Comment on "Experimental Observation of Electrons Accelerated in Vacuum to Relativistic Energies by a High-Energy Laser" by G. Malka, E. Lefebvre and J.L. Miquel, Phys. Rev. Lett. 78, 3314 (1997)

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
(August 22, 1997, Version 1.1)

The interpretation of the results of this paper were based on electromagnetic fields that do not satisfy Maxwell's equations and neglect the small but important longitudinal electric field near a focus. The intricate question of vacuum laser acceleration of free electrons deserves more rigorous analysis.

The experimental results presented by Malka *et al.* address an issue that has been debated with vigor and controversy for decades: under what conditions can a free electron extract energy from an electromagnetic field that consists entirely of waves that obey the vacuum dispersion relation $\omega = kc$? Here ω is the angular frequency and k is the wave number of the wave and c is the speed of light. The only previous experimental result for the interaction of a laser pulse with a free electron (one not ionized from an atom by the same laser pulse) showed an electron-energy gain of at most 0.2 eV [1], compared to the 900-keV gain reported by Malka *et al.*

The interpretation by Malka *et al.* of their results is called into question by the claimed agreement with a theory based on their eq. (1) which does not satisfy the free-space Maxwell equation $\nabla \cdot \mathbf{E} = 0$, and which is known to overestimate greatly the possible energy transfer between an electron and a laser beam due to neglect of the longitudinal electric field near a focus [2]. While third-order [3] and fifth-order [4] approximations to Gaussian laser beams are available, no Gaussian approximation reproduces such classic features as the Airy rings in the focal plane of a wave focused through a circular aperture (in contrast to approaches utilizing integrals of Bessel functions [5]) and hence a Gaussian approximation may never provide a sufficiently accurate model of the electromagnetic fields relevant to vacuum laser acceleration.

As is well known, a free electron cannot gain energy simply by absorbing photons, but only by scattering [6,7]. The oscillation of an electron in a wave is argued to be the result of a kind of stimulated scattering process in which photons scatter from one occupied laser mode to another, generally resulting in momentum transfer to the electron [1]. However, if the laser pulse is 'long' in time then the oscillatory motion can be described by the (conservative) ponderomotive potential $U = mc^2\sqrt{1+\eta^2}$, and no net energy transfer occurs for an electron that is free both before and after it encounters the pulse [8]. In the above, the dimensionless, invariant parameter η equals $e\sqrt{\langle A_\mu A^\mu \rangle}/mc^2 = eE_{\rm rms}/m\omega c$ where the wave has frequency ω, root-mean-square electric field $E_{\rm rms}$, and four-vector potential A_μ; e and m are the charge and mass of the electron, respectively. (Parameter η is $1/\sqrt{2}$ times the parameter a of Malka *et al.*).

However, for 'short' linearly polarized laser pulses it is argued that the large, oscillatory energy transfers that occur between the electron and wave in each cycle can result in small net energy transfer per cycle, and in large net energy transfer during the complete interaction [1]. The laser pulse of Malka *et al.* may be 'short' in this sense.

In contrast, the so-called Lawson-Woodward theorem [9–11] states that when the superposition principle holds, *i.e.*, for relativistic electrons in which case the $\mathbf{v}/c \times \mathbf{B}$ force is effectively linear, free electrons cannot extract energy from interaction with waves each of which obeys the dispersion relation $\omega = kc$. This argument does not preclude an electron being accelerated from nonrelativistic energies to energy of order mc^2, as reported by Malka *et al.* It does suggest that an electron with MeV-scale kinetic energy would gain very little energy in additional encounters with similar laser pulses.

There seems to be no convincing, detailed analysis (one that obeys Maxwell's equations) as to the possibility of acceleration of nonrelativistic free electrons by intense laser pulses, *i.e.*, ones with $\eta \gtrsim 1$. The experimental results of Malka *et al.* should stimulate interest in vacuum laser acceleration, but their theoretical analysis should not be accepted uncritically.

[1] P.H. Bucksbaum *et al.*, Phys. Rev. Lett. **58**, 349 (1987).
[2] E. Esarey *et al.*, Phys. Rev. E**52**, 5443 (1995); P. Sprangle *et al.*, Opt. Commun. **124**, 69 (1996).
[3] L.W. Davis, Phys. Rev. A**19**, 1177 (1979).
[4] J.P. Barton and D.R. Alexander, J. Appl. Phys. **66**, 2800 (1989).
[5] A. Boivin and E. Wolf, Phys. Rev. **138**, B1561 (1965).
[6] A. Gover and A. Yariv, Appl. Phys. **16**, 121 (1979).
[7] R.H. Pantell, *Free-electron Generation of Coherent Radiation*, ed. by S.F. Jacobs *et al.* (Addison-Wesley, Reading, MA, 1980), p.1.
[8] T.W.B. Kibble, Phys. Rev. **138**, B740, (1965); Phys. Lett. **20**, 627 (1966); Phys. Rev. Lett. **16**, 1054 (1966); Phys. Rev. **150**, 1060 (1966); *Cargèse Lectures in Physics*, Vol. 2, ed. by M. Lévy (Gordon and Breach, New York, 1968), p. 299.
[9] J.A. Edighoffer and R.H. Pantell, J. Appl. Phys. **50**, 6120 (1979),
[10] R.B. Palmer, Part. Accel. **11**, 81 (1980); A.I.P. Conf. Proc. **335**, 90 (1994).
[11] E.J. Bochove *et al.*, Phys. Rev. A**46**, 6640 (1992).