Signal and Noise modeling for the PSI APD detector data.

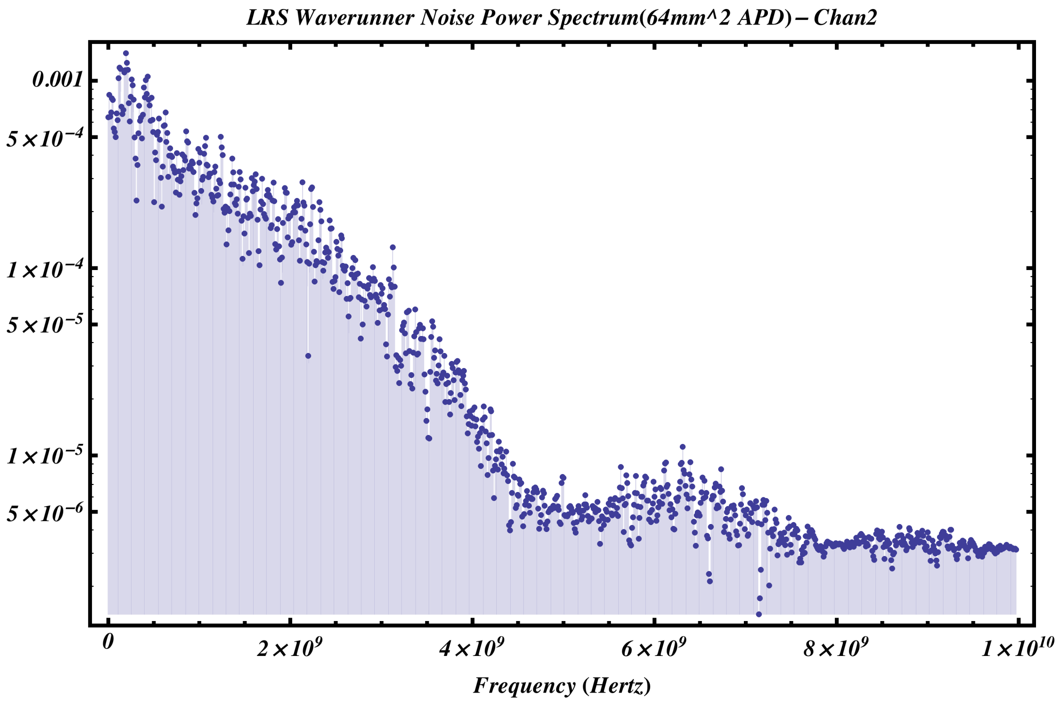
SNW - July, 2013

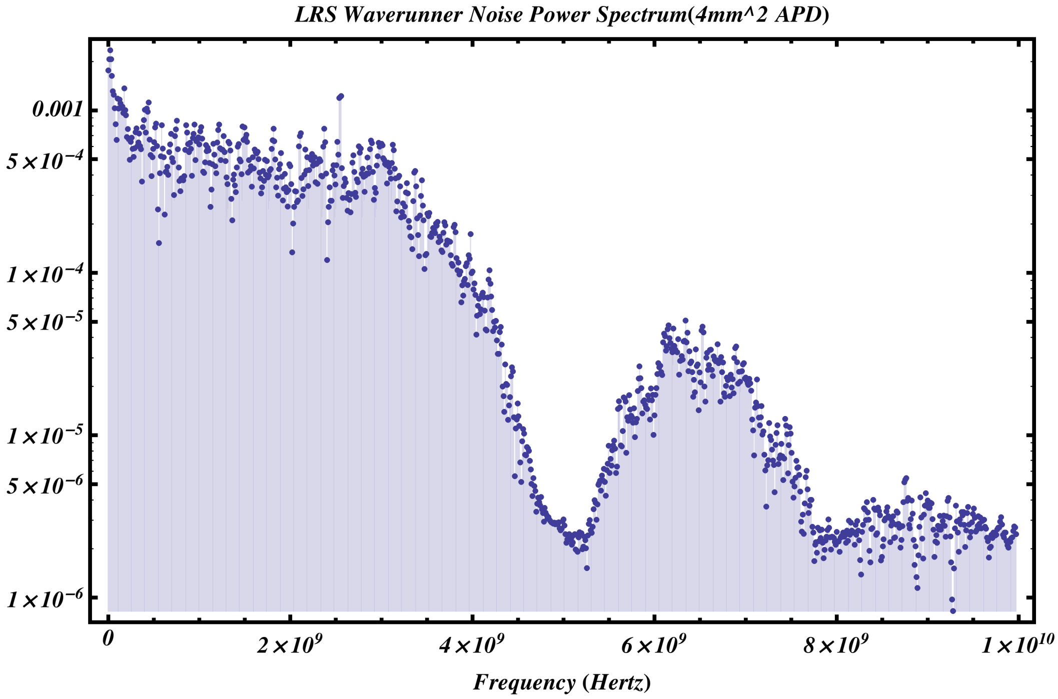
This notebook is initialized to correspond to detectors (4 mm^2, 64 mm^2) which should have a 0.6 pC signal for 1 MIP-ie 6000 e/MIP and an internal APD gain of 600. The detector capacitances are ~4 and 60 pF, respectively. The amplifier 3 dB bandwidth is 500 MHz and the amp gain is 20 dB (factor of 10 in voltage). The amplifier model is modified from the original one by J. Kaplon.

All data were taken with a Lecroy 2.5 GHz bandwidth scope recording at 20 Gsa/s. The scope full scale was 100 mV and the rms noise, independent of whether or not there was an input connection, was ~300 µV. This is roughly what is expected for this scope, given that it is an 8 bit scope.

The actual noise power spectrum is calculated using a fast fourier transform from data taken during the same run. This too is not a surprising distribution according to Xavier Boissier, Lecroy - Teledyne.

There is, however , some evidence that the noise spectrum is affected also by the input. This can be seen by comparing the following 2 plots. The first is from the 64 mm^2 detector followed by a 500 MHz, 20 dB voltage amplifier and the second is from the 4 mm^2 detector followed by a 3 GHz, 13 dB voltage amplifier.





In[182]:= **Q0 = 1.6 * 10^{-19} * 6000. * 600.**

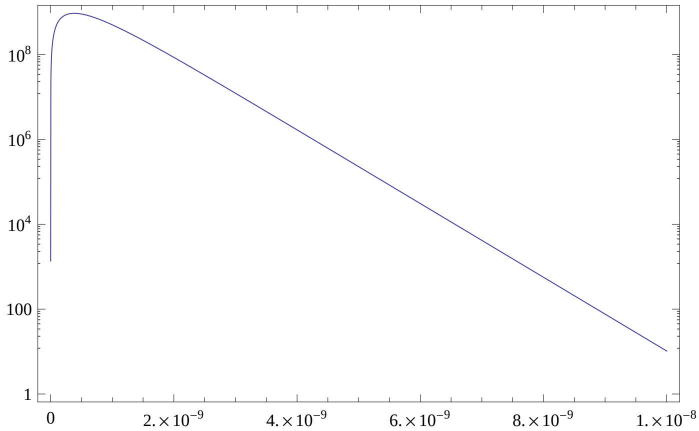
Out[182]= 5.76×10^{-13}

Input signal time distribution, approximate with an exponential. Start from Laplace transform of the signal, which gives a reasonable shape in time domain. Signal normalization (integral eq. to Q0). However as shown later the pulse shape at the preamp output is reproduced quite well assuming delta dirac signal from the detector.

In[183]:=
```
Clear[taud, taui];
sigs = 1/(1 + taui s) 1/(1 + taud s);
sig = InverseLaplaceTransform[sigs, s, t]
taud = .5 × 10^-9; taui = 0.3 × 10^-9;
LogPlot[sig , {t, 0, 10 × 10^-9}, Frame → True]
```

Out[185]= $\frac{e^{-\frac{t}{taud}}}{taud - taui} - \frac{e^{-\frac{t}{taui}}}{taud - taui}$

Out[187]=



In[188]:= FullSimplify[$\int_0^\infty$ sig ⅆt, taui > 0 && taud > 0]

Out[188]= 1.

Time domain response of the preamp (vo) to dirac delta signal to charge Qi. taup0 is determined by the amplifier bandwidth and we use 320 picosec for 500 MHz BW. ku is the amplifier voltage gain, which we take to be 10 (20 dB amp).

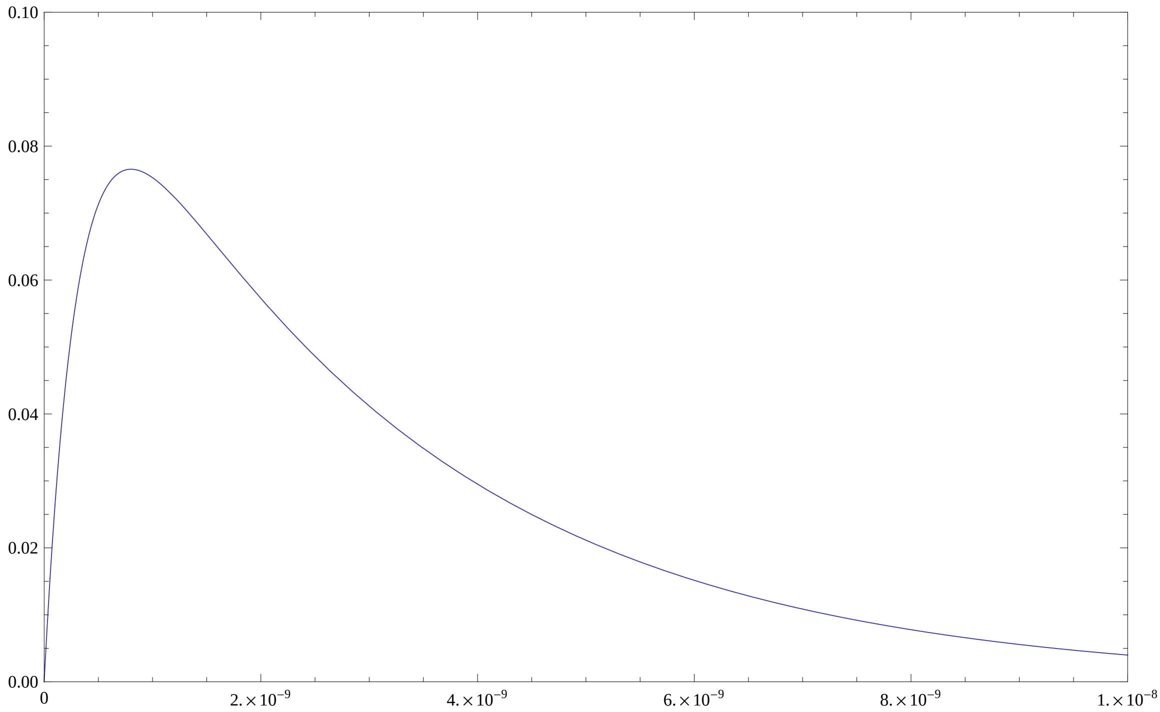
In[189]:=
```
Clear[Qi, id, cd, ri, taud, taui, vin, vpo, vo, vot, ku, taup0];
vin = id ri/(1 + s cd ri) Qi;
vo = vin ku/(1 + s taup0) ;
id = 1;
vot = InverseLaplaceTransform[ vo, s, t]
```

Out[193]= $ku\,Qi\,ri\left(\frac{e^{-\frac{t}{cd\,ri}}}{cd\,ri - taup0} - \frac{e^{-\frac{t}{taup0}}}{cd\,ri - taup0}\right)$

In[194]:=
```
Qi = 0.6 × 10^-12; cd = 60 × 10^-12; ri = 50; ku = 10; taup0 = 0.32 × 10^-9;
```

In[195]:=
```
Plot[vot , {t, 0, 10 × 10^-9}, Frame → True, PlotRange → {{0, 10 × 10^-9}, {0, .1}}]
```

Out[195]=



Solve for the peak amplitude and plot it vs. Capaciatance of the detector.

In[196]:=
```
Clear[Qi, id, cd, ri, taud, taui, vin, vpo, vo, vot, ku, taup0];
```

$$\text{resp}[t\_] := \text{ku Qi ri} \left(\frac{e^{-\frac{t}{\text{cd ri}}}}{\text{cd ri} - \text{taup0}} - \frac{e^{-\frac{t}{\text{taup0}}}}{\text{cd ri} - \text{taup0}} \right)$$

```
FullSimplify[D[resp[t], t]]
```

Out[198]=

$$\frac{\text{ku Qi} \left(-\frac{e^{-\frac{t}{\text{cd ri}}}}{\text{cd}} + \frac{e^{-\frac{t}{\text{taup0}}}\,\text{ri}}{\text{taup0}} \right)}{\text{cd ri} - \text{taup0}}$$

In[199]:=
```
Solve[D[resp[t], t] == 0, t]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

Out[199]=

$$\left\{\left\{t \to \frac{\text{cd ri taup0 Log}\left[\frac{\text{cd ri}}{\text{taup0}}\right]}{\text{cd ri} - \text{taup0}}\right\}\right\}$$

In[200]:=
$$\text{resp}\left[\frac{\text{cd ri taup0 Log}\left[\frac{\text{cd ri}}{\text{taup0}}\right]}{\text{cd ri} - \text{taup0}}\right]$$

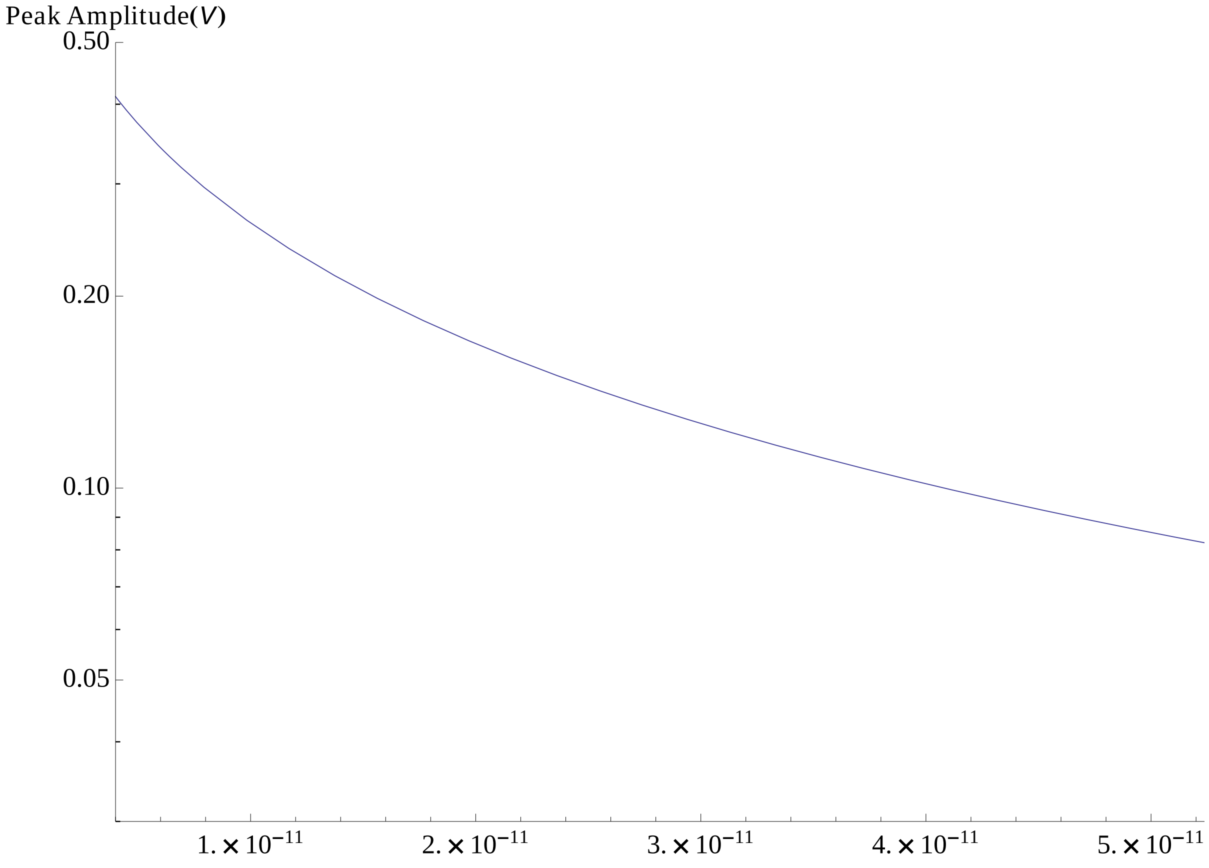
Out[200]=
$$\text{ku Qi ri}\left(-\frac{\left(\frac{\text{cd ri}}{\text{taup0}}\right)^{-\frac{\text{cd ri}}{\text{cd ri}-\text{taup0}}}}{\text{cd ri} - \text{taup0}} + \frac{\left(\frac{\text{cd ri}}{\text{taup0}}\right)^{-\frac{\text{taup0}}{\text{cd ri}-\text{taup0}}}}{\text{cd ri} - \text{taup0}}\right)$$

In[238]:=
$$\text{maxresp} = \text{ku Qi ri}\left(-\frac{\left(\frac{\text{cd ri}}{\text{taup0}}\right)^{-\frac{\text{cd ri}}{\text{cd ri}-\text{taup0}}}}{\text{cd ri} - \text{taup0}} + \frac{\left(\frac{\text{cd ri}}{\text{taup0}}\right)^{-\frac{\text{taup0}}{\text{cd ri}-\text{taup0}}}}{\text{cd ri} - \text{taup0}}\right)$$

```
Qi = Q0; ri = 50; ku = 10; taup0 = 0.32 × 10^-9;

LogPlot[maxresp , {cd, 4 * 10^-12, 100 × 10^-12},
 AxesLabel → { C_d[Farad], Peak Amplitude[V]},
 PlotRange → {{4 × 10^-12, 60 × 10^-12}, {.03, .5}},
 LabelStyle → Directive[Bold, Larger]]
```

Out[238]=
$$2.88\times10^{-10}\left(\frac{1.5625\times10^{11^{-\frac{3.2\times10^{-10}}{-3.2\times10^{-10}+50\,\text{cd}}}}\ \text{cd}^{-\frac{3.2\times10^{-10}}{-3.2\times10^{-10}+50\,\text{cd}}}}{-3.2\times10^{-10}+50\,\text{cd}} - \frac{1.5625\times10^{11^{-\frac{50\,\text{cd}}{-3.2\times10^{-10}+50\,\text{cd}}}}\ \text{cd}^{-\frac{50\,\text{cd}}{-3.2\times10^{-10}+50\,\text{cd}}}}{-3.2\times10^{-10}+50\,\text{cd}}\right)$$

Out[240]=


so for C_d below fraction of pF the amplitude response depends on ri (50 Ohm), for higher detector capacitances it depends on inverse of C_d