ATF rates from Scattering Foils

SNW, Jan '10

This is an update on scattering rates we would expect from practical foils in the ATF. If we take 80 MeV beam energy and 10^9 e/bunch as standard then we can get ~1 scatter per bunch with a plasic target 1 mm thick at angles <~45 degrees. Similar rates with a Gold target 0.12 mm thick.

In order to get 1 Hz at a 75^0angle it would take a ~ 7 mm thick plastic radiator tilted at 45^0. Maybe this is practical.

We start from Hofstadter's derivation in RMP 28 #3 p 214 and his Nobel lecture(it turns out he also used 80 MeV electrons). The relativistic correction and contribution of electron spin is applied to the Rutherford form as given by Mott and then the form factors for different nuclei are included.

At the end we compare this calculation with Hofstadter's results at Stanford.

In[857]:=
```
<< PhysicalConstants`
<< Units`
Needs["PlotLegends`"]
```

In[860]:=
```
Ee-MeV = 80;
```

- **Size parameters used by Hofstadter in exponential form : rms charge radius = $2*a\sqrt{3}$**

In[861]:=
```
a_Au = 2.3; a_C = 0.6;
Z_C = 6; Z_Au = 79;
α_EM = FineStructureConstant;
ℏc = Convert[PlanckConstantReduced*SpeedOfLight,
    Mega*ElectronVolt*Fermi][[1]];
ℏc1 = Convert[PlanckConstantReduced*SpeedOfLight,
    Mega*ElectronVolt*Centimeter][[1]];
```

Out[862]= Null$_{Null}$

In[866]:= $\text{Rutherford}[\theta\_, Z\_] := 1/4\,(Z*\alpha_{EM})^2 \dfrac{\hbar c1^2}{E_{e-MeV}{}^2}\,\text{Csc}[\theta/2]^4$

In[867]:= $\text{Mott}[\theta\_, Z\_] := \text{Rutherford}[\theta, Z]*\text{Cos}[\theta/2]^2$

In[868]:= $Q[\theta\_] := 2*E_{e-MeV}\,\text{Sin}[\theta/2]/\hbar c$

In[870]:= $\rho[r\_, a\_] := \dfrac{1}{8\pi\,(a)^3}\,\text{Exp}[-r/a]$

- **Form Factor normalization. We require it->1 as q->0. Then we have 4 π/Q[θ]*ρ[r] Sin[Q[θ] r] r and Sin[qr]/q->r**

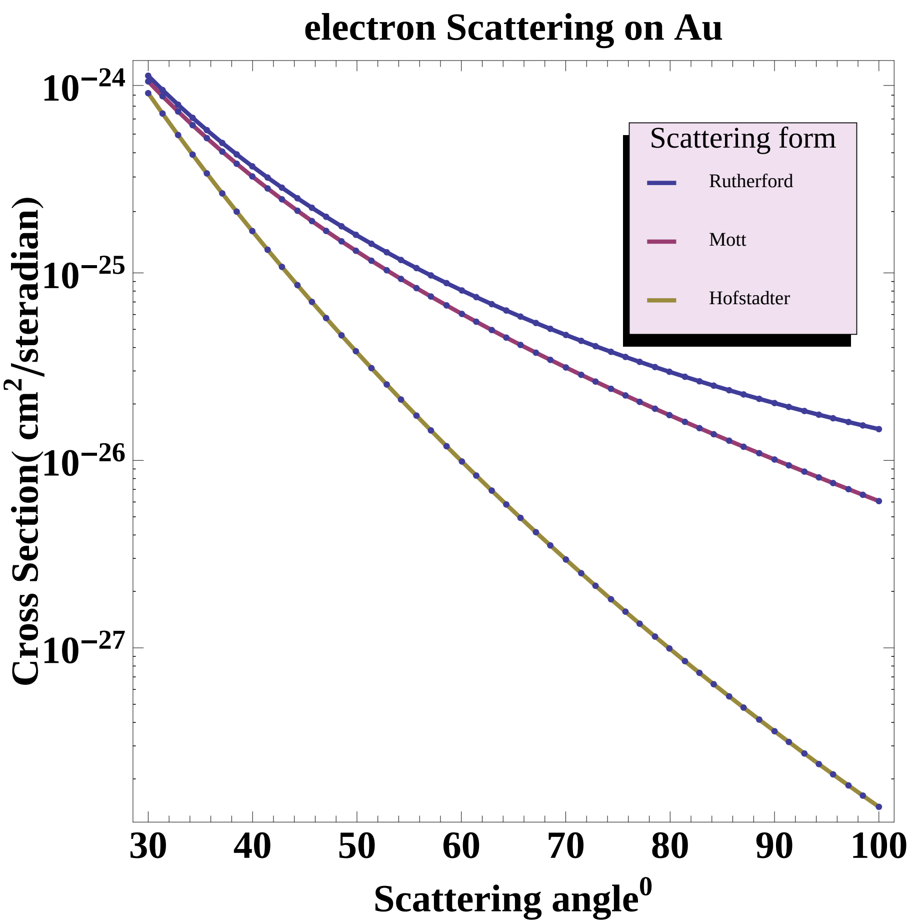
In[871]:= $\text{Normalization} = \text{Integrate}\left[4\pi r^2 \rho(r, a), \{r, 0, \infty\}, \text{Assumptions} \to \text{Re}(a) > 0\right]$

Out[871]= 1.

In[872]:= $\text{FormFactor}(\theta\_, a\_) := \dfrac{4\pi \int_0^\infty r\,\rho(r, a)\sin(r\,Q(\theta))\,dr}{Q(\theta)}$

FudgeFactor = 2; (*needed to agree with Hofstadter data on Au*)

In[874]:=
```
Hofstadter[θ_, Z_, a_] :=
  Mott[θ, Z] * FormFactor[θ, a]^2 * FudgeFactor
```



Now fetch properties of targets

In[879]:=
```
ChemicalData["Polystyrene", "ElementTally"]
```

Out[879]=
```
{{C, 9}, {H, 12}}
```

In[880]:=
```
N_C = ChemicalData["Polystyrene", "ElementTally"][[1, 2]];
N_H = ChemicalData["Polystyrene", "ElementTally"][[2, 2]];
ρ_PS = ChemicalData["Polystyrene", "Density"] / 1000;
ρ_Au = ChemicalData["Gold", "Density"] / 1000;
M_p = Convert[ProtonMass, Gram][[1]];
Molecules = ρ_PS / (N_C * 12 * M_p + N_H * M_p);
Atoms_H = N_H * Molecules;
Atoms_C = N_C * Molecules;
Atoms_Au = ρ_Au / (197 * M_p);
```

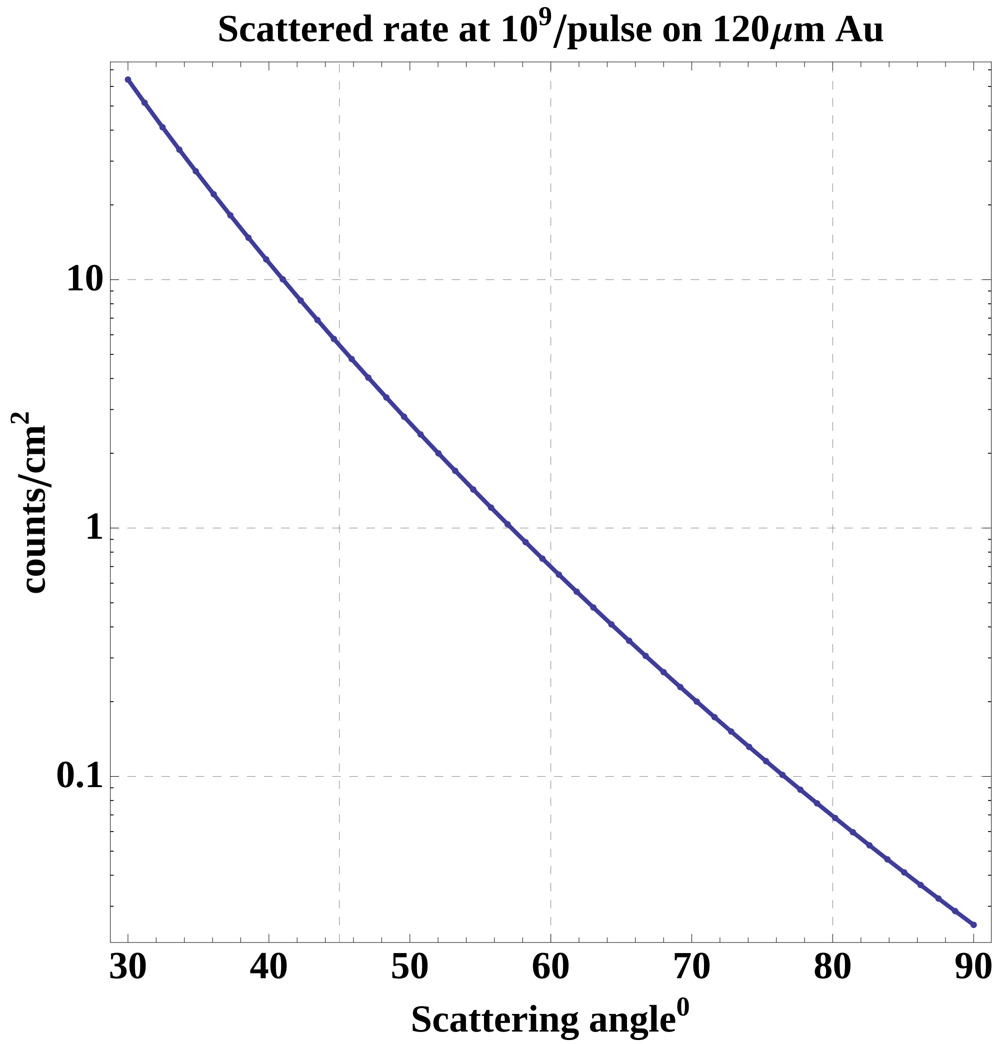
The number of scattered electrons from a bunch of 10^9 on the gold foil
Detector : $0.3 \to 1\ cm^2$ area 1 m away. -> $d\Omega$ =

$$\frac{Area}{4\pi * R^2} * 4\pi \text{ steradians} = 10^{-4}$$

So rate = flux * atoms / cm^3 * t * sigma * $d\Omega$.

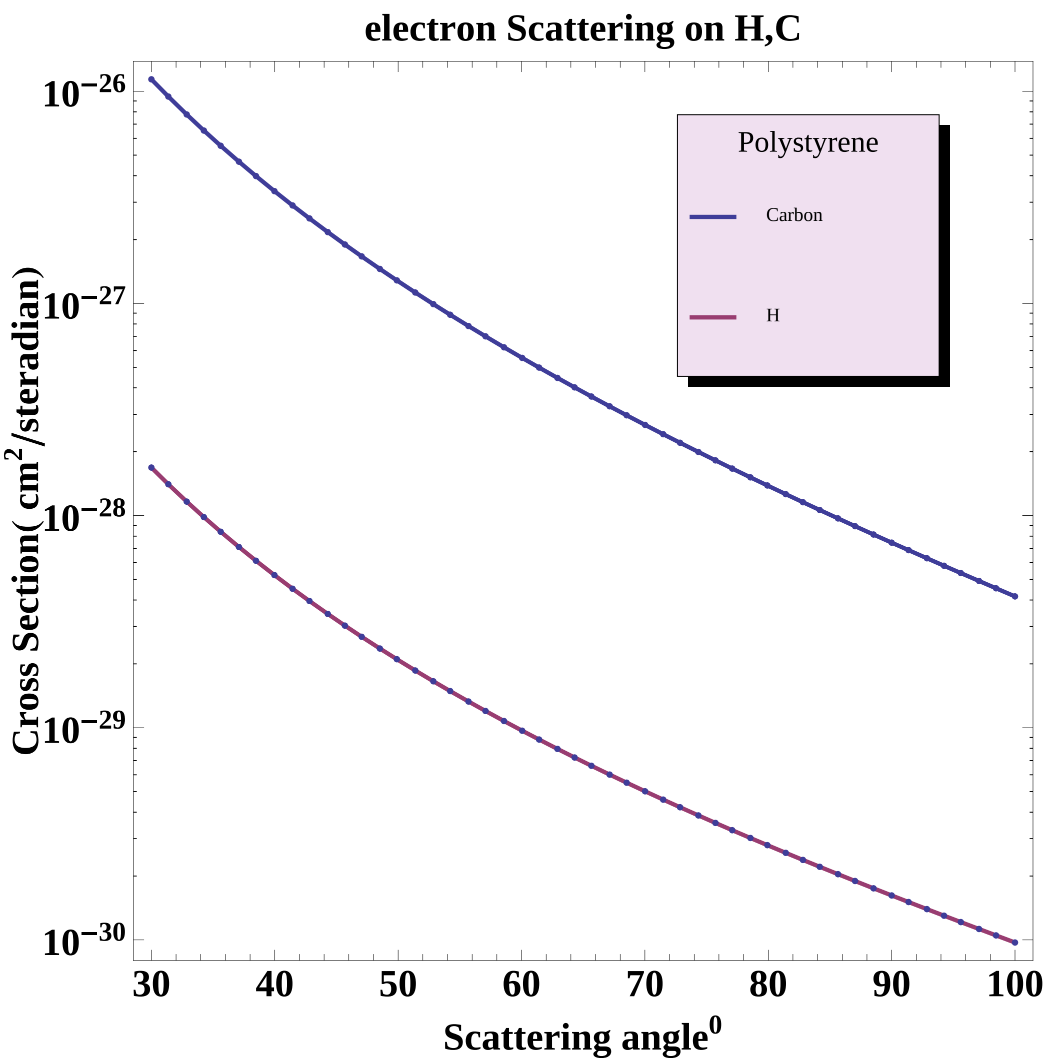
In[889]:=
```
t_Au = 120 * 10^-4;
domega = 10^-4;
coeff = 10^9 Atoms_Au * domega * t_Au;
```

Out[892]=



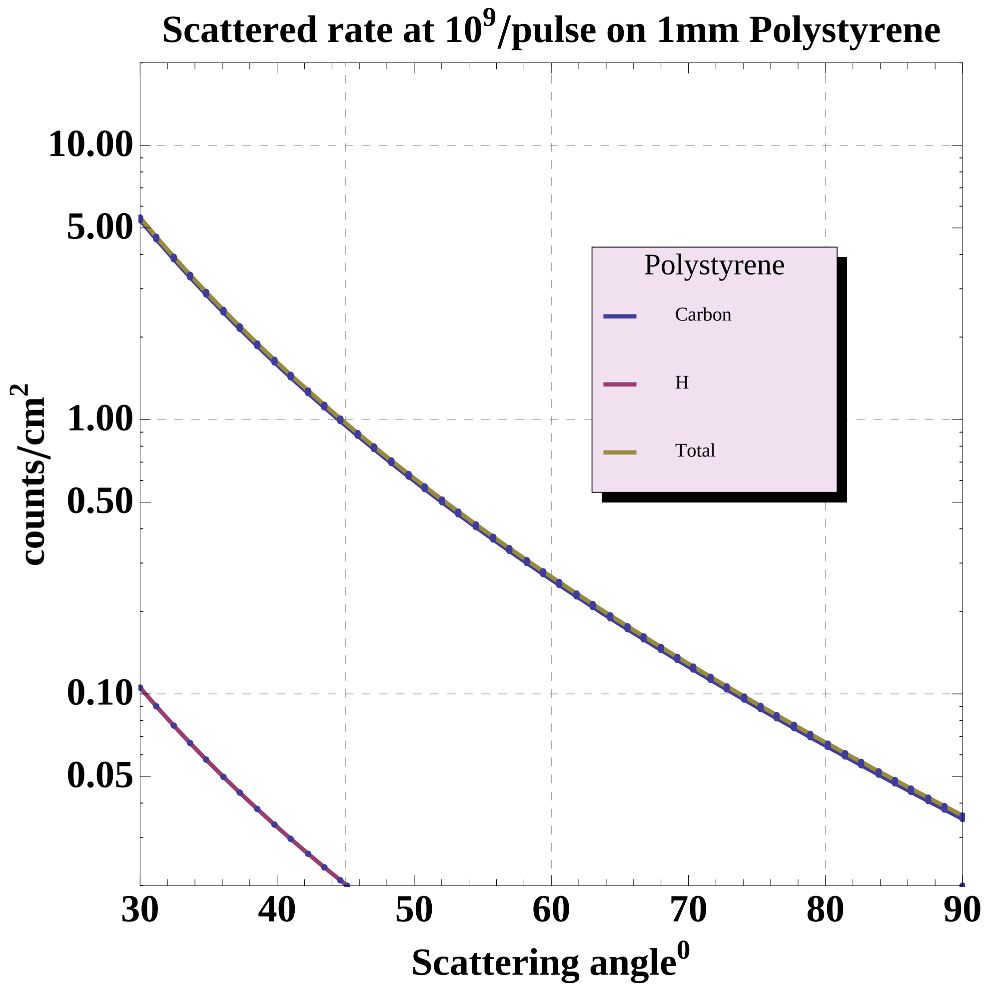
Calculate cross sections for Polystyrene

In[893]:=
```
Carbon[θ_] := Hofstadter[θ, 6, aC]
Hydrogen[θ_] := Mott[θ, 1]
```



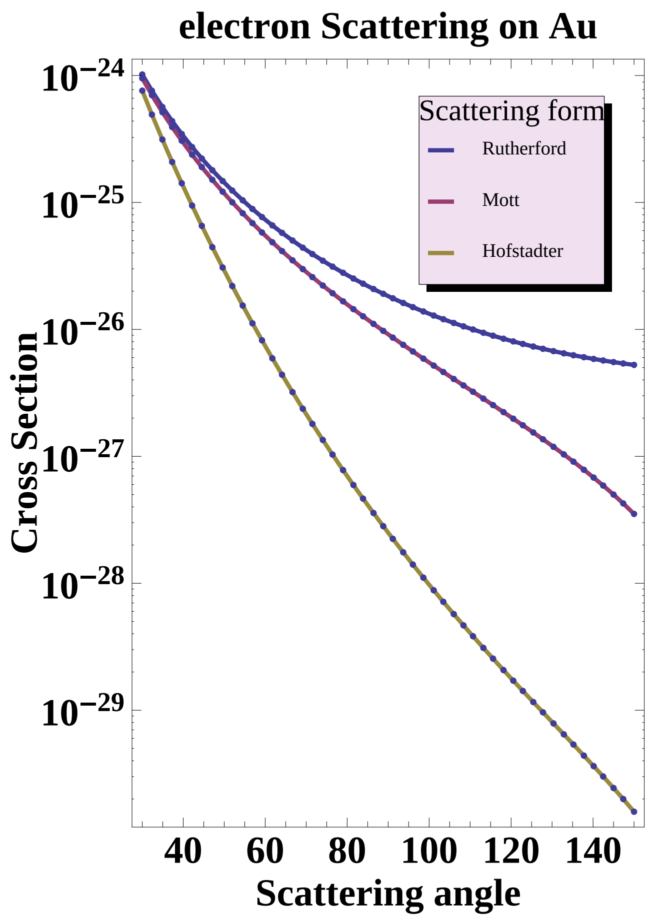
In[897]:=
```
tPS = 10^-1;
coef = 10^9 * tPS * domega;
```

Out[899]=



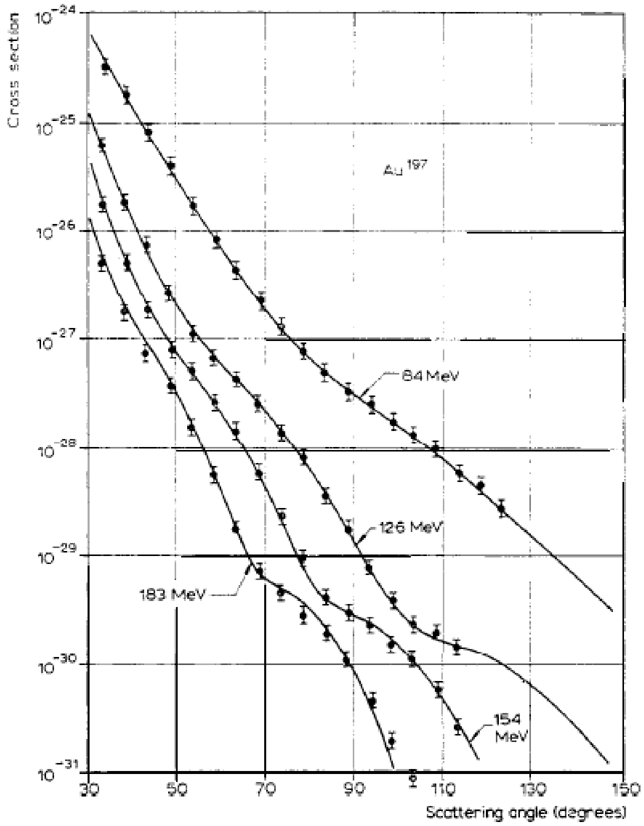
Tests of this calculation on e-Nucleus scattering data:

This calculation is compared to Hofstadter at 84 MeV below. The overall agreement is good except for an overall factor of 2 that I couldn' t find.



{"7.59863" × 10^"-25", "2.50336" × 10^"-28"}

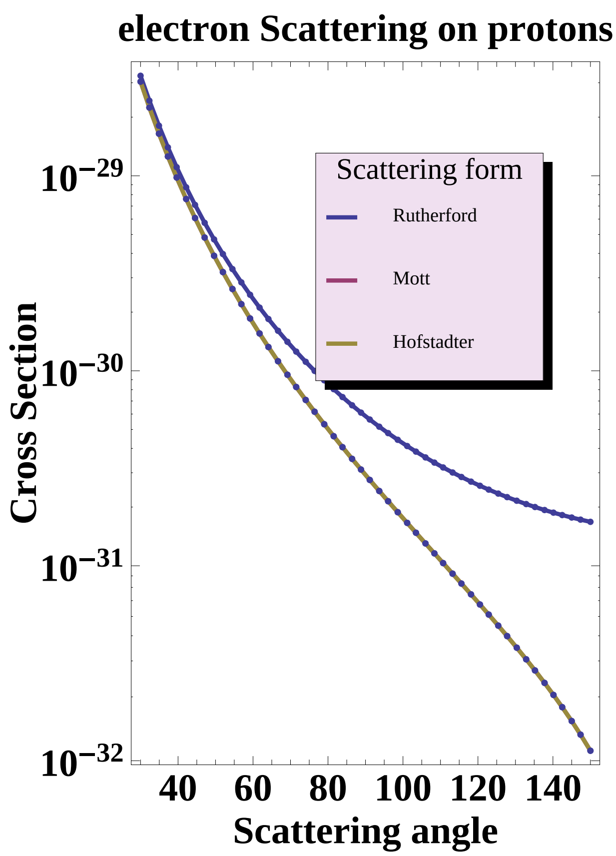
Out[902]=



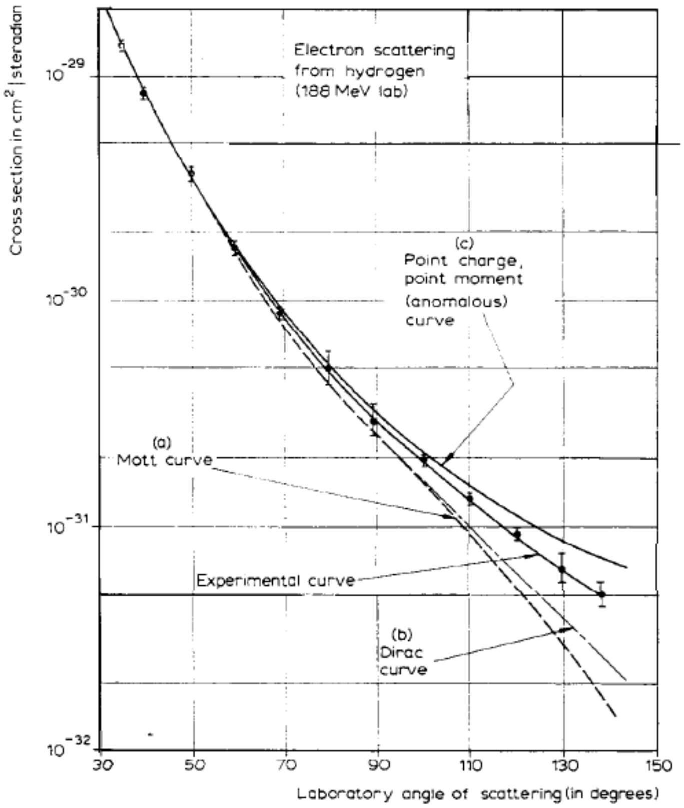
Cross section
10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
Au 197
84 MeV
126 MeV
183 MeV
154 MeV
30
50
70
90
110
130
150
Scattering angle (degrees)

- **A similar comparison was made for protons at 188 MeV and there agreement is good.**

In[903]:=



In[904]:=


Electron scattering
from hydrogen
(188 MeV lab)
Cross section in cm² | steradian
10⁻²⁹
10⁻³⁰
10⁻³¹
10⁻³²
(c)
Point charge,
point moment
(anomalous)
curve
(a)
Mott curve
Experimental curve
(b)
Dirac
curve
30
50
70
90
110
130
150
Laboratory angle of scattering (in degrees)