CORRESPONDENCE

A Note on the Aerodynamic Effects of Internal Circulations in Small Raindrops [1]

JAMES E. McDONALD

University of Arizona

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In an analysis (McDonald, 1954) of the aerodynamics of large raindrops, I have given quantitative reasons for believing that, if surface shear-stress acts to induce internal circulations within falling rain-

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drops, the attendant centrifugal pressure fields are small enough to be ignored in explaining drop deformation itself. In that paper, however, I point out that it is possible that internal circulations may be stronger in smaller raindrops (diameter under about 0.5 mm) because of absence of strong wake circulations over the upper hemispheres of these small drops. (On a large drop, the reverse circulation of boundary-layer air in the well-developed wake tends to oppose internal circulations induced by surface friction acting on the lower hemisphere of the drop.) In the present note, I wish to cite some quantitative evidence for existence of some degree of internal motion within drizzle drops and small raindrops.

The method of showing this is quite elementary, yet it had not occurred to me in the course of my earlier analysis. One may simply compare the carefully observed terminal velocities reported by Gunn and Kinzer (1949) with velocities calculated on the assumption that the drops are *solid* spheres of density equal to that of water. As is explicitly pointed out by Gunn and Kinzer (1949, p. 245), attempts to predict the fall velocities on the above basis, using empirical drag coefficients, lead directly to an equation that can only be solved by successive approximations. However, I have recently pointed out, in a paper to be published elsewhere, a method of removing this mathematical indeterminacy.[2] Using this method, I have calculated terminal velocities for "solid" water drops falling in sea-level air at 20C (conditions of the Gunn and Kinzer measurements). In fig. 1, these predicted values are compared with those observed by Gunn and Kinzer.

As may be seen from fig. 1, the solid-drop prediction yields velocities which are a few per cent under the Gunn-Kinzer experimental values over the range from 0.1 to about 0.7 mm. This range is bounded below by the Stokes' law regime and above by the regime in which drop deformation leads to excessive drag (compared with that of a sphere of same volume). This latter effect is clearly shown in fig. 1. An estimate, based only on pressure analysis (McDonald, 1954) that deformation effects should become discernible at diameters near 1.0 mm is well confirmed by fig. 1.

Although the steps leading to the solid-sphere prediction involve two graph-reading steps that lead to random errors which I estimate to be of the order of a few per cent, the fact that the plotted points for all of the lowest seven drop sizes fall uniformly below the Gunn-Kinzer curve by an average of a few per cent constitutes a weak, but probably real, indication of drag-reduction due to internal circulations.

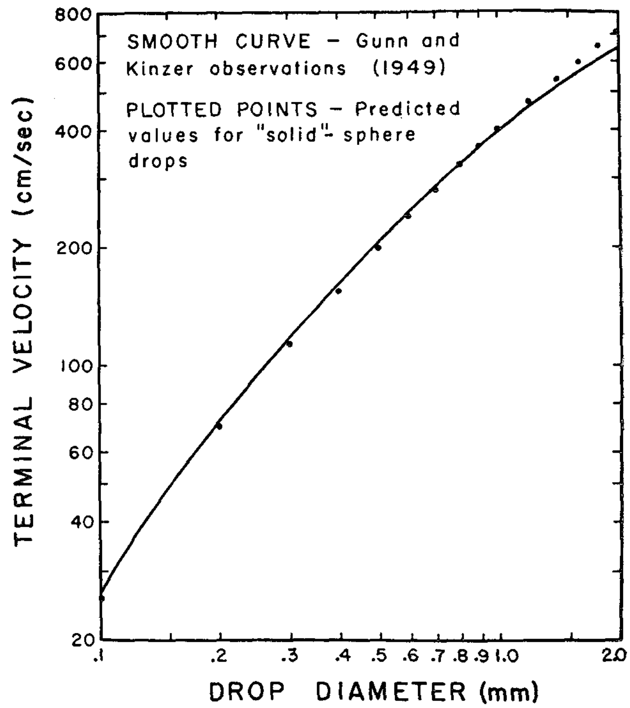

FIG. 1. Predicted and observed values.

The mechanism by which this drag-reduction would operate is as follows: any downstream (*i.e.*, upward) motion of the *water* at the water-air interface will require that the surface *air* layers (which are at rest relative to the surface water layer, by the no-slip condition) also be drifting toward the upper pole of the drop. This latter air motion at the base of the air boundary layer will serve to lower slightly the mean gradient through the boundary layer. Thus, the total skin-friction drag will be reduced and the pressure drag will also be lowered by virtue of delayed separation of the boundary layer.

The actual magnitudes of the velocity differences of fig. 1 are probably not of sufficient precision to yield useful quantitative data concerning the intensity of the internal circulations. The chief point of interest is the weak evidence for at least some internal circulation, confirming previous predictions (McDonald, 1954). In problems involving gas exchange between raindrops and atmosphere or involving internal solute concentration gradients inside evaporating drops, it is of value even to know only that some degree of stirring does take place within rain and drizzle drops.

[2] I am indebted to Dr. L. J. Battan for the suggestion that I test this method by computing fall velocities of raindrops and comparing them with the Gunn and Kinzer data. It was only after doing so that I noted the obvious implications relative to internal circulation effects.

REFERENCES

Gunn, R., and G. D. Kinzer, 1949: The terminal velocity of fall for water droplets in stagnant air. *J. Meteor.*, **6**, 243–248.
McDonald, J. E., 1954: The shape and aerodynamics of large raindrops, *J. Meteor.*, 11, 478–494.