On the Calibration of the Antineutrino Detectors

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Abstract

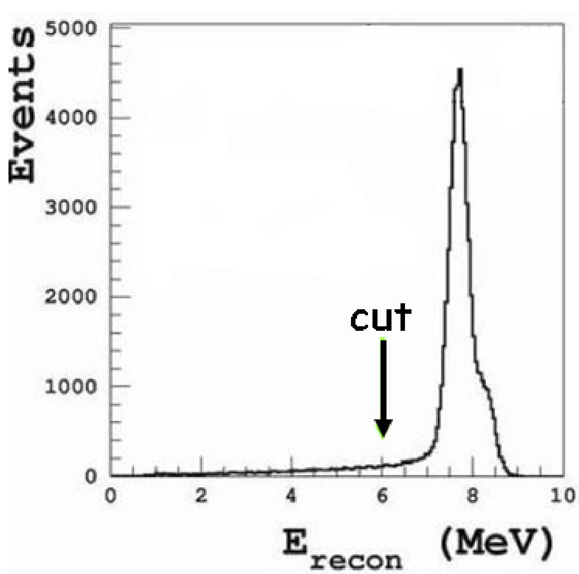
The requirement that the energy scale of scintillation in the antineutrino detectors be known to 1% at 6 MeV leads to a requirement that the "gain" of each of the 224 PMTs in a detector be known to 10%. This surprisingly modest requirement could be meet by a calibration based on as few as 40 thermal neutron captures from reactor antineutrino interactions,permitting continual recalibration of the detectors every 10-12 hours. Occasional source-based calibrations will provide an important check on the antineutrino calibrations.

1 How Well Do We Need to Know the PMT Gains?

In the Daya Bay Reactor Neutrino experiment, $\overline{\nu}_e$'s of energy $\approx$ 1-10 MeV will be detected via the inverse β-decay reaction,

$$\overline{\nu}_e + p \to n + e^+, \tag{1}$$

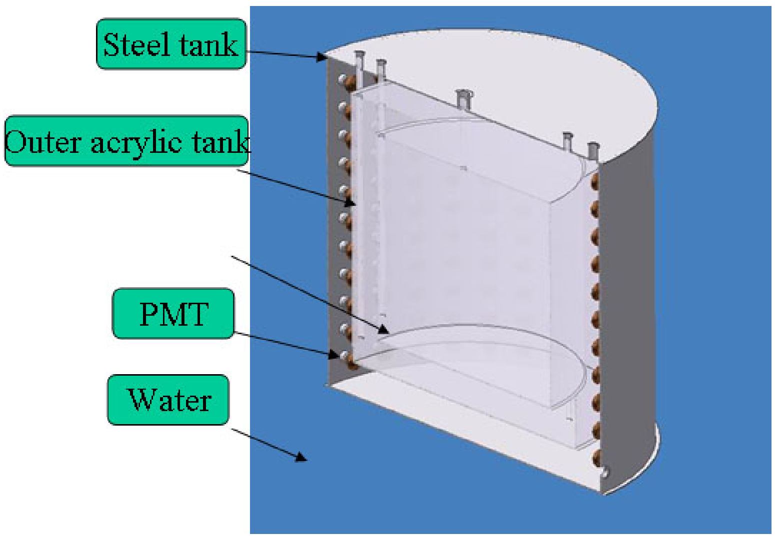
on free protons in Gd-loaded liquid scintillator in 8 antineutrino detectors. The energy $E_{\overline{\nu}}$ of the antineutrino is inferred from the observed scintillation energy $E_{e^+} > 1.022$ MeV of the positron and its decay products after annihilation. The signature of inverse β-decay is confirmed by the observation of the delayed capture of the (thermalized) neutron on a Gd nucleus, which releases $\approx$ 8 MeV energy in the form of 3-4 γ's. A simulated spectrum of the scintillation energy observed following a neutron capture is shown below. The tail to low energies is due to events in which one or more of the γ's from the neutron capture deposits energy outside the active scintillation volume of the detector.

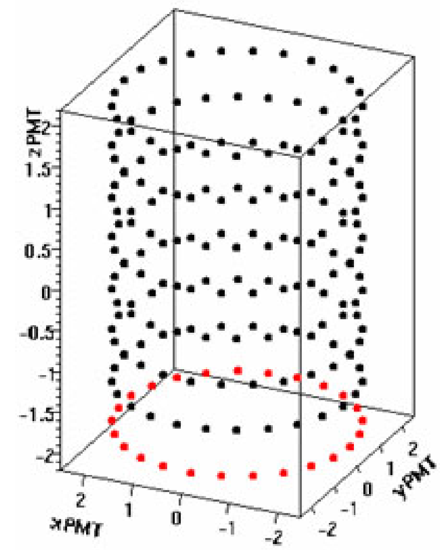


The strongest requirement on the calibration of the scintillation process is that the energy scale at 6 MeV be known to an accuracy of 1% so that the effect of a cut at this energy on the efficiency of neutron detection is less than 0.2%.

Each antineutrino detector will have $N = 224$ $8''$ photomultiplier tubes (PMTs), arrayed in 7 rings of 32 PMTs on the inside of the vertical surface of the detector tank, as sketched below. The effective photocathode coverage is about 12% of the surface area of the detectors, resulting in about 75 photoelectrons per MeV of deposited energy. The number n of photoelectrons per PMT is,

$$n \approx \frac{75}{224} E \approx \frac{E(\text{MeV})}{3}. \tag{2}$$





We label the PMTs by the index i which runs from 1 to $N = 224$. The number of photoelectrons observed (during some appropriately narrow time window) in PMT i is denoted n_i, and the "gain" of PMT i in units of MeV per photoelectron is G_i.[1] The signal S in MeV from a scintillation process such as neutron capture is,

$$S = \sum_{i=1}^{N} G_i n_i \approx G \sum_{i=1}^{N} n_i \approx \frac{GNE(\text{MeV})}{3} \approx 75GE(\text{MeV}), \tag{3}$$

where G is the typical value of the PMT gains G_i, and we use eq. (2) to approximate n_i. The root-mean-square uncertainty σ_S on the signal (3) due to the uncertainties σ_{G_i} in the PMT gains and due to the fluctuations in the numbers n_i of observed photoelectrons from a given energy deposition E is related by,

$$\begin{aligned} \sigma_S &= \sqrt{\sum_{i=1}^{N} [\sigma_{G_i}^2 n_i^2 + G_i^2 \sigma_{n_i}^2]} \approx \sqrt{\sigma_G^2 \sum_{i=1}^{N} n_i^2 + G^2 \sum_{i=1}^{N} n_i} \\ &\approx \sqrt{\sigma_G^2 N \left(\frac{E^2(\text{MeV})}{9} + \frac{E(\text{MeV})}{3} \right) + \frac{G^2 NE(\text{MeV})}{3}}, \end{aligned} \tag{4}$$

[1] In more detail, a PMT "multiplies" each photoelectron by a factor of $g_i \approx 10^6$ so that the observed charge is $g_i n_i e$, where e is the charge of an electron. Each PMT must be associated with a calibration constant k_i in units of MeV per electron charge, such that $G_i = k_i g_i$. In practice, there is no need to know g_i and k_i separately, so we discuss only the single "gain" factor G_i.

where σ_G is the typical uncertainty in the PMT gains G_i, and we use the estimate $\sigma_{n_i} = \sqrt{n_i}$ for the fluctuations in the number of observed photoelectrons.[2] The relative uncertainty in the signal S due only to the uncertainty σ_G in the PMT gains is,

$$\left.\frac{\sigma_S}{S}\right|_{\rm Gain} = \frac{\sqrt{\sum_{i=1}^N \sigma_{G_i}^2 n_i^2}}{\sum_{i=1}^N G_i n_i} \approx \frac{1}{\sqrt{N}}\frac{\sigma_G}{G}\sqrt{1+\frac{3}{E({\rm MeV})}} \approx 0.08\frac{\sigma_G}{G}\,, \tag{5}$$

while the relative uncertainty in S due to fluctuations in the numbers of photoelectrons is,

$$\left.\frac{\sigma_S}{S}\right|_{\rm statistics} = \frac{\sqrt{\sum_{i=1}^N G_i^2 \sigma_{n_i}^2}}{\sum_{i=1}^N G_i n_i} \approx \sqrt{\frac{3}{NE({\rm MeV})}} \approx \frac{0.12}{\sqrt{E({\rm MeV})}}\,. \tag{6}$$

The energy resolution of the detector is $12\%/\sqrt{E({\rm MeV})}$ according to eq. (6), assuming that the gains of the PMTs are well known. For example, at the 8-MeV peak in the scintillation spectrum from neutron capture on Gd, the energy resolution can be $\sigma_S/S = 0.042$, provided that the uncertainty (5) due to uncertainty in the PMT gains is small compared to this. However, this consideration leads only to the weak requirement that $\sigma_G/G \lesssim 0.5$.

A stronger requirement is that the uncertainty in the scintillation energy scale be less than 1% at the 6-MeV cut on the neutron-capture spectrum shown on p. 1. From eq. (5), we deduce the requirement that,

$$\frac{\sigma_G}{G} \lesssim 0.1 \tag{7}$$

for each of the $8 \times 224 = 1792$ PMTs in the antineutrino detectors.[3]

2 PMT Calibration via Neutron Capture on Gd Nuclei

One way to calibrate the PMT gains G_i is via observation of thermal neutron capture on Gd nuclei in the liquid scintillator, either from localized neutrons sources, of from the uniform spatial distribution of inverse β-decay due to reactor antineutrinos.

Either way, we must have additional knowledge of the expected spatial variation of the signal $S_i(\mathbf{x})$ in PMT i due to a neutron emitted (or created) at location $\mathbf{x}$, such that the total signal,

$$S = \sum_{i=1}^N S_i(\mathbf{x}), \tag{8}$$

is independent of $\mathbf{x}$. The design of the antineutrino detector includes an oil buffer around the PMTs so that the expected signal $S_i(\mathbf{x})$ in PMT i is a weak function of $\mathbf{x}$, and we can rely on a Monte Carlo simulation for a calculation of $S_i(\mathbf{x})$.

[2]Equation (5) includes the approximation that $\sum_{i=1}^N n_i^2 = N\langle n_i^2\rangle = N(\langle n_i\rangle^2 + \sigma_{n_i}^2) \approx N(\langle n_i\rangle^2 + \langle n_i\rangle)$. Thanks to L. Littenberg for pointing this out.

[3]The requirement (7) is based on the assumption that the signal is roughly equal in all 224 PMTs of a detector. For a neutron capture close to a particular PMT, the signal in that PMT will be larger than that of eq. (2) by a factor k. In the approximation that this PMT is the only one whose signal departs from eq. (2), the resulting modification to eq. (5) is $(\sigma_S/S)_{\rm Gain} \approx (1/\sqrt{N})(\sigma_G/G)(1+k^2/2N)$. For example, $k = 5$ results in a requirement that is 5% stronger than eq. (7).

If reactor antineutrinos are used as the source of thermal neutron captures, then the location $\mathbf{x}$ of the capture must first be reconstructed from the PMT data before the $S_i(\mathbf{x})$ can be calculated. Simulations indicate that the spatial resolution in the reconstruction of the location of the neutron capture will be $\sigma_x \approx 15$ cm, which is small compared to the length scale of variation of $S_i(\mathbf{x})$.

Suppose that a total of M neutron captures are observed at known or reconstructed positions $\mathbf{x}_j,\ j = 1, ...M$, and that $S_{ij} = S_i(\mathbf{x}_j)$ are the expected energy signals in PMT i. The actual signal associated with PMT i is $G_i n_{ij}$, where n_{ij} is the number of photoelectrons observed in PMT i during event j. To extract the gains G_i from this data we can form a χ^2 for each PMT,

$$\chi_i^2 = \sum_{j=1}^{M} \frac{(S_{ij} - G_i n_{ij})^2}{\sigma_{ij}^2} = \sum_{j=1}^{M} \frac{(S_{ij} - G_i n_{ij})^2}{G_i^2 n_{ij}} = \frac{1}{G_i^2} \sum_{j=1}^{M} \frac{S_{ij}^2}{n_{ij}} - \frac{2}{G_i} \sum_{j=1}^{M} S_{ij} + \sum_{j=1}^{M} n_{ij}. \tag{9}$$

The needed derivatives are,

$$\frac{d\chi_i^2/2}{dG_i} = -\frac{1}{G_i^3} \sum_{j=1}^{M} \frac{S_{ij}^2}{n_{ij}} + \frac{1}{G_i^2} \sum_{j=1}^{M} S_{ij}, \tag{10}$$

and,

$$\frac{d^2\chi_i^2/2}{dG_i^2} = \frac{3}{G_i^4} \sum_{j=1}^{M} \frac{S_{ij}^2}{n_{ij}} - \frac{2}{G_i^3} \sum_{j=1}^{M} S_{ij}. \tag{11}$$

Setting the first derivative, eq. (10), to zero, we obtain the best estimate of the gain G_i,

$$G_i = \frac{\sum_{j=1}^{M} \frac{S_{ij}^2}{n_{ij}}}{\sum_{j=1}^{M} S_{ij}}. \tag{12}$$

Using the fit value for G_i in eq. (11), we obtain the estimate of the uncertainty σ_{G_i},

$$\frac{1}{\sigma_{G_i}^2} = \frac{d^2\chi_i^2/2}{dG_i^2} = \frac{(\sum_{j=1}^{M} S_{ij})^4}{\left(\sum_{j=1}^{M} \frac{S_{ij}^2}{n_{ij}}\right)^3}, \tag{13}$$

so that,

$$\frac{\sigma_{G_i}}{G_i} = \frac{\sqrt{\sum_{j=1}^{M} \frac{S_{ij}^2}{n_{ij}}}}{\sum_{j=1}^{M} S_{ij}} \approx \frac{1}{\sqrt{M n_i}} \approx \sqrt{\frac{3}{M E(\mathrm{MeV})}} = \frac{0.61}{\sqrt{M}}, \tag{14}$$

where the last form holds for thermal neutron capture on Gd with $E = 8$ MeV. A calibration of the gains to accuracy $\sigma_{G_i}/G_i = 0.1$ can be obtained with only $M = 37$ events.

The expected rate of neutron captures from antineutrino interactions in a single detector module at the Far site is 90 per day. Hence, a calibration of the gains of all 224 PMTs in each module can be obtained every 10 hours to the desired accuracy of $\sigma_{G_i}/G_i = 0.1$. This calibration depends on the reconstruction of the position $\mathbf{x}_j$ of each neutron capture, and reliable modeling of the spatial dependence $S_{ij} = S_i(\mathbf{x}_j)$ of the expected signal in PMT i. Also, the data used in this calibration will include a few percent of background events with nearly flat energy distribution in the needed window around the 8-MeV neutron-capture peak.

We can also perform occasional calibrations in which a neutron source is lowered into the detector and placed at a fixed location. These calibrations will have a much smaller background component than those based on reactor antineutrinos, as well as a different dependence on the model for the PMT signals $S_i(\mathbf{x}_j)$, and will provide a cross check on the continual energy calibration based on the antineutrino data itself.