ATF rates from Scattering Foils

SNW, Jan '10

This is an update on scattering rates we would expect from practical foils in the ATF. If we take 80 MeV beam energy and 10^9 e/bunch as standard then we can get ~1 scatter per bunch with a plasic target ~2 mm thick(1.5 mm if it's tilted 45^0) at angles <~45 degrees. Similar rates with a Gold target 0.12 mm thick. Since a comfortable range is 10^8 to 10^{10}e/pulse maybe 45 degrees is the right choice.

In order to get 1 Hz at a 75^0angle it would take a ~ 7 mm thick plastic radiator tilted at 45^0. The emittance blowup might then be a problem.

We start from Hofstadter's derivation in RMP 28 #3 p 214 and his Nobel lecture(it turns out he also used 80 MeV electrons). The relativistic correction and contribution of electron spin is applied to the Rutherford form as given by Mott and then the form factors for different nuclei are included.

At the end we compare this calculation with Hofstadter's results at Stanford.

In[71]:=
```
<< PhysicalConstants`
<< Units`
Needs["PlotLegends`"]
```

- **Size parameters used by Hofstadter in exponential form : rms charge radius = $2*a\sqrt{3}$**

In[74]:=
```
a_Au = 2.3; a_C = 0.6;
Z_C = 6; Z_Au = 79;
(*M_p=938;
M_Au=197*M_p;
M_C=12*M_p;
M=M_C;*)
a = a_C; Z = Z_C;
α_EM = FineStructureConstant;
ħc = Convert[PlanckConstantReduced*SpeedOfLight,
    Mega*ElectronVolt*Fermi][[1]];
ħc1 = Convert[PlanckConstantReduced*SpeedOfLight,
    Mega*ElectronVolt*Centimeter][[1]];
```

In[80]:= $\text{Rutherford}[\theta\_, Z\_, \text{EeMeV}\_] := 1/4\,(Z*\alpha_{EM})^2\,\frac{\hbar c1^2}{\text{EeMeV}^2}\,\text{Csc}[\theta/2]^4$

In[81]:= $\text{Mott}[\theta\_, Z\_, \text{EeMeV}\_] := \text{Rutherford}[\theta, Z, \text{EeMeV}]*\text{Cos}[\theta/2]^2\left(1+\frac{\text{Pi}*Z*\alpha_{EM}*\text{Sin}[\theta/2]*(1-\text{Sin}[\theta/2])}{\text{Cos}[\theta/2]^2}\right)$

In[82]:= $Q[\theta\_, \text{EeMeV}\_] := \frac{\frac{2*\text{EeMeV}}{\hbar c}\,\text{Sin}[\theta/2]}{\sqrt{1+(2\,\text{EeMeV}/(197*938))\,\text{Sin}[\theta/2]^2}}$

In[83]:= $\rho[r\_, a\_] := \frac{1}{8\pi\,(a)^3}\,\text{Exp}[-r/a]$

- **Form Factor normalization. We require it->1 as q->0. Then we have 4 π/Q[θ]*ρ[r] Sin[Q[θ] r] r and Sin[qr]/q->r**

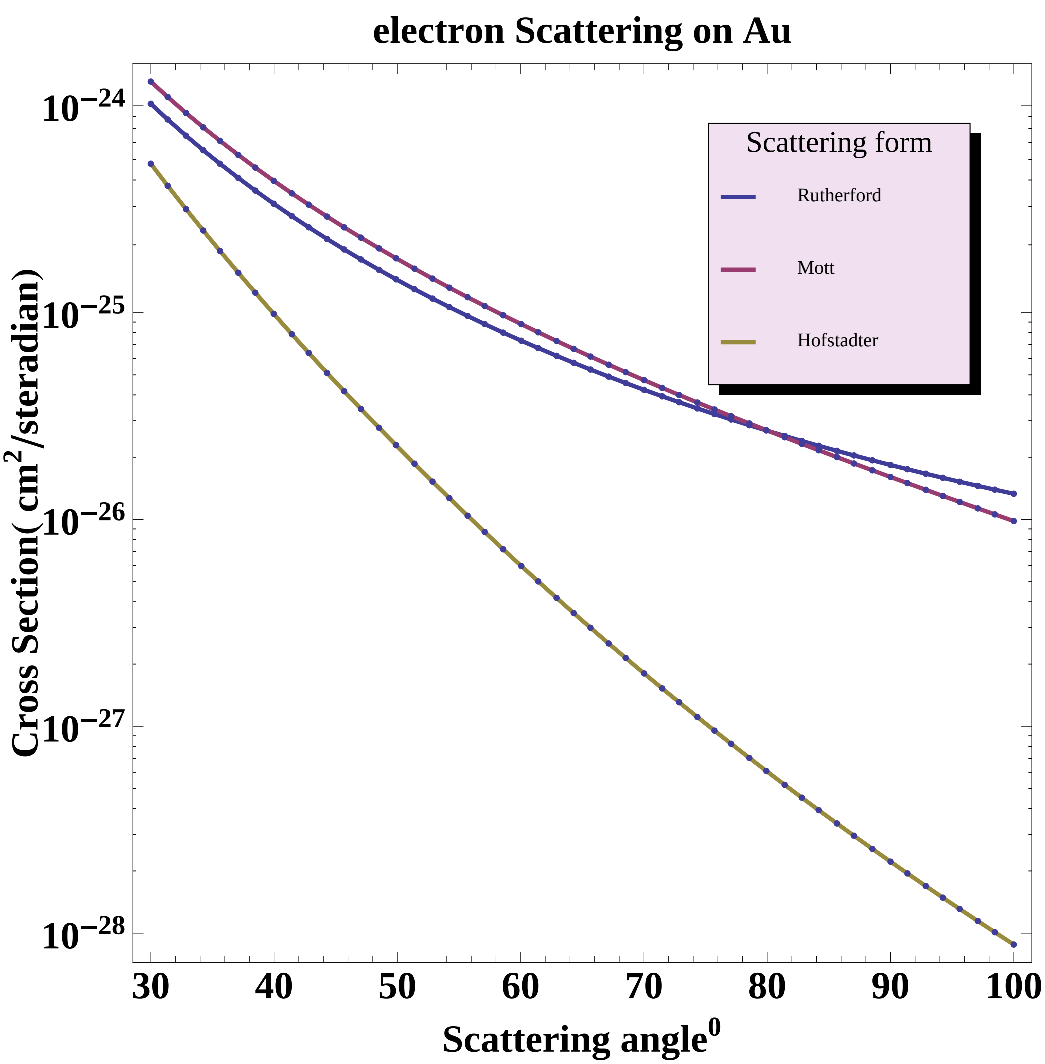
In[84]:= $\text{Normalization} = \text{Integrate}\left[4\pi r^2\rho(r, a), \{r, 0, \infty\}, \text{Assumptions}\to \text{Re}(a)>0\right]$

Out[84]= 1.

$$\text{FormFactor}(\theta\_, \text{a}\_, \text{EeMeV}\_) := \frac{4\pi\int_0^\infty r\,\rho(r, a)\sin(r\,Q(\theta, \text{EeMeV}))\,dr}{Q(\theta, \text{EeMeV})}$$

```
Hofstadter[θ_, Z_, EeMeV_, a_] :=
 Mott[θ, Z, EeMeV] * FormFactor[θ, a, EeMeV]^2

In[88]:= LogPlot[{Rutherford[θ * Degree, Z_Au, 84],
   Mott[θ * Degree, Z_Au, 84], Hofstadter[θ * Degree, Z_Au, 84, a_Au]},
  {θ , 30, 100}, PlotPoints → 50, MaxRecursion → 0, Mesh → All,
  PlotLegend → {"Rutherford", "Mott", "Hofstadter"},
  PlotStyle → {Thick}, Frame → True, ImageSize → {500, 500},
  AspectRatio → 1, LabelStyle → Directive[Black, Bold, FontSize → 18],
  FrameLabel → {Style["Scattering angle^0", 18],
    Style["Cross Section( cm^2/steradian)", 18],
    Style["electron Scattering on Au", 18]},
  LabelStyle → Directive[Black, Bold, FontSize → 18],
  LegendPosition → {.2, -.0}, LegendTextSpace → 2.5,
  LegendLabel → Style["Scattering form", 14], LegendLabelSpace → .5,
  LegendOrientation → Vertical, LegendBackground → LightPurple,
  LegendShadow → {.02, -.02}, LegendSize → {0.5, 0.5}]
```



Now fetch properties of targets

In[91]:=
```
ChemicalData["Polystyrene", "ElementTally"]
```

Out[91]=
```
{{C, 9}, {H, 12}}
```

In[92]:=
```
NC = ChemicalData["Polystyrene", "ElementTally"][[1, 2]];
NH = ChemicalData["Polystyrene", "ElementTally"][[2, 2]];
ρPS = ChemicalData["Polystyrene", "Density"] / 1000;
ρAu = ChemicalData["Gold", "Density"] / 1000;
Mp = Convert[ProtonMass, Gram][[1]];
Molecules = ρPS / (NC * 12 * Mp + NH * Mp);
AtomsH = NH * Molecules;
AtomsC = NC * Molecules;
AtomsAu = ρAu / (197 * Mp);
```

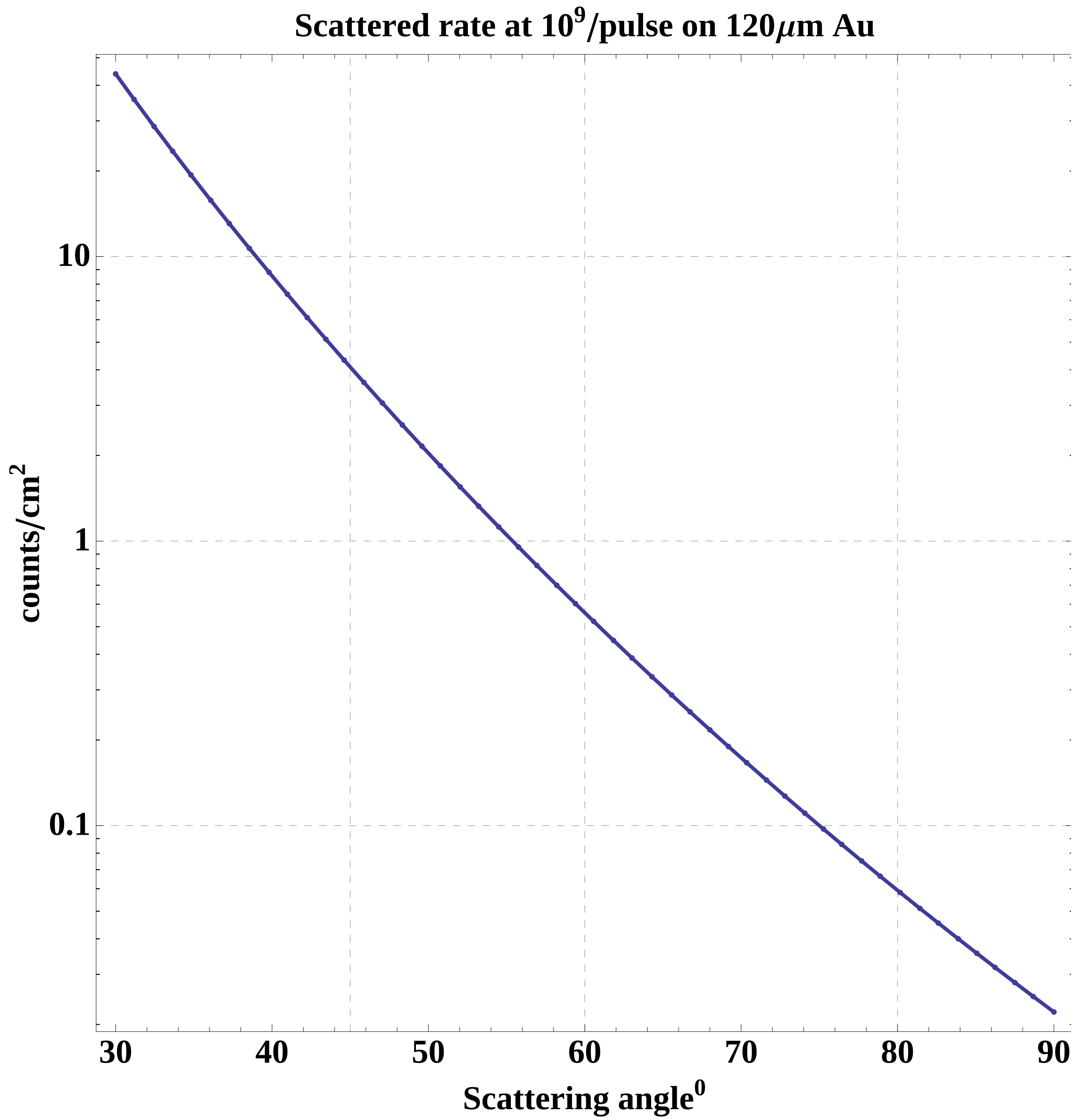
The number of scattered electrons from a bunch of 10^9 on the gold foil
Detector : 0.3 → 1 cm^2 area 1 m away. -> dΩ =

$$\frac{Area}{4\pi * R^2} * 4\pi \text{ steradians} = 10^{-4}$$

So rate = flux * atoms / cm^3 * t * sigma * dΩ.

In[101]:=
```
t_Au = 120 * 10^-4;
domega = 10^-4;
coeff = 10^9 Atoms_Au * domega * t_Au;
```

Out[104]=

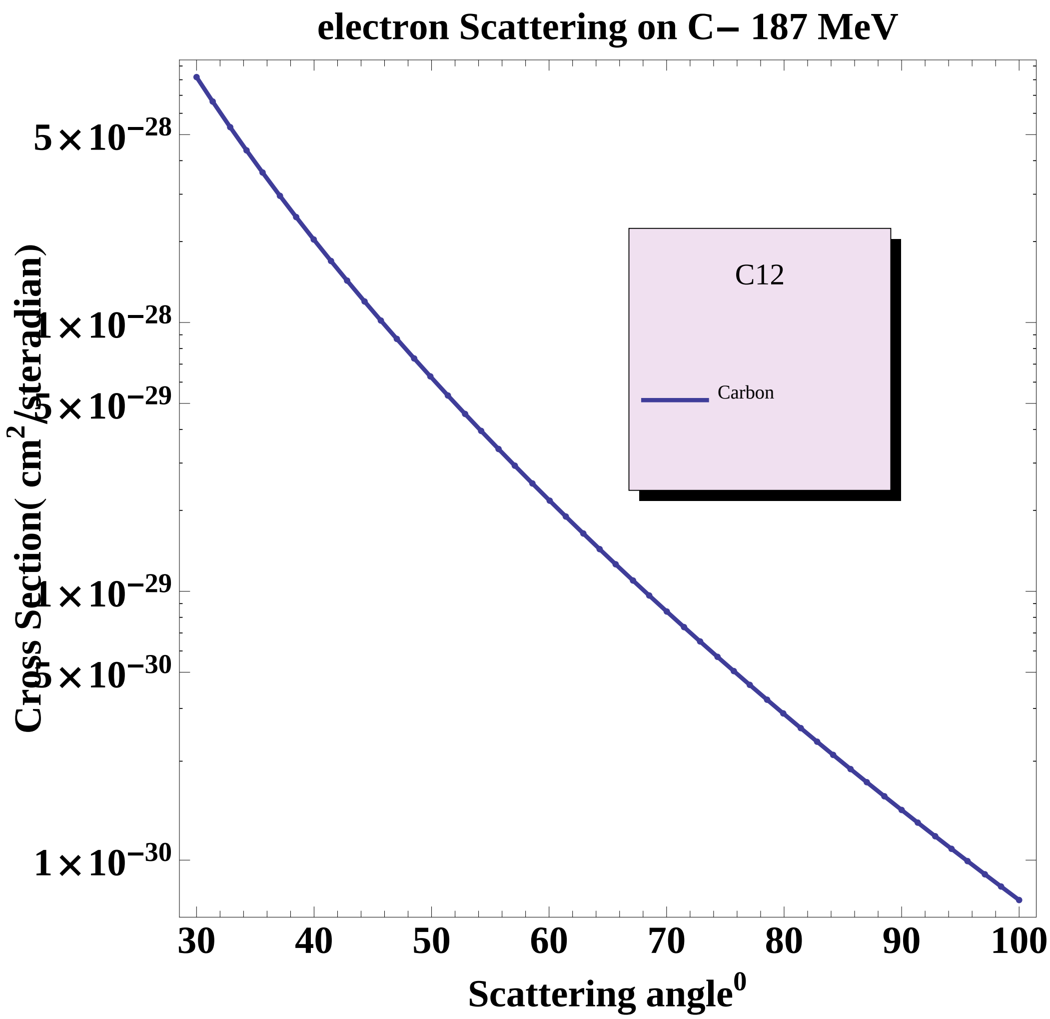


- **Calculate cross sections for Carbon (this is a check with Hofstadter's data. for this the recoil correction uses the Carbon mass)**

In[105]:=
```
Carbon[θ_] := Hofstadter[θ, 6, 187, aC]
```

In[106]:=
```
LogPlot[{Carbon[θ*Degree]}, {θ , 30, 100},
 PlotPoints → 50, MaxRecursion → 0, Mesh → All,
 PlotLegend → {"Carbon"}, PlotStyle → {Thick},
 Frame → True, ImageSize → {500, 500}, AspectRatio → 1,
 LabelStyle → Directive[Black, Bold, FontSize → 18],
 FrameLabel → {Style["Scattering angle^0", 18],
   Style["Cross Section( cm^2/steradian)", 18],
   Style["electron Scattering on C- 187 MeV", 18]},
 LabelStyle → Directive[Black, Bold, FontSize → 18],
 LegendPosition → {.2, -.0}, LegendTextSpace → 2.5,
 LegendLabel → Style["C12", 14], LegendLabelSpace → .5,
 LegendOrientation → Vertical, LegendBackground → LightPurple,
 LegendShadow → {.02, -.02}, LegendSize → {0.5, 0.5}]
```

Out[106]=

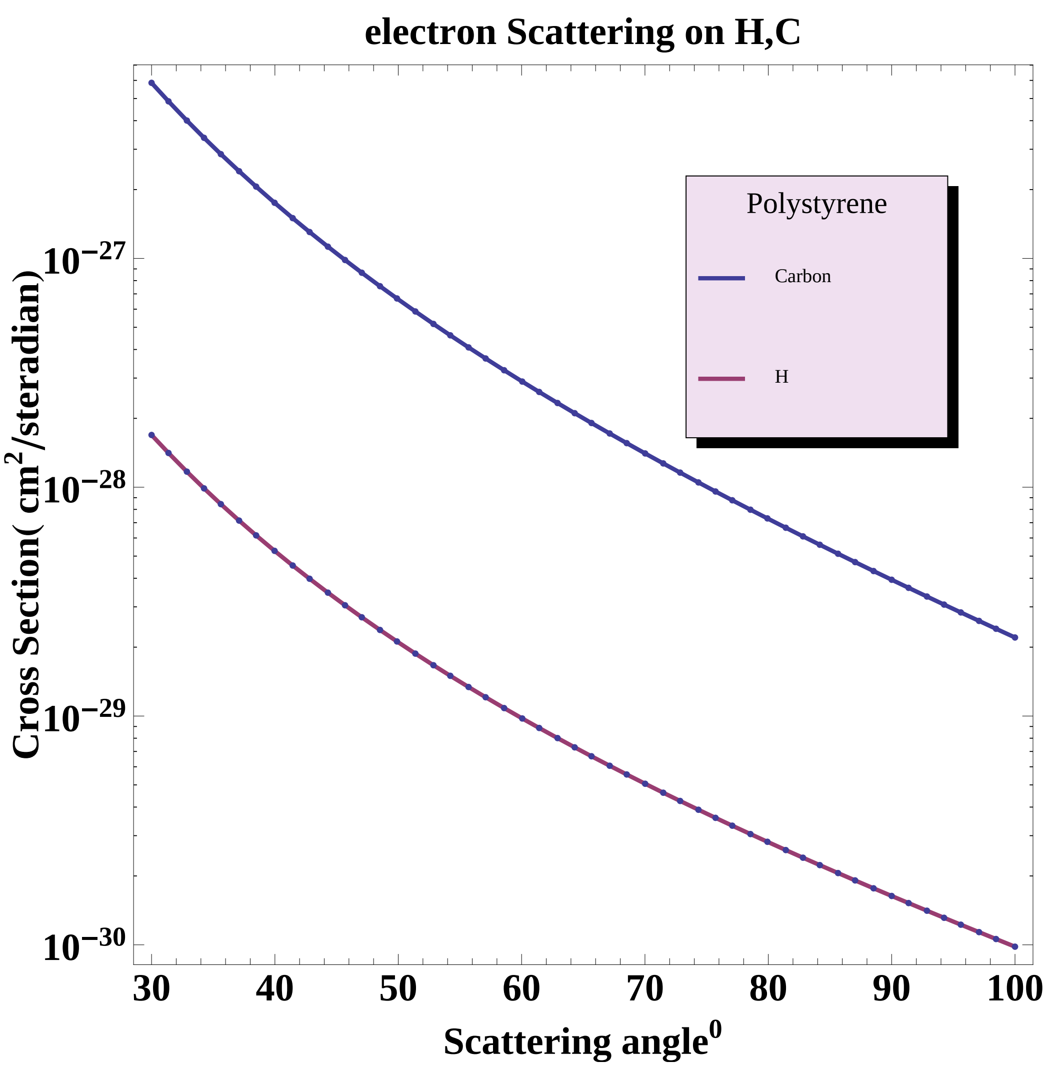


In[107]:=

In[108]:=

```
Hydrogen[θ_] := Mott[θ, 1, 80]
Carbon[θ_] := Hofstadter[θ, 6, 80, aC]
```

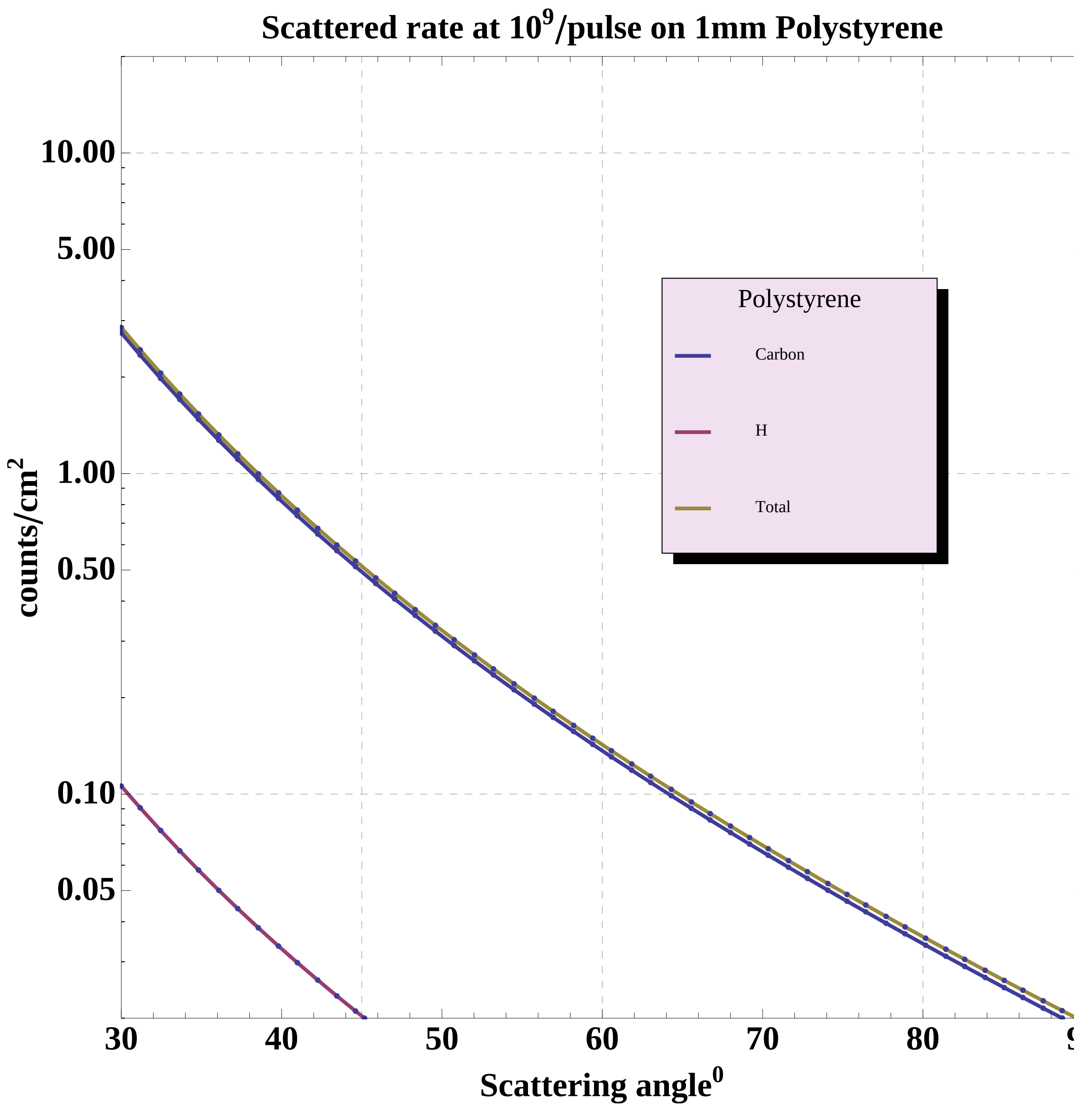
In[110]:=
```
LogPlot[{Carbon[θ*Degree], Hydrogen[θ*Degree]},
 {θ , 30, 100}, PlotPoints → 50, MaxRecursion → 0, Mesh → All,
 PlotLegend → {"Carbon",  "H"}, PlotStyle → {Thick},
 Frame → True, ImageSize → {500, 500}, AspectRatio → 1,
 LabelStyle → Directive[Black, Bold, FontSize → 18],
 FrameLabel → {Style["Scattering angle⁰", 18],
   Style["Cross Section( cm²/steradian)", 18],
   Style["electron Scattering on H,C", 18]},
 LabelStyle → Directive[Black, Bold, FontSize → 18],
 LegendPosition → {.2, -.0}, LegendTextSpace → 2.5,
 LegendLabel → Style["Polystyrene", 14], LegendLabelSpace → .5,
 LegendOrientation → Vertical, LegendBackground → LightPurple,
 LegendShadow → {.02, -.02}, LegendSize → {0.5, 0.5}]
```



In[111]:=

In[112]:=
```
tPS = 10^-1;
coef = 10^9 * tPS * domega;
```

Out[114]=

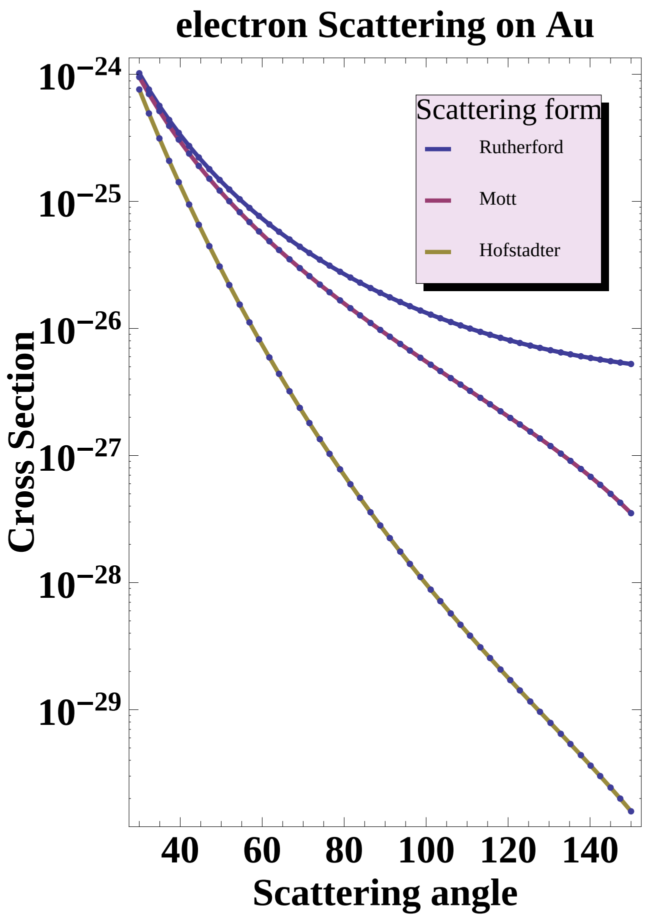


In[115]:=

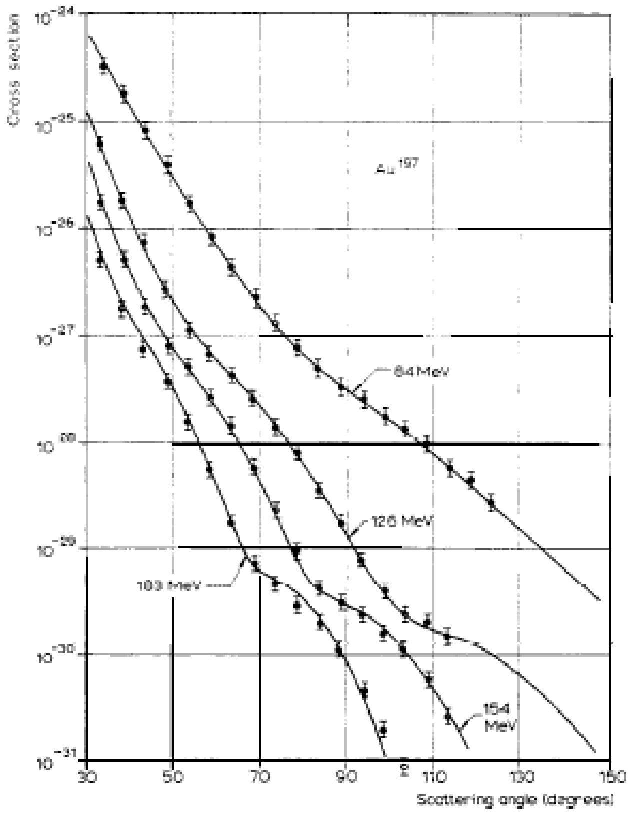
In[116]:=

Tests of this calculation on e - Nucleus scattering data :
This calculation is compared to Hofstadter below. The overall agreement is good at the factor of 2 level.

In[117]:=

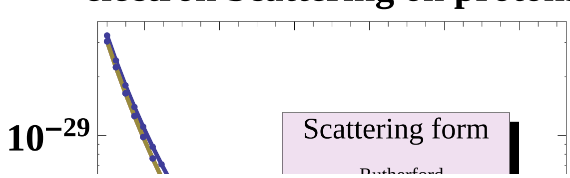


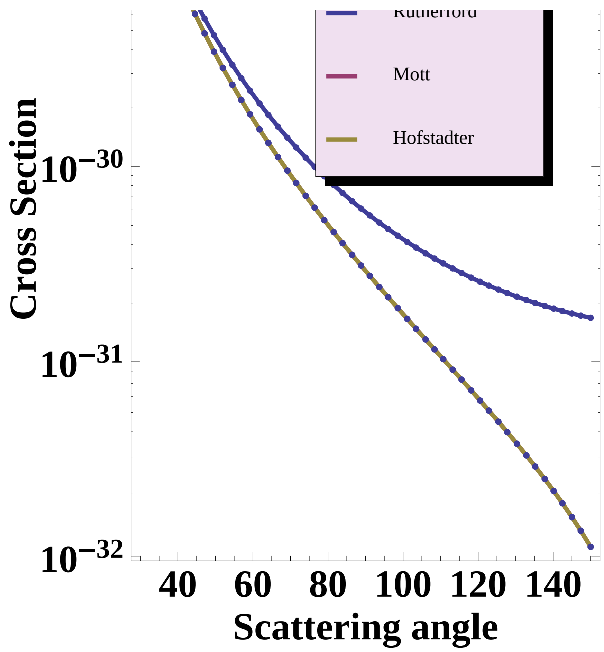
{"7.59863" × 10"-25", "2.50336" × 10"-28"}

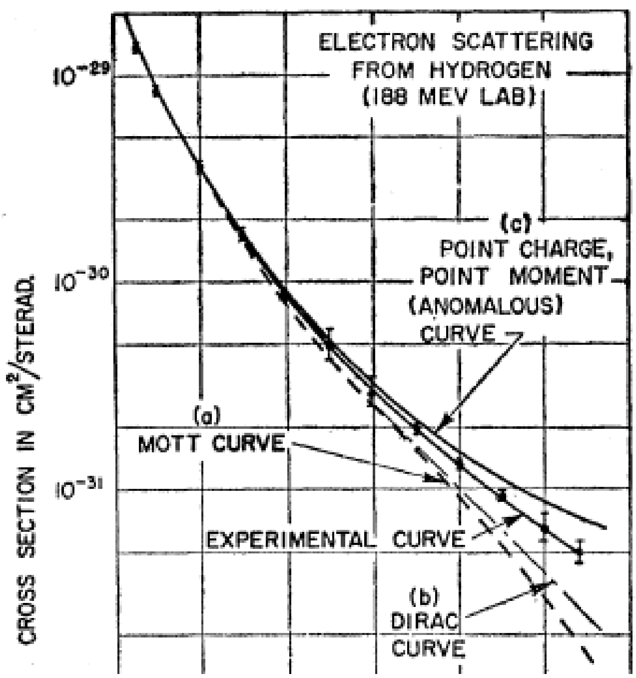


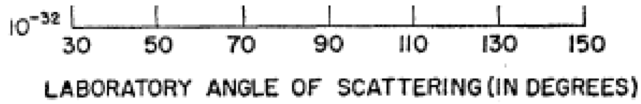
- A similar comparison was made for protons at 188 MeV and there agreement is good.

electron Scattering on protons




Rutherford
Mott
Hofstadter
Cross Section
10^{-30}
10^{-31}
10^{-32}
40 60 80 100 120 140
Scattering angle


ELECTRON SCATTERING
FROM HYDROGEN
(188 MEV LAB)
(c)
POINT CHARGE,
POINT MOMENT
(ANOMALOUS)
CURVE
(a)
MOTT CURVE
EXPERIMENTAL CURVE
(b)
DIRAC
CURVE
10^{-29}
10^{-30}
10^{-31}
CROSS SECTION IN CM^2/STERAD.



and Carbon (see above):