ON RADAR DETECTION OF NON-SPHERICAL ICE PARTICLES[1]

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1. Introduction

In theoretical analyses aimed at predicting whether development of precipitation-size ice particles in convective clouds can be detected by given radar equipment under various specified conditions, it is obviously simpler to approximate the real problem of growth and detection of non-spherical particles by the spherical case. In doing so, it is desirable to have some notion of how much error might be made through such convenient approximations. Two factors contribute to the error: first, growth rates are greater in the non-spherical case, and second, radar reflectivities are also greater in the non-spherical case; so both factors tend to increase radar-echo intensities at given *times* after appearance of the ice phase. Since it was shown by Houghton (1950) that ice growth by the Bergeron process is faster than growth by accretion up to equivalent drop diameters of the order of 250 to 300 microns, and since this order of ice-particle-diameter is roughly that required for just-detectable radar echoes under commonly occurring operating conditions, the question posed above warrants attention. Use will be made of the electrostatic analogy (Houghton, 1950) in examining this question here.

2. Analysis

The first step required here is that of formulation of the relative mass growth-rates of ice spheroids and ice spheres of the same mass. Since characteristics of natural ice-nuclei populations are such as to imply that appreciable amounts of ice will not be present until air parcels are cooled by ascent to about −15C to −20C, and because this temperature range favors either dendritic types or hexagonal plates (*cf.* Mason, 1957), it will be sufficient here to consider just the oblate spheroid case.

The mass of an oblate spheroid of major and minor semi-axes a and c is $4\pi a^2 c\rho_i/3$ where ρ_i is the density of the ice. By introducing the *fineness ratio* $h = c/a$ of the elliptical cross-section of the spheroid, we have $r = ah^{\frac{1}{3}}$ as the radius of the ice *sphere* of equivalent mass. In terms of the fineness ratio, the capacitance C_0 of an oblate spheroid may be written as

$$C_0 = r[(1 - h^2)^{\frac{1}{2}}/h^{\frac{1}{3}} \arcsin\,(1 - h^2)^{\frac{1}{2}}]$$

where a has been replaced by $rh^{-\frac{1}{3}}$. If one calls the bracketed factor A, the mass growth-rate for the oblate ice spheroid becomes $dm/dt = 4\pi DrkA\Delta$ and that of the equivalent sphere $dm'/dt = 4\pi Drk'\Delta$, where D is the vapor-diffusion coefficient, k and k' are the respective ventilation factors for the two cases, and Δ is the vapor-density difference between the surface of the ice particle and regions of the vapor phase far from the ice particle. The greater drag-weight ratio of oblate spheroids probably implies the inequality $k > k'$; but this remains a poorly understood matter, so as a simplification we will put $k \doteq k'$. That the same value of vapor-density difference Δ should be used in the two cases follows from the fact that Δ is independent of both shape and size, to first approximation. The ratio of oblate spheroidal mass growth rate to that of a sphere of equal mass is found from the above to be simply the quantity A, a function only of the fineness ratio. In table 1 are shown values of A calculated for a number of values of h spanning the range of interest.

If we next formulate the growth laws for our two cases and integrate them from an initial condition where r and a are each only of the order of, say, tens of microns at time zero to such a time t that the particle radii are about an order of magnitude greater, Houghton's parabolic growth relation gives simply $A^{\frac{3}{2}}$ as the approximate ratio of the final masses of spheroidal and spherical particles at time t. Table 1 shows values of this quantity for various h.

The effect of particle asphericity on radar backscattering has been treated by Atlas, Kerker and Hitschfeld (1953). Their values of the radar return of randomly oriented oblate ice spheroids expressed

Table 1. Dependence of A, $A^{\frac{3}{2}}$, and S on fineness ratio $h = c/a$.

h	0.01	0.05	0.10	0.20	0.50	1
A	3.0	1.8	1.5	1.2	1.1	1.0
$A^{\frac{3}{2}}$	5.2	2.4	1.8	1.3	1.1	1.0
S	—	—	1.5	1.3	1.1	1.0
N (db)	—	—	6.9	3.4	1.3	0

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as multiples of the return of ice spheres of the *same mass* (*cf.* Battan, 1959, p. 39) are shown as the quantity S in table 1 for h down to 0.1, the lowest value to which Atlas *et al* extended their computations.

For any given time following inception of ice growth in a cloud, the relative radar return (for commonly used radar wavelengths) for the oblate *vs.* the spherical case is given by SA^3 since radar scattering cross-sections of spheres vary as the particle-volume squared. Converting these to decibels of difference N in radar return, by using the relation $N = \log_{10}(SA^3)$, yields the values shown in the bottom row of table 1.

In Houghton's (1950) computations, he used results of Nakaya and Terada and of Schaeffer that indicated that tabular crystals tend to have constant thicknesses (10 μ for dendrites, 40 μ for hexagonal plates). Subsequent experimental studies by Reynolds (1952) and Mason (1953) indicate, however, that such constancy of plate-thickness is not assured. Rather, both of their results suggest roughly constant fineness ratio for hexagonal plates. Reynolds found $h = 0.3$ for hexagonal plates and $h = 0.15$–0.20 for dendritic crystals. By contrast, the fineness ratio of an hexagonal crystal of 10 micrograms mass (order of magnitude of interest in first-echo studies) and of thickness 40 microns, as considered by Houghton, would be about 40/1800 or 0.02. If table 1 could be extended to values of N corresponding to such small h values, the differences in return would evidently be quite large. However, it is very likely that we will be closer to reality to go only slightly below the smallest fineness ratio reported by Reynolds and take $h = 0.1$ as a lower limit to be expected under most conditions.

For this limit, table 1 indicates that there might be about 7 db difference in radar-echo intensity between equally distant cloud regions containing the same concentrations of spheroidal and spherical ice particles that have grown for the *same length of time* under the same ambient conditions.

Inclusion of some allowance for differences in ventilation factors for the two cases would probably lower this difference to a somewhat smaller value, while if the Nakaya-Terada and Schaeffer fineness ratios are more typical of real crystals the shape effect would be rather larger than the 7-db value suggested here.

A difference of 7 db in theoretical return, depending upon whether one assumes spherical or oblate spheroidal ice particles, is not extremely large, yet it is by no means insignificant by radar meteorological standards. It is, for example, about two to three times the magnitude of the gain-steps employed in echo-contouring studies. And, all other conditions being equal, a 7-db excess in scattering cross-section will be associated with an increase in range of detectability of about 2.5 if the cloud fills the radar beam (inverse square case).

But the implications of the above-estimated 7-db difference become most significant of all in certain studies based on first-detection techniques. When "first echo" methods are used in an effort to distinguish between operation of water- *versus* ice-processes in the growth of precipitation (Battan, 1953; Braham, 1958; Ackerman, 1959), it has been customary to base the final decision as to mechanisms on the temperature and hence the altitude of the cloud region containing the first echo. Another method, which does not appear to have been used quantitatively, would be that of relating altitude of first echo to time of growth through updraft speeds and then comparing observations with *theoretical predictions* of rates of growth of ice and of water particles. It was in this context that the author encountered the question treated in the present section. That a 7-db theoretical difference in radar return is, in such context, of crucial importance can be shown through arguments of the following type: if ice *spheres* of 250-micron diam can be just detected at some representative range and cloud ice content with a given radar, a 7-db advantage would make *oblate spheroids* of 0.1 fineness ratio detectable for the same conditions when their equivalent spherical diameter became about 240 microns (*cf.* table 1). If we assume that heterogeneous nucleation took place, on the average, when the parent water drops had diameters of 100 microns in each case, which is in reasonable accord with natural nuclei activation spectra and typical cloud-growth histories, then 20 min of subsequent diffusional growth would be predicted on the *spherical* assumption but only 7 min of subsequent diffusional growth would, by table 1, be demanded on the *oblate spheroidal* assumption in order to get the particles to detectable size. This is almost a *threefold* difference in the theoretical prediction of time required. Put in terms of difference of height attained at time of detection, and by using an updraft speed of 1000 ft min^{-1}, we get about 13,000 ft difference in theoretically predicted altitude of first-echo for our two models. Thus, a 7-db difference may become extremely important in any arguments of this type, aimed at trying to disentangle the two competing particle-growth processes through theoretical analyses of observed radar histories of natural clouds.

In the past, asphericity of ice particles has been regarded as unimportant in radar meteorology, especially when compared to asphericity of water drops or water-coated ice particles. The objective of the present note has been to show, however, that when the scattering anomalies are added to the growth-rate anomalies of oblate spheroids the *combined* effects on radar detectability are of greater significance than has been recognized.

3. Concluding remark

Existing observational data on the degree of oblateness of natural ice crystals is evidently inadequate. There is almost an order-of-magnitude difference between the fineness ratios implied in the observations of Nakaya and Terada and of Schaeffer and the fineness ratios indicated in the experimental work of Reynolds and Mason. Reynolds' fineness ratios were smaller than Mason's, consistent with the fact that his growth conditions were nearer those promoting tabular and dendritic growth. The coincidences (1) that tabular growth is favored in the temperature range from about −15C to −20C for which growth rates are intrinsically fastest for typical pressures and (2) that natural nuclei activation spectra also imply that significant numbers of ice crystals will often be enjoying depositional growth in cloud regions at just these temperatures both combine to make it very desirable to secure better observational information on fineness ratios of crystals growing naturally under such conditions, in order to make better use of radar and growth theory in distinguishing operation of the ice and the all-water processes of growth of precipitation particles in convective clouds.

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