of climatic prediction through use of, first, persistence and, secondly, the best weighted combination of climatology and persistence (regression) and compares these two. When the coefficient of correlation exceeds 0.80, persistence serves nearly as well as the combination, and both techniques provide appreciable improvement over climatology. Persistence rapidly loses its value at lower coefficients down to no help at 0.5, but for the best combination even a coefficient of 0.3 will provide nearly 5 per cent improvement over climatology.

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AN AID TO COMPUTATION OF TERMINAL FALL VELOCITIES OF SPHERES [1]

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There is frequent need in meteorological studies to compute the terminal velocity of smooth spheres falling freely through air. The problem of falling raindrops (smaller than the size suffering aerodynamic distortion) is a very familiar case and one for which good tabular data exist. But if one seeks to calculate terminal velocities for water drops at high altitudes or terminal velocities for smooth spheres of density other than that of water, a mathematical difficulty arises.

This difficulty enters in the following manner: on equating the aerodynamic drag force to the weight W of the sphere (or the buoyant force in the case of balloons), we obtain

$$\tfrac{1}{2}\rho_m V^2 C_d \pi r^2 = W = \tfrac{4}{3}\pi r^3 \rho_s g \qquad (1)$$

where ρ_m is the density of the fluid medium through which the sphere of radius r and density ρ_s is falling with uniform velocity V under the influence of the gravitational acceleration g, and C_d is the drag coefficient. Here and throughout the following, it will be assumed that ρ_s is so much greater than ρ_m that no buoyancy corrections must be made. To allow for buoyancy effects involves only quite simple changes, when required. Solving (1) for the desired velocity gives

$$V^2 = 8\rho_s g r / 3\rho_m C_d. \qquad (2)$$

The above-mentioned mathematical difficulty results from the fact that (2) is not amenable to analytic manipulation because C_d is itself a function of V through its dependence on the Reynolds number R, conventionally defined for spheres as

$$R \equiv 2Vr/\nu \qquad (3)$$

in which ν is the kinematic viscosity of the fluid medium. But the C_d-R dependence is only an empirical relation not expressible in terms of any elementary functions, so trial and error method of solution, involving reference to graphical C_d-R data, is the standard approach for predicting V. However, the difficulty may be removed by combining (1) and (3) to obtain

$$C_d R^2 = 8W/\pi\rho_m \nu^2. \qquad (4)$$

By plotting $C_d R^2$ as a function of R, we can obtain a curve that may be interpreted as relating R to

$$Q \equiv 8W/\pi\rho_m \nu^2.$$

Hence, a determinate method of finding V is as follows: compute Q from the given specifications of the problem, enter the R-Q plot to find the value R corresponding to this Q, and solve for V by using (2) rewritten in the form

$$V = \nu R/2r. \qquad (5)$$

In some terminal-velocity problems, the range of Reynolds numbers involved may extend above and below the upper limit of validity of Stokes' law; namely, $R \approx 1$. The procedure to be used in such cases is straightforward: one still begins by computing Q. But now if Q is equal to or less than 24 ($Re \leq 1$), one dispenses with the R-Q plot and computes V directly from the familiar Stokes' law relation

$$V^2 = 2\rho_s g r^2/9\mu \qquad (6)$$

where μ is the absolute viscosity of the medium, and where buoyancy effects are assumed unimportant as before. The value of V obtained by using (6) at the limit $R = 1$ can be shown to be in error, relative to the more accurate value found from (5), by only about 0.5 per cent. As R decreases from unity, (6) rapidly becomes even more accurate. If $Q \geq 24$, one proceeds

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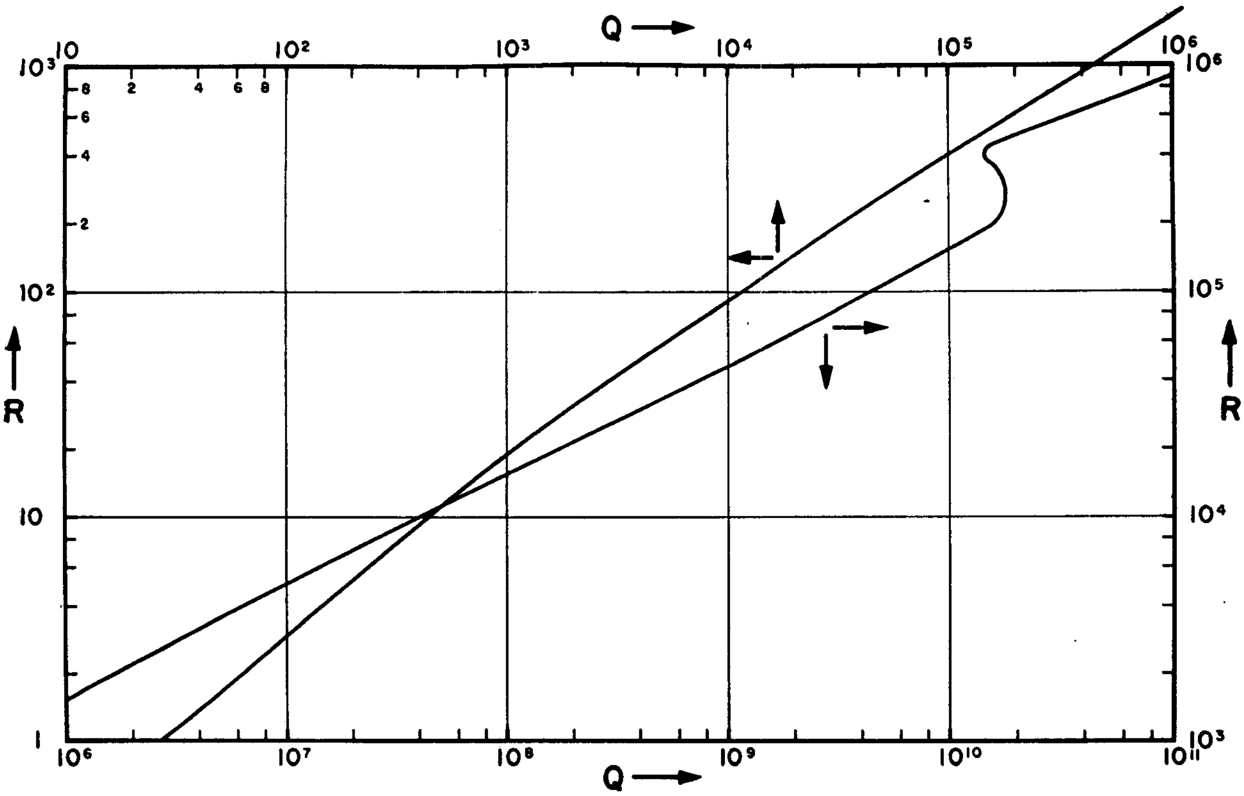

FIG. 1. The R-Q relation for the range from the upper R-limit of validity of Stokes' law to beyond the R-value for transition from laminar to turbulent boundary layer. Arrows indicate the scales to be used with each of the two sections of the R-Q curve.

via the R-Q plot as explained. Thus, trial-and-error methods are completely eliminated.

The above approach[2] is particularly convenient when a large number of terminal-velocity computations must be made, as in the case of predicting rates of descent of solid spherical fallout particles carried to great heights in thermonuclear explosions. For purposes such as the latter, one requires values of ν and ρ_m to altitudes extending into the stratosphere, so attention is called to recent publication of an extensive tabulation of just such data (U. S. Air Force, 1957). The present method is, of course, equally applicable to the prediction of terminal velocities of spheres moving through media other than air (*e.g.*, modelling experiments where water is used in place of air).

Fig. 1 presents a graph of the R-Q relation for values of Q ranging from 24 to 10^{11} (R ranging from 1 to 10^6). This graph was prepared from a fairly large-scale plot of C_d *versus* R given by Schilling (1955, p. 16). In the Q-interval from 1.4×10^{10} to 1.8×10^{10}, it will be noted that R is triple-valued with respect to Q. This corresponds to the well-known anomalous behavior of drag near the critical Reynolds number for transition from a laminar to a turbulent boundary layer. If Q for any given problem falls in this interval, real physical indeterminacy and attendant erratic

[2] While the present manuscript was in preparation, a paper by Sinha (1959) was received. The same basic method that is outlined here is discussed by Sinha with reference to the prediction of pilot-balloon ascent rates. The pilot-balloon problem is dynamically equivalent to the terminal-velocity problem, of course. Sinha presents what has here been called the R-Q relation for only a very limited range of R (about 10^4 to 10^5 as compared with the range of 10^0 to 10^6 considered here), and he displays it in a form directly applicable only to the balloon problem.

Footnote added in press. While the present paper was in press, the author discovered that Langmuir (*J. Meteor.*, **5**, p. 179) has already discussed a method identical in principle to that proposed here for handling the terminal velocity problem. Furthermore, Mason (*Physics of Clouds*, London, Oxford, 1957, p. 420), in summarizing what appears to be Langmuir's treatment, suggests use of an empirical equation (due to Langmuir and Blodgett) relating R and C_dR^2. I find, however, that the latter equation gives results about 20 per cent *low* near R = 100, about 20 per cent *high* near $R = 10^5$, and is inapplicable in the important region near the point of boundary-layer transition at about 2×10^5. Hence, it may be said that fig. 1 offers at least improved accuracy and range in use of a method originated by Langmuir and independently developed here.

TABLE 1. Basic data for the R-Q relation.

R	C_d	Q	R	C_d	Q
1	24	24	2×10^3	0.41	1.64×10^6
2	16	64	4×10^3	0.40	6.40×10^6
4	8.8	140	6×10^3	0.40	1.44×10^7
6	6.4	230	1×10^4	0.41	4.10×10^7
10	4.2	420	2×10^4	0.44	1.76×10^8
20	2.9	1.16×10^3	4×10^4	0.46	7.33×10^8
40	1.8	2.88×10^3	6×10^4	0.46	1.65×10^9
60	1.5	5.40×10^3	1×10^5	0.44	4.40×10^9
100	1.2	1.20×10^4	2×10^5	0.41	1.64×10^{10}
200	0.80	3.20×10^4	3×10^5	0.20	1.80×10^{10}
400	0.61	9.76×10^4	4×10^5	0.09	1.44×10^{10}
600	0.54	1.94×10^5	6×10^5	0.10	3.60×10^{10}
1×10^3	0.46	4.60×10^5	1×10^6	0.14	1.40×10^{11}

aerodynamic behavior must be expected. By coincidence, typical pilot-balloon conditions fall in this very R-region, with consequences unfavorable to single-theodolite accuracies (Lettau, 1939, p. 97).

Direct use of fig. 1 will yield adequate accuracy for many problems; but, for the convenience of investigators seeking the slightly higher precision that the original drag data afford, the original values of R, C_d, and Q from which fig. 1 was plotted are reproduced in table 1. From them, a larger-scale plot can be prepared if needed.

To save other investigators the trouble of computing values of the quantity $Q/W = 8/\pi\rho_m\nu^2$ needed in applying the above methods to specific problems, Q/W values for selected altitudes Z in the currently accepted I.C.A.O. Standard Atmosphere have been computed and are listed in table 2. For additional

TABLE 2. Values of ν and $Q/W \equiv 8/\pi\rho_m\nu^2$ for selected altitudes in the I.C.A.O. Standard Atmosphere.

Z (km)	ν (cm²/sec)	Q/W (sec²/gm cm)	Z (km)	ν (cm²/sec)	Q/W (sec²/gm cm)
0	1.46×10^{-1}	9.53×10^{4}	30	8.40	2.02×10^{3}
2	1.72×10^{-1}	8.59×10^{4}	40	4.13×10^{1}	3.71×10^{2}
4	2.03×10^{-1}	7.57×10^{4}	50	1.63×10^{2}	8.87×10^{1}
6	2.42×10^{-1}	6.61×10^{4}	60	4.69×10^{2}	3.31×10^{1}
8	2.91×10^{-1}	5.73×10^{4}	70	1.43×10^{3}	1.23×10^{1}
10	3.53×10^{-1}	4.95×10^{4}	80	6.05×10^{3}	3.21×10^{1}
15	7.30×10^{-1}	2.45×10^{4}	90	3.28×10^{4}	5.90×10^{-1}
20	1.60×10^{-1}	1.12×10^{4}	100	1.91×10^{5}	9.81×10^{-2}
25	3.50	5.13×10^{3}			

reference use, the corresponding values of ν itself are shown (U. S. Air Force, 1957). It is a point of considerable importance in a variety of high-altitude aerodynamic problems that air near the base of the ionosphere has a kinematic viscosity about a million times greater than air near sea level. As increasing numbers of high-altitude vehicles come into use, opportunities to use the above fall-velocity-prediction technique will probably become fairly common even up to or above the 100-km level—hence, the extension of table 2 to that altitude.

In certain problems, it is helpful to know, prior to undertaking detailed calculations, what heights and velocities are associated with the Stokes' law limit for spheres of specified nature. These can readily be found. From a graph of the Q/W data of table 2, read out the value of height Z associated with Q/W equal to $24/W$, where W is the weight of the sphere in question. Then find, from a plot of the ν data of table 2, the value of ν at this height, and use it to compute V from (5), with R equal to unity by hypothesis. As an illustration, suppose one seeks the Stokes' law limit for spheres of 1-mm diameter and density 2.5 g per cm³. W is then 1.28 dynes, so $24/W$ is 18.7 sec² per g cm. Hence, from a graph based on table 2, one finds Z to be 66 km at the Stokes' law limit. At this height, the sphere falls with terminal velocity of 90 m per sec. This speed is about 14 times greater than the same sphere's sea-level terminal velocity that is associated with a Reynolds number of 450. At 66 km, ν is about 6100 times greater than at sea level, so the sphere can fall 6100/450 times faster and still have $R = 1$ at this altitude. The example thus illustrates not only the method in question but also the somewhat peculiar properties of the upper atmosphere considered as an aerodynamic medium.

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ON A DIURNAL VARIATION OF STRATOSPHERIC WINDS [1]

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It has been observed from the trajectories of constant-altitude balloons that a large part of total variability of the stratospheric summer easterlies is produced by oscillations of shorter period than would be associated with motions of synoptic scale (Mantis, 1959). In an effort to determine the origin of the observed motions, it was decided to investigate both semidiurnal and diurnal periodicities of the observed motions.[2]

Mainly to make the data more homogeneous and reduce the amount of background variability, the study was restricted to data at altitudes of 95,000 ft

[1] This research was supported by the Office of Naval Research under Contract Nonr-710(22).

[2] This was done despite the fact that the resonance theory for the semidiurnal surface pressure oscillation predicts a nodal surface very near the balloon altitudes. (Weekes, K., and M. V. Wilkes, 1947: Atmospheric oscillations and the resonance theory. *Proc. r. Soc. of London*, Ser. A, **192**, 80–99.)