The interaction region of high energy protons

Igor M. Dremin[1,2], **Sebastian N. White**[3]

[1]Lebedev Physics Institute, Moscow 119991, Russia

[2]National Research Nuclear University "MEPhI", Moscow 115409, Russia

[3]CERN/Princeton University

Abstract

The spatial view of the interaction region of colliding high energy protons is considered. It is shown that the region of inelastic collisions has a very peculiar shape. It saturates for central collisions at an energy of 7 TeV. Its further evolution with energy is speculated and the assumption about the black disk shape is discussed.

1 Introduction

The search ever deeper into the interior of matter successfully started by Rutherford's discovery of atomic structure is going on now at much lower scales of order 10^{-13} cm at high energy accelerators. The interaction region of colliding protons can be quantitatively explored with the help of the unitarity condition if experimental data about their elastic scattering are used. With only these two ingredients at hand we are able to show that the energy evolution of inelastic interaction region demonstrates quite surprising features.

2 The unitarity condition

From the theoretical side, the most reliable information comes from the unitarity condition. The unitarity of the S-matrix SS^+=1 relates the amplitude of elastic scattering $f(s,t)$ to the amplitudes of inelastic processes M_n. In the s-channel they are subject to the integral relation (for more details see, e.g., [1, 2, 3]) which can be written symbolically as

$$\mathrm{Im}f(s,t) = I_2(s,t) + g(s,t) = \int d\Phi_2 f f^* + \sum_n \int d\Phi_n M_n M_n^*. \qquad (1)$$

The variables s and t are the squared energy and transferred momentum of colliding protons in the center of mass system $s = 4E^2 = 4(p^2 + m^2)$, $-t = 2p^2(1 - \cos\theta)$ at the scattering angle θ. The non-linear integral term represents the two-particle intermediate states of the incoming particles. The second term represents the shadowing contribution of inelastic processes to the imaginary part of the elastic scattering amplitude. Following [4] it is called the overlap function. This terminology is ascribed to it because the integral there defines the overlap within the corresponding phase space $d\Phi_n$ between the matrix element M_n of the n-th inelastic channel and its conjugated counterpart with the collision axis of initial particles deflected by an angle θ in proton elastic scattering. It is positive at $\theta = 0$ but can change sign at $\theta \neq 0$ due to the relative phases of inelastic matrix elements M_n's.

At $t = 0$ it leads to the optical theorem

$$\mathrm{Im}f(s,0) = \sigma_{tot}/4\sqrt{\pi} \tag{2}$$

and to the general statement that the total cross section is the sum of cross sections of elastic and inelastic processes

$$\sigma_{tot} = \sigma_{el} + \sigma_{in}, \tag{3}$$

i.e., that the total probability of all processes is equal to one.

3 The geometry of the interaction region

Here, we show that it is possible to study the space structure of the interaction region of colliding protons using the information about their elastic scattering within the unitarity condition. The whole procedure is simplified because in the space representation one gets the algebraic relation between the elastic and inelastic contributions to the unitarity condition in place of the more complicated non-linear integral term I_2 in Eq. (1).

To define the geometry of the collision we must express all characteristics presented by the angle θ and the transferred momentum t in terms of the transverse distance between the trajectories of the centers of the colliding protons called the impact parameter b. This is easily carried out using the Fourier – Bessel transform of the amplitude f which retranslates the momentum data to the transverse space features and is written as

$$i\Gamma(s,b) = \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| f(s,t) J_0(b\sqrt{|t|}). \tag{4}$$

The unitarity condition in the b-representation reads

$$G(s,b) = 2\mathrm{Re}\Gamma(s,b) - |\Gamma(s,b)|^2. \tag{5}$$

The left-hand side (the overlap function in the b-representation) describes the transverse impact-parameter profile of inelastic collisions of protons. It is just the Fourier – Bessel transform of the overlap function g. It satisfies the inequalities $0 \leq G(s,b) \leq 1$ and determines how absorptive the interaction region is depending on the impact parameter (with $G = 1$ for full absorption and $G = 0$ for the complete transparency). The profile of elastic processes is determined by the subtrahend in Eq. (5). If $G(s,b)$ is integrated over the impact parameter, it leads to the cross section of inelastic processes. The terms on the right-hand side would produce the total cross section and the elastic cross section, correspondingly, as it should be according to Eq. (3). The overlap function is often discussed in relation with the opacity (or the eikonal phase) $\Omega(s,b)$ such that $G(s,b) = 1 - \exp(-\Omega(s,b))$. Thus, full absorption corresponds to $\Omega = \infty$ and complete transparency to $\Omega = 0$.

The most prominent feature of elastic scattering is the rapid decrease of the differential cross section with increasing transferred momentum $|t|$ in the diffraction peak. As a first approximation at present energies, it can be described by the exponential shape with the slope $B(s)$:

$$\frac{d\sigma}{dt} = \frac{\sigma_{tot}^2}{16\pi} \exp(-B(s)|t|). \tag{6}$$

The diffraction cone contributes predominantly to the Fourier - Bessel transform of the amplitude. Using the above formulae, one can write the dimensionless Γ as

$$i\Gamma(s,b) = \frac{\sigma_t}{8\pi} \int_0^\infty d|t| \exp(-B|t|/2)(i+\rho) J_0(b\sqrt{|t|}). \tag{7}$$

Here, the diffraction cone approximation (6) is inserted. Herefrom, one calculates

$$\mathrm{Re}\Gamma(s,b) = \zeta \exp(-\frac{b^2}{2B}), \tag{8}$$

where we introduce the dimensionless ratio of the cone slope (or the elastic cross section) to the total cross section

$$\zeta = \frac{\sigma_{tot}}{4\pi B} \approx \frac{4\sigma_{el}}{\sigma_{tot}}. \tag{9}$$

Table. The energy behavior of ζ and $G(s,0)$.

$\sqrt{s}$, GeV	2.70	4.11	4.74	7.62	13.8	62.5	546	1800	7000
ζ	1.56	0.98	0.92	0.75	0.69	0.67	0.83	0.93	1.00
$G(s,0)$	0.68	1.00	0.993	0.94	0.904	0.89	0.97	0.995	1.00

The ratio σ_{el}/σ_{tot} defines the survival probability of initial protons. The approximation sign refers to the neglected factor $1+\rho^2$ where ρ is the ratio of the real to imaginary part of the amplitude in the diffraction cone. In what follows we neglect ρ according to experimental data (with $\rho(7\ TeV,0)\approx 0.12$) and theoretical considerations which favor its decrease inside the diffraction cone. Thus one gets

$$G(s,b)=\zeta\exp(-\frac{b^2}{2B})[2-\zeta\exp(-\frac{b^2}{2B})]. \tag{10}$$

The inelastic profile scales as a function of $b/\sqrt{2B}$. For central collisions with $b=0$ one gets

$$G(s,b=0)=\zeta(2-\zeta). \tag{11}$$

This formula is very significant because it follows herefrom that the darkness at the very center is fully determined by the only one parameter ζ, i.e. by the ratio of experimentally measured characteristics - the width of the diffraction cone B (or σ_{el}) to the total cross section. Their energy evolution defines the evolution of the absorption value. The interaction region becomes completely absorptive $G(s,0)=1$ in the center only at $\zeta=1$ and the absorption diminishes for other values of ζ. However for small variations of $\zeta=1\pm\epsilon$ the value of $G(s,0)=1-\epsilon^2$ varies even less.

In the Table, we show the energy evolution of ζ and $G(s,0)$ for pp and $p\bar{p}$ scattering as calculated from experimental data about the total cross section and the diffraction cone slope at corresponding energies. Let us point out that starting from ISR energies the value of ζ increases systematically and at LHC energies becomes equal to 1 within the accuracy of measurements of B and σ_{tot}.

The impact parameter distribution of $G(s,b)$ (10) has its maximum at $b_m^2=2B\ln\zeta$ with full absorption $G(b_m)=1$. Its position depends both on B and ζ.

Note, that, for $\zeta<1$, one gets incomplete absorption $G(s,b)<1$ at any

physical $b \geq 0$ with the largest value reached at $b = 0$ because the maximum appears at non-physical values of b. The disk is semi-transparent.

At $\zeta = 1$, the maximum is positioned exactly at $b = 0$, and the absorption is absolutely strong there $G(s, 0) = 1$. The disk center becomes impenetrable (black). The strongly absorptive core of the inelastic interaction region grows in size as we see from expansion of Eq. (10) at small impact parameters:

$$G(s, b) = \zeta[2 - \zeta - \frac{b^2}{B}(1 - \zeta) - \frac{b^4}{4B^2}(2\zeta - 1)]. \tag{12}$$

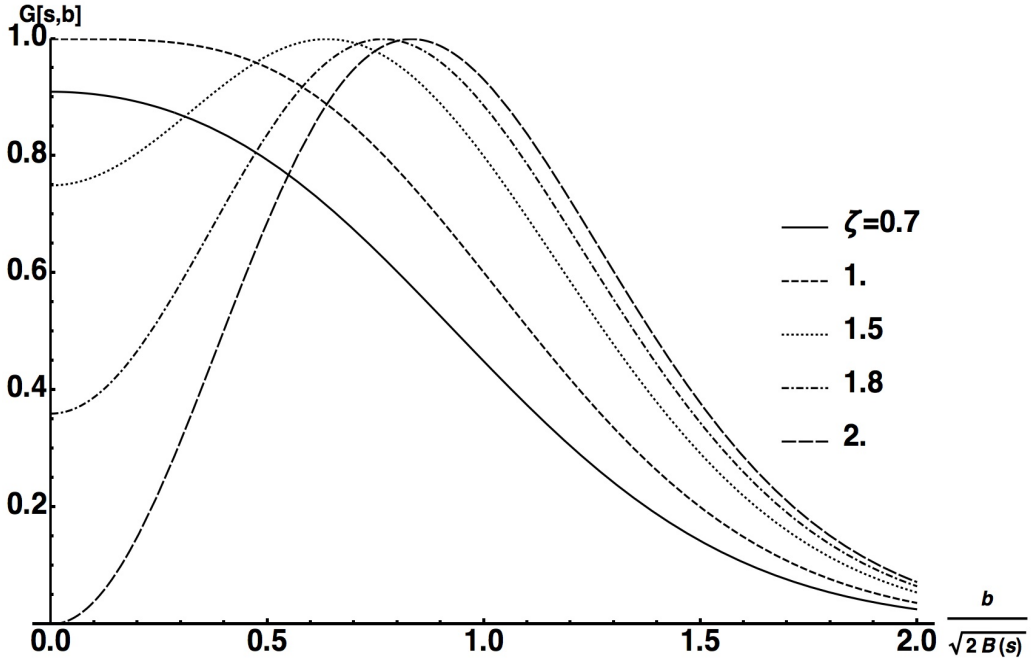
The second term proportional to b^2 vanishes at $\zeta = 1$, and $G(b)$ develops a plateau which extends to quite large values of b about 0.5 fm. The plateau is very flat because the third term starts to play a role at 7 TeV (where $B \approx 20$ GeV^{-2}) only for larger values of b.

At $\zeta > 1$, the maximum shifts to positive physical impact parameters. A dip is formed at b=0 leading to the concave shaped inelastic interaction region -approaching a toroidal shape. This dip becomes deeper at larger ζ. The limiting value $\zeta = 2$ leads to complete transparency at the center $b = 0$.

All these cases are demonstrated in Fig. 1 where $G(s, b)$ is plotted as a function of the scaling variable $b/\sqrt{2B}$ for different values of the parameter ζ according to Eq. (10). The line with $\zeta = 0.7$ corresponds to ISR results and with $\zeta = 1$ to LHC. Earlier it was shown that the results of analytical calculations according to (10) and the computation with experimental data directly inserted in the unitarity condition practically coincide (see Fig. 1 in [8]). The line with $\zeta = 1.5$ describes the shape of the inelastic interaction region according to asymptotic expectations predicted in [5, 6] where the successful fits to present experimental data are reported. The new proposal of [7] is shown at $\zeta = 1.8$. The dip increases at larger ζ and reaches the very bottom for $\zeta = 2$. Strangely enough this situation with $\sigma_{el} = \sigma_{inel} = 0.5\sigma_{tot}$ is usually referred to as the "black disk" limit [9].

It is quite opposite to our expectations from electromagnetic interactions. Collisions of two billiard balls with extremely high energies would be completely inelastic splitting them into small pieces. However we know that strong interactions become weaker at large transferred momenta. They differ from electromagnetic forces by the property of asymptotic freedom. Nevertheless at low transferred momenta they are still strong. One could hardly ascribe the survival of elastic processes at central collisions (penetration of protons through each other) to this property because the parton collisions usually proceed with small transferred momenta and are strong.

Figure 1: The evolution of the inelastic interaction region in terms of the survival probability. The values $\zeta = 0.7$ and 1.0 correspond to ISR and LHC energies and agree well with the result of detailed fitting to the elastic scattering data [10]. A further increase of ζ leads to the toroid-like shape with a dip at $b = 0$. The values $\zeta = 1.5$ are proposed in [5, 6] and $\zeta = 1.8$ in [7] as corresponding to asymptotical regimes. The value $\zeta = 2$ corresponds to the "black disk" regime ($\sigma_{el} = \sigma_{inel} = 0.5\sigma_{tot}$).



Another possibility is that protons start reflecting their partners after becoming so dark at 7 TeV. This results in the elastic backward scattering of protons like backward scattering of billiard balls in head-on collisions. This is often referred to as the reflective mode [10].

In conclusion, we can state that, analyzing the unitarity condition, we have found the special role of the ratio of elastic to total cross sections being equal to 1/4 at 7 TeV pp-interactions. That could be attributed to the equal share of processes with exchange and no-exchange by quantum numbers in particle collisions if elastic processes contibute a half of this share. Another half would be attributed to inelastic diffraction processes. That would lead to the saturation of the Pumplin-Miettinen bound [13] which states that their sum is less or equal to 0.5 of the total cross section. Usually diffractive processes are defined by the separation of inelastic events with large rapidity gaps. According to CMS data [14] their sum does not saturate this limit leading to a gap cross section of about 15 mb at 7 TeV. That would require a no-gap diffractive cross section of about 10 mb. Unfortunately, there are no proposals how they can be separated from ordinary inelastic processes since both interfere within the overlap function. This quantum mechanical interference can also be at the origin of damping the blackness at low impact parameters.

Acknowledgments

I.D. is grateful for support by the RFBR-grant 14-02-00099 and the RAS-CERN program.

References

[1] PDG group, China Phys. C **38**, 090513 (2014).

[2] I.V. Andreev, I.M. Dremin, ZhETF Pis'ma **6** (1967) 810

[3] I.M. Dremin, Physics-Uspekhi **56** (2013) 3; **58** (2015) 61

[4] L. Van Hove, Nuovo Cimento **28** (1963) 798

[5] A.K. Kohara, E. Ferreira, T. Kodama, pp elastic scattering at LHC energies; arXiv:1408.1599

[6] D.A. Fagundes, M.J. Menon, P.V.R.G. Silva, Exploring central opacity and asymptotic scenarios in elastic hadron scattering; arXiv:1509.04108

[7] S.M. Roy, A two component picture for high energy scattering: unitarity, analitycity and LHC data, arXiv: hep-ph 1602.03627

[8] M.Yu. Azarkin, I.M. Dremin, M. Strikman, Jets in multiparticle production in and beyond geometry of proton-proton collisions at the LHC. Phys. Lett. B 735 (2014) 244; arXiv:1401.1973

[9] M.M. Bloch, F. Halzen, Experimental confirmation that the proton is asymptotically a black disk. Phys. Rev. Lett. 107 (2011) 212002

[10] S.M. Troshin, N.E. Tyurin, Phys. Lett. B **316** (1993) 175

[11] A. Alkin, E. Martynov, O. Kovalenko, S.M. Troshin, Phys. Rev. D **89** (2014) 091501(R)

[12] I.M. Dremin, V.A. Nechitailo, Nucl. Phys. A **916** (2013) 241.

[13] H.I. Miettinen, J. Pumplin, Phys. Rev. D 18 (1978) 1696

[14] CMS Collaboration, Phys. Rev. D 92 (2015) 012003