{L^3_{i,j,\rm Far}\sum_{j'=1}^6 \frac{\overline{\phi}_{j'}}{L^2_{i,j',\rm Far}}} \approx -\frac{2\rho_i}{L^3_{i,j,\rm Far}\sum_{j'=1}^6 \frac{1}{L^2_{i,j',\rm Far}}} \equiv \rho_i B_{ij,\rm Far}, \tag{34}$$

and similarly,

$$\frac{\partial \rho_i}{\partial L_{i,j,\rm Near}} = \frac{2\rho_i \overline{\phi}_j}{L^3_{i,j,\rm Near}\sum_{j'=1}^6 \frac{\overline{\phi}_{j'}}{L^2_{i,j',\rm Near}}} \approx \frac{2\rho_i}{L^3_{i,j,\rm Near}\sum_{j'=1}^6 \frac{1}{L^2_{i,j',\rm Near}}} \equiv \rho_i B_{ij,\rm Near}, \tag{35}$$

where we suppose that all 6 reactor fluxes $\overline{\phi}_j$ are close to a common value ϕ.

The dependence of the measurement (25) of $\sin^2 2\theta_{13}$ on the ratios ρ_i is such that the contribution to its uncertainty due to the uncertainty in the distances $L_{i,j}$ is,

$$\begin{aligned}
\delta_{\sin^2 2\theta_{13}} &= \sqrt{\sum_{i=1}^8\sum_{j=1}^6 \left(w_i \frac{\partial \rho_i}{\partial L_{i,j,\rm Far}}\delta_{L_{i,j,\rm Far}}\frac{N_i^{\rm Far}}{N_i^{\rm Near}}\right)^2 + \left(w_i \frac{\partial \rho_i}{\partial L_{i,j,\rm Near}}\delta_{L_{i,j,\rm Near}}\frac{N_i^{\rm Far}}{N_i^{\rm Near}}\right)^2} \\
&\approx \sqrt{\sum_{i=1}^8\sum_{j=1}^6 \left(w_i B_{ij,\rm Far}\delta_{L_{i,j,\rm Far}}\rho_i\frac{N_i^{\rm Far}}{N_i^{\rm Near}}\right)^2 + \left(w_i B_{ij,\rm Near}\delta_{L_{i,j,\rm Near}}\rho_i\frac{N_i^{\rm Far}}{N_i^{\rm Near}}\right)^2} \\
&\approx \sqrt{\sum_{i=1}^8\sum_{j=1}^6 \left(w_i B_{ij,\rm Far}\delta_{L_{i,j,\rm Far}}\right)^2 + \left(w_i B_{ij,\rm Near}\delta_{L_{i,j,\rm Near}}\right)^2} \\
&\approx \delta_L\sqrt{\sum_{i=1}^8\sum_{j=1}^6 (w_i B_{ij,\rm Far})^2 + (w_i B_{ij,\rm Near})^2},
\end{aligned} \tag{36}$$

assuming that the uncertainty in all distances is that same, namely δ_L.

The location of the reactor cores will be determined to a precision of about 30 cm. We assume that the location *uncertainties* $\delta_{L_{i,j}}$ are uncorrelated. The resulting *uncertainty* in *the measurement of* $\sin^2 2\theta_{13}$ is shown in Table 8.1.

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