

## DIRECT ABSORPTION OF SOLAR RADIATION BY ATMOSPHERIC WATER VAPOR<sup>1</sup>

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### ABSTRACT

A review of Fowle's solar-absorption studies reveals that no pressure corrections must be applied to the familiar Fowle band-absorptivity curves since these were obtained by methods that automatically corrected them to sea-level pressures. Using these absorptivities, along with recent solar-energy-distribution data, total insolational absorptivities are obtained. These are fitted (with relative accuracy of about 1 per cent over the range 0.3 to 8.0, precipitable water) to an exponential function similar to that used by Mügge and Möller to obtain working relations for use in calculating daily heating amounts due to insolational absorption by water vapor. It is found that variation of the pressure exponent from zero to unity in trial pressure-correction laws yields only a six-per-cent variation in total columnar absorption for a concrete case. Estimates accurate to within about ten per cent with respect to the Fowle absorption data are attainable for levels below about 400 mb in humid regions. However, clarification of the method employed by Houghton in arriving at his roughly thirty per cent greater absorptivities leaves doubt as to the status of the familiar Fowle absorptions, so it is concluded that we cannot now claim to be able to predict insolational heating rates to better than about thirty per cent absolute accuracy.

### 1. Introduction

In studies of the atmospheric heat balance, it is necessary to have quantitative estimates of the rate of free-air heating due to absorption of sunlight by water vapor and other gases. I recently encountered need for the same type of heating estimates in the course of an analysis of radiosonde radiation errors and diurnal temperature variations in the middle troposphere. On reviewing the literature of the subject of insolational heating, I found that certain improvements and simplifications might be made in existing methods of estimating absorptive heating rates. This paper summarizes my results, and it also contains a brief discussion of certain aspects of the basic Fowle absorption data that have not previously received adequate attention.

### 2. Discussion of earlier work

It should be stressed that all but a single effort to compute free-air insolation absorption during the past forty years depend ultimately upon the observational work of a single investigator, F. E. Fowle (1912, 1915). The present paper falls into the same category. Kimball (1927, 1930) made the first attempt to put Fowle's results into a form more directly useful to meteorologists. Kimball, unfortunately, gave a very inadequate description of his exact method of converting Fowle's data into the curve that has subsequently been so widely reproduced (see, for example,

List, 1951, p. 437, curve number 16), and unexplained differences exist between his 1927 and 1930 versions of this curve. Houghton (1954) has expressed doubts concerning the validity of Kimball's curve, and has presented an absorptivity function considerably different from Kimball's and all other published curves. Houghton's work is very important in that it appears to be the sole effort to arrive at absorption values by use of data other than those in Fowle's papers specifically concerned with absorption itself (see footnote 5 below).

Mügge and Möller (1932) employed Fowle's basic data, again in a manner whose details were not made clear to the reader, to derive an empirical function fitted to the Fowle absorptivities. They implied that their absorption function's validity was limited primarily to a range of precipitable water vapor of 0.5 to 8 cm, apparently unaware that Fowle's entire work rested ultimately upon *laboratory* measurements confined to path lengths of less than 0.5 cm, but extended, by indirect methods involving Mt. Wilson bolometric insolation measurements, to the range 0.5 to 8 cm. Tanck (1940) applied the Mügge-Möller function to an extended discussion of solar heating rates over Hamburg.

Elsasser (1942) very briefly discussed the solar absorption problem. For the benefit of readers who might consult Elsasser's monograph it should be noted that in his formulation of the final working relation (his equation 10.2) he omitted the mixing ratio as a factor. In addition, Elsasser quite incorrectly implied that Kimball's absorption curve was

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based directly on an average of a large number of pyr heliometric observations. Elsasser apparently took his Kimball curve from a paper by Hoelper, who may have mistranslated Kimball's paper with its large bulk of pyr heliometric data immediately preceding a too-brief discussion of the derivation of the absorption curve from the Fowle data. Elsasser added confusion to the question of pressure-correction of the solar absorption data when he misinterpreted Fowle's original discussion (1912, p. 160) of the implications of spectral measurements due to V. Bahr. Fowle's point was that use of pressure-corrections similar to those suggested by Bahr would change an uncorrected *transmission* of 88 per cent to values of about 89 per cent in winter and 90 per cent in summer. Elsasser erroneously construed this to mean that application of such a pressure-correction reduces the effective precipitable water-vapor content to 89–90 per cent of the uncorrected amounts.

Yamamoto and Onishi (1952) have presented a discussion of insolation absorption by water vapor, applying Elsasser's method of generalized transmission coefficients to the original Fowle data and obtaining a graphic method of determining columnar absorption rates. One may question the justification of such elaborate mathematical manipulations of the Fowle data which, as will be noted below, were put by Fowle himself into a form fairly readily adaptable to computations of the type needed in heat balance investigations. In addition, when one is specifically seeking heating rates rather than columnar energy-accumulation rates, the absorption-chart method of Yamamoto and Onishi is considerably less convenient than the analytic approach of Mücke-Möller, after which the method outlined below is patterned. Finally, as will be elaborated in section 6 below, there appear to exist unexplainable discrepancies between the basic Fowle data and the Yamamoto-Onishi chart values.

### 3. Fowle's absorption data

There appears to be no published discussion of the question of whether the original Fowle absorption data are to be interpreted as applicable directly at sea-level pressure. The question warrants attention because these absorption data were based upon measurements made from a station (Mt. Wilson) at a pressure of only 830 mb, using an atmospheric path length over which the pressures varied from 830 mb to zero. Regrettably, Fowle himself was not at all specific on this point. However, a careful reexamination of all of the steps involved in Fowle's approach leads me to conclude that no pressure adjustment is required.

A complete justification of this conclusion would require too detailed a recapitulation of Fowle's

methods to be given here. However, it should be noted that the crux of the justification rests on the important fact that Fowle calibrated his Mt. Wilson observations against his own careful laboratory measurements conducted at sea-level pressure in Washington, D. C. The latter measurements, made with folded optical path lengths of up to 800 ft, permitted him to relate precipitable water-vapor amounts (optical masses) of up to 0.5 cm to their resultant absorption effects as manifested in the  $\rho$ ,  $\Phi$ , and  $\psi$  bands of the water-vapor spectrum. He used these calibrations at Mt. Wilson (without pressure-corrections of any sort) to extend his curves of band absorptivity *versus* optical mass from 0.5 to 8 cm, by a method whose details must also here be omitted (Fowle, 1912, 1915). Briefly, this method automatically gave Fowle absorptivities expressed in terms of precipitable water-vapor amounts *effective at sea level*. This latter conclusion is in accord with a recent sea-level test of Fowle's method which Aldrich (1949) has carried out using path lengths of up to 1.5 cm precipitable water, although Aldrich, like Fowle, makes no explicit mention of the fact that Fowle's curves were automatically pressure corrected to sea level.<sup>2</sup>

When using Fowle's absorption curves, one must know the exact wave-length limits which he associated, slightly arbitrarily, with each absorption band. Fowle appears to have given these explicitly only in a single table (1915, p. 408), and two of the band limits are there quite obviously in error (namely, the upper limit of the  $\rho$  band which, as given, does not even contain the band-center, and the upper limit of the  $\Omega$  band). The limits given by List (1951, p. 425) appear to have been corrected for both bands in that List's limits closely match band-limits indicated graphically in fig. 2 of Fowle's 1915 paper; therefore, I shall assume here that List's band-limits are correct—*i.e.*, that they are the ones Fowle actually employed in reducing his data.

The following question arises in using Fowle's absorptions to get solar heating rates: should one regard the absorption as occurring first, followed by scattering, as did Houghton (1954), or should one follow Fowle and let the scattering coefficients get first cut at the solar beam? Clearly, neither assumption is physically correct, since the two processes go on more or less concurrently. For simplicity and for reasons cited in the following two paragraphs, I shall

<sup>2</sup> The present conclusion that the Fowle absorptivities, as reproduced for example by List (1951, p. 426), are intrinsically pressure-corrected to sea level despite their being derived from direct solar measurements using optical paths lying far above sea level must not be construed as insuring correctness of Fowle's inferences as to the actual precipitable water-vapor contents overhead at Mt. Wilson during his observations. Fowle (1915, p. 398) appeared to understand this point in a general way but did not discuss it adequately in his papers.

TABLE 1. Computation of insolation absorptivities summed over six water-vapor bands. Data from Fowle (1915).

Absorption band	Wave-length interval ( $\mu$ )	Solar flux in band (ly/min)	Absorbed flux (ly/min) per band for specified values of precipitable water vapor $u$							
			$u = 0.5$ cm	$u = 1$ cm	$u = 2$ cm	$u = 3$ cm	$u = 4$ cm	$u = 5$ cm	$u = 6$ cm	$u = 8$ cm
$a$	0.70–0.74	0.0784	0.0016	0.0024	0.0047	0.0055	0.0071	0.0086	0.0102	0.0133
$0.8 \mu$	0.79–0.84	0.0777	0.0019	0.0031	0.0054	0.0062	0.0077	0.0093	0.0108	0.0140
$\rho$	0.86–0.99	0.1578	0.0142	0.0205	0.0300	0.0363	0.0426	0.0490	0.0520	0.0584
$\Phi$	1.03–1.23	0.1648	0.0165	0.0230	0.0329	0.0379	0.0445	0.0478	0.0510	0.0560
$\Psi$	1.24–1.53	0.1432	0.0538	0.0601	0.0686	0.0758	0.0801	0.0830	0.0844	0.0858
$\Omega$	1.53–2.10	0.1209	0.0350	0.0386	0.0423	0.0447	0.0458	0.0471	0.0496	0.0508
Total absorbed flux in all six bands			0.1230	0.1477	0.1839	0.2064	0.2278	0.2448	0.2580	0.2783
Total insolation absorptivity			0.063	0.076	0.095	0.106	0.117	0.126	0.133	0.144
Absorptivities calculated from (1)			0.062	0.077	0.095	0.107	0.117	0.125	0.132	0.144

follow Houghton and assume that absorption comes first.

A closely related question that must be considered is the following: what additional heating arises from water-vapor absorption of the scattered radiation itself, particularly that which is back-scattered from fairly low altitudes and hence has a long return path to outer space? As noted by Houghton (1954), the scattered flux is chiefly at the blue, the absorbed at the red and infrared end of the solar spectrum; yet some scattered flux is surely absorbed. It would be most difficult to assess this contribution quantitatively. In assuming the absorption to occur before any scattering, one tends to cancel errors due to neglect of absorption of the scattered flux. Indeed, one probably overcancels this error in such a way as to help offset the additional small negative error incurred by neglect of dry-air absorption (see next paragraph).

Finally, when it is *total* atmospheric heating that one seeks (true of most heat-balance studies and diurnal temperature-variation studies), some small allowance ought to be made for absorption by dry air. Because the water-vapor mixing ratio is strongly variable with height, no simple analytic or graphic method can ever correctly embrace this small additional term due to two electronic bands of the  $O_2$  molecule. Fowle's results (1915) show that, by and large, the magnitude of the dry-air absorption amounts to around five per cent of that of the water-vapor absorption (more in dry-air columns, less in moist-air columns). As was noted above, it is assumed here that the error of underestimate of total free-air heating incurred by neglecting the oxygen absorption probably is offset by other small errors, so no specific allowance for dry-air absorption will be made here.

It should be clear from the comments of this section, as well as from comments below on residual uncertainties as to correct form of the pressure-correction, that we are still unable to predict very accurately the solar-heating rates in the free air. Probably errors of at least ten per cent must be assumed to be present in

past and present estimates of insolational-heating rates due solely to the several approximations just cited. (See also footnote 5 below for sources of still greater uncertainty.) For this very reason, it seems out of proportion to overelaborate any one portion of the treatment of this problem, and hence I turn next to a simple and straightforward method of using the Fowle absorption data.

#### 4. Insolational-heating equation

By using the Fowle absorption-band intervals as specified by List (1951, p. 425) and data (*ibid.*, p. 416) on the energy distribution of solar radiation outside the earth's atmosphere (in keeping with the decision to assume the absorption to precede any other atmospheric depletion processes), I have computed the solar flux lying within each of the six water-vapor bands of importance in free-air absorption, as shown in table 1. Then, for each of a number of values of precipitable water vapor,  $u$ , ranging from 0.5 to 8 cm, I have read from the Fowle curves (List, p. 426) the band absorptivities. Multiplying the several absorptivities into the appropriate fluxes yields actually absorbed flux amounts for each of the six bands, as displayed in table 1.<sup>3</sup> Since the solar-energy distribution here employed is based on a solar constant of 1.94 ly per min, the spectrally weighted total water-vapor absorptivity for each value of  $u$  is obtained simply by dividing the total absorbed flux by 1.94 ly per min to obtain the weighted total absorptivities shown in the next to the last row of table 1.

These total insolational absorptivities, it should be noted, will not accurately apply to a beam of radiation whose energy distribution is of significantly different shape from that of the solar-energy curve at the top of the atmosphere. It is of physical interest to note from table 1 that between a third and a half of all

<sup>3</sup> Values of flux absorbed within individual bands are shown in table 1 to the fourth decimal place only to reduce round-off error. Limitations in reading absorptivities from Fowle's curves hold the precision down to two significant digits, in general.

sunlight absorbed by water vapor is removed by the great  $\psi$  band centered near  $1.4 \mu$ .<sup>4</sup>

The absorptivities were next fitted to an empirical exponential function to get the relation

$$a(u) = 0.077u^{0.30} \quad (1)$$

where  $a(u)$  is the total insolation absorptivity for an optical path whose precipitable water-vapor content is  $u$  cm (effective at sea-level pressure). In the last line of table 1, the values of  $a(u)$  computed from (1) are shown below each corresponding Fowle value to display the goodness of fit of (1). It will be seen that agreement is about one per cent over the 0.5–8 cm range.

Because one is frequently interested in midday insolational heating rates at levels in the middle troposphere for which the pressure-corrected precipitable water-vapor values are often below 0.5 cm, it is of considerable interest to know how reliable (1) may be for  $u$  less than 0.5 cm. In view of the fact, stressed above, that all of Fowle's work rested ultimately on laboratory measurements made with optical mass of less than this value, it is disturbing to find that he never published in any of his numerous papers any summary of absorptivities of all six bands in this low- $u$  range. The only pertinent datum I have succeeded in finding is a curve of total band areas (expressed in the kind of arbitrary units Fowle all too often used) that shows plotted points to about  $u = 0.5$  cm plus an extrapolated curve extending to  $u = 0.12$  cm (Fowle, 1915, p. 399, curve  $e$ ). I find, on dividing each ordinate by  $u^{0.30}$  for selected points from  $u = 8$  cm down to 0.12 cm, that the functional form of (1) appears to satisfy the curve to better than one per cent at  $u = 0.33$  cm and to about five per cent at the lower limit of  $u = 0.12$  of this particular curve.

As further indication of the validity of the empirical relation (1) for values of  $u$  below 0.5 cm, consider fig. 1. The plotted points represent the weighted total insolational absorptivities summed over all six bands, taken from the next to the last line of table 1. The abscissa is laid out on a scale of  $u^{0.30}$ , so a plot of the relation (1) becomes a straight line passing through the origin, as shown. The point of present interest is the fact that this line does seem to represent quite unequivocally the extrapolation of the plotted points. A similar plot (not shown) of the six individual bands reveals that the latter are *not* all linear when their absorptivities are plotted against  $u^{0.30}$ , particularly the important  $\psi$  and  $\Omega$  bands. But when reasonable extrapolations of these individual bands were drawn on

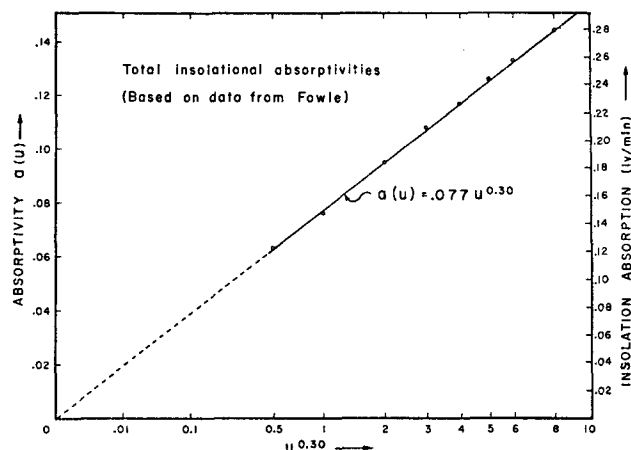


FIG. 1. Total insolational absorptivities as a function of  $u^{0.30}$ . The plotted points are taken from table 1. The straight line through the origin is the empirically fitted exponential function representing the Fowle absorptivities. Ordinate scale at right gives total columnar insolation absorption in ly per min for columnar effective precipitable water-vapor  $u$ , assuming solar constant of 1.94 ly per min.

such a plot and used, after the manner of table 1, to estimate the absorptivities below 0.5 cm, the results fell very close to the dashed extrapolation in fig. 1. For example, at  $u = 0.1$  cm, such a procedure gave 0.041, as compared with the value 0.039 predicted by (1). In those parts of the atmosphere where such low pressure-corrected precipitable water-vapor amounts prevail, the unreliability of radiosonde humidity data will usually be the limiting factor, rather than absorptivity data. Needless to say, no simple theoretical justification for a relation of form (1) may be given since it represents a sum over broad bands of quite different absorptivities and also involves the shape of the solar-energy curve itself. Inasmuch as one cannot, in view of various approximations already cited, regard the entire approach to estimating insolational heating rates as accurate, relative to the Fowle data, to better than about ten per cent, it seems in order to say that the empirical relation (1) is quite compatible with overall accuracy for values of  $u$  down to about 0.1 cm, and probably even lower. For high-sun conditions, this limit will apply to pressure-corrected  $u$ -values at about the 400-mb level in humid regions in summer. In winter or in dry air masses, this limit will occur somewhat lower in the atmosphere, a point to be kept in mind in applying (1).

In order to use (1), it is necessary to substitute for  $u$  the pressure-corrected precipitable water-vapor  $u'$ , which from here on is to be understood to be measured vertically as in hydrometeorological work. The question of what pressure law should be employed is a question in a state of some present uncertainty. Elsasser (1942) recommended a square-root pressure correction to precipitable water amounts for reasons

<sup>4</sup> This band is actually an overlapping composite of two distinct vibration-rotation bands involving transitions from the vibrational ground state to the 1, 0, 1 and the 0, 2, 1 vibration states, respectively.

that Kaplan (1952) has subsequently rejected. Kaplan indicated that the linear pressure-correction law was preferable to the square root if any attempt at pressure adjustment is to be made at all. Howard, Burch, and Williams (1956) have more recently presented laboratory measurements suggesting that corrections involving the 0.30-power of the pressure be used. Fortunately, as will be shown in section 5 below, varying the pressure exponent from zero to unity produces a surprisingly small variation in the final calculated heating rates, due to operation of two opposing influences of the pressure-broadening effects. The linear law will be used here on the rough basis of Kaplan's (1952) discussion.

On this basis, the pressure-corrected precipitable water-vapor content  $u'$  (cm) of the atmospheric column extending from the top of the atmosphere down to the level at pressure  $p$  is given by

$$u'(p) = \rho_w \int_0^p \frac{pw}{p_0 g} dp \quad (2)$$

where  $w$  is the mixing ratio expressed in g per kg,  $g$  is 980 cm per sec<sup>2</sup>,  $p$  is expressed in mb, and  $\rho_w$  is the density of liquid water.

Assuming then that  $u'$  has been obtained by application of the linear pressure correction, using  $p_0 = 1000$  mb as an adequate approximation to sea-level pressure and taking account of the fact that, in general, the sun is not in zenith but stands at some zenith angle  $Z$ , we may rewrite (1) in terms of the rate of energy absorption  $A(u', Z)$  in the *slant* optical path from the top of the atmosphere down to the level where the effective precipitable water vapor is  $u'$ , as follows:

$$A(u', Z) = 1.94[0.077(u' \sec Z)^{0.30}]$$

where  $A$  is in ly per min due to introduction of the solar constant. Note carefully that  $A$  is the absorbed power in a cylinder of unit cross-sectional area inclined to the vertical at angle  $Z$ .

The local rate of energy absorption per unit horizontal area within a *horizontal* slab of the atmosphere of mass  $dm$  per unit area can then be written

$$c_p \dot{T} dm = \cos Z dA = 2.7(u' \sec Z)^{-0.70} du'$$

where  $c_p$  is the specific heat of air at constant pressure and  $\dot{T}$  is the rate of temperature rise in deg C/hr. Replacing  $dm$  by  $10^3 dp/g$  and  $du'$  from the differential form of (2) yields as the final working equation

$$\dot{T} = 0.011(p/p_0)w(u' \sec Z)^{-0.70} (\text{deg C hr}^{-1}). \quad (3)$$

In heat-balance applications, one typically seeks the daily heating rate (more accurately, the amount by which a given free-air parcel's temperature would rise over 24 hr if no infrared emissive losses were opposing the daylight accumulation of absorbed insolation),

which requires, effectively, that (3) be integrated from sunrise to sunset. The form of (3), containing as it does the factorable quantity  $(\sec Z)^{-0.70}$ , permits very convenient numerical integration over the daylight period. Experience reveals that more than adequate accuracy is obtained by evaluating the zenith-angle factor at the midpoints of one-hour intervals, so the daily heating rate is expressible as

$$\Delta T(u') = 2[0.011(p/p_0)wu'^{-0.70}] \sum \{(\sec Z)^{-0.70}\} \quad (4)$$

where  $\Delta T(u')$  is the *daily* absorptive contribution to temperature change at the level  $u'$ . The summation is taken over values of the secant factor at one-hour intervals of local solar-hour angle (namely, 0.5, 1.5, 2.5... ) extending back to the sunrise-sunset hour angle, but making, of course, due allowance for any fractional parts of an hour near the sunrise-sunset time. The factor 2 on the right side of (4) follows from the morning-afternoon symmetry of the heating functions. In a check made on the sample calculation of section 5 below, it was found that the simple hourly secant-factor summation of (4) was less than two per cent larger than the planimetrically evaluated integral of the secant function, a wholly negligible error.

Two practical recommendations may be made to workers who may wish to apply the present results in heat-balance studies:

(1) If one seeks only total absorptive-energy accumulations for the entire atmospheric column from the top of the atmosphere down to the surface, it is suggested that from equation (1) an enlarged version of fig. 1 be prepared, preferably with linear  $u$ -scale to facilitate interpolation. Then, using an energy-absorption scale of the type displayed at the right-hand edge of fig. 1, one can read out directly the columnar absorptions for assumed *zenith sun* by entering the graph with the pressure-corrected total precipitable water at the surface level. And, if desired, the succeeding step of taking account of the zenith-angle factor can also be incorporated by proceeding to construct an "absorption chart" of the same form as that presented by Yamamoto and Onishi (1952) so that the complete answer may be read out for any given total effective precipitable water and solar zenith angle. However, if heating rates at specific levels are sought, use of (4) itself is very much more convenient (and precise) than the method of Yamamoto and Onishi.

(2) Since most of the labor of calculating a series of absorptive heating profiles is spent in calculation of the relation between pressure (or height) and  $u'$ , attention may be called to the fact that this problem may easily be solved graphically in any of several ways sufficiently obvious to require no details here. In a large series of calculations, preparation of such a graphical aid would effect considerable time-saving.

### 5. Sample calculation

To illustrate the height-variations of absorptive heating rates through a deep column of tropospheric air, a sounding having humidity data to a maximum height is desirable, so the mean monthly sounding for Phoenix, Arizona for July 1957 was chosen. This sounding, as published in the U. S. Weather Bureau *National Summary Climatological Data*, included mean humidities up to the 300-mb level. To extend the computation somewhat higher, it was arbitrarily assumed that the 300-mb relative humidity (39 per cent) also prevailed at the 250- and 200-mb levels.

In table 2, computational steps in calculating the mean July, 1957, Phoenix absorptive heating amounts are shown. After determining the true values of the 50-mb increments  $\Delta u$  of precipitable water vapor, these are linearly pressure corrected and summed downwards to get  $u'$  for each pressure-level. Hypothetical zenith-sun heating rates given by the quantity within brackets in (4) are then computed as displayed in the next to last column of table 2. The

TABLE 2. Computation of clear-sky daily insolational heating rates in the free atmosphere over Phoenix, Arizona for mean July 1957 sounding.

$p$ (mb)	$w$ (g/kg)	$\Delta u$ (cm)	$\Delta u(p/p_0)$ (cm)	$u'$ (cm)	$0.11w(p/p_0)u'^{-0.70}$	Daily heating (°C)
200	(0.05)					
250	(0.15)	0.005	0.001	0.001	0.052	0.50
300	0.31	0.012	0.003	0.004	0.042	0.40
350	0.56	0.022	0.007	0.011	0.052	0.50
400	0.90	0.037	0.014	0.025	0.054	0.52
450	1.6	0.064	0.027	0.052	0.064	0.61
500	2.4	0.102	0.048	0.100	0.068	0.65
550	3.5	0.151	0.079	0.179	0.072	0.69
600	4.7	0.209	0.120	0.299	0.073	0.70
650	6.1	0.275	0.172	0.471	0.075	0.72
700	7.0	0.334	0.225	0.696	0.071	0.68
750	7.9	0.380	0.276	0.972	0.068	0.65
800	8.6	0.421	0.326	1.30	0.064	0.61
850	8.9	0.446	0.368	1.67	0.059	0.56
900	9.7	0.474	0.414	2.08	0.059	0.56
950	10.9	0.525	0.486	2.57	0.060	0.57

remaining factor of equation (4),  $2\Sigma(\sec Z)^{-0.70}$ , is computed as indicated in table 3 for mean July conditions at Phoenix, using standard astronomical formulae. In the last column of table 2, multiplication of the zenith-sun hourly heating rates by this day-long

TABLE 3. Computation of  $2\Sigma(\sec Z)^{-0.70}$  for July at Phoenix, Arizona. (Sunrise-sunset solar hour angle 7.0 hr.)

Hour angle (hr)	$Z$ (degrees)	$(\sec Z)^{-0.70}$
0.5	13	0.982
1.5	23	0.947
2.5	35	0.870
3.5	48	0.755
4.5	60	0.616
5.5	73	0.424
6.5	85	0.180
		$2\Sigma(\sec Z)^{-0.70} = 9.55$

sum of the secant factors yields the desired daily heating estimates for each level.

In order to establish, for a concrete case, the magnitude of the changes in daily heating amounts imposed by changing from one pressure-correction law to another, all of the steps of table 2 were repeated for three other pressure-law assumptions. In fig. 2, plots are shown of the resulting four profiles of daily

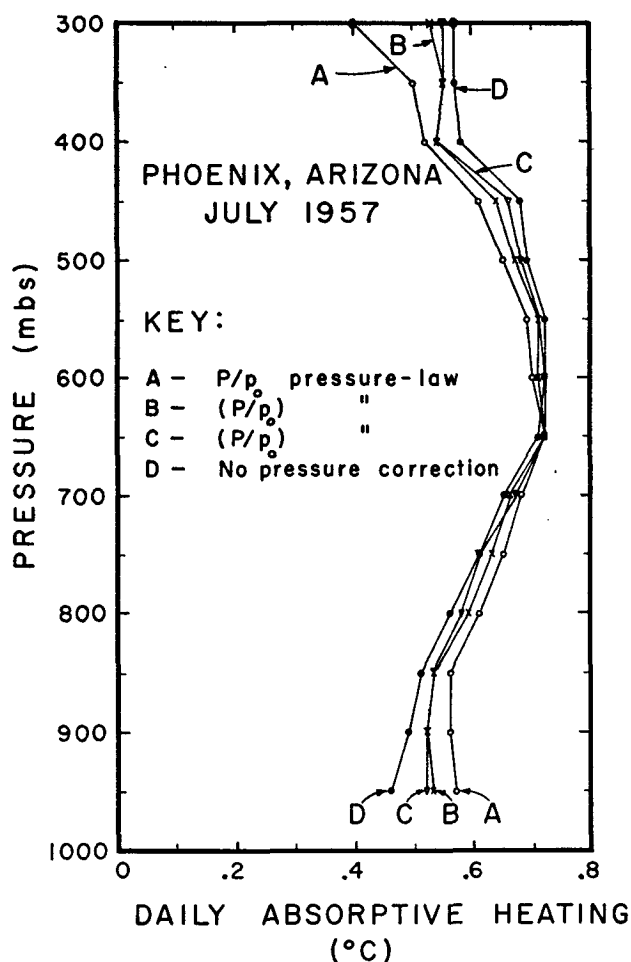


FIG. 2. Vertical profiles of daily temperature increase due to insolation absorption for four assumed pressure-correction laws. Data for mean July 1957 sounding for Phoenix, Arizona. (Errata: In the key,  $(p/p_0)$  should be raised to the exponent 0.5 for B and to 0.3 for C.)

heating amounts for laws ranging from no pressure-correction at all to the most extreme form that has been suggested up to the present, the linear law. It will be seen that the insolational-heating values are relatively insensitive to the form of the pressure law, especially in the 500- to 700-mb region. Specifically, the percentage excesses of the estimates based on no pressure correction over the estimates based on the linear pressure correction run as follows for every hundred-millibar level from 300 to 900: 42, 11, 6, 3, -4, -8, -13. Only in the upper troposphere is it likely that errors due to inadequacy of pressure correction can exceed the roughly ten per cent error inherent in such other approximations as neglect of scattered radiation, *etc.*

Furthermore, it is noteworthy that the several curves cross each other in such a way that the columnar total energy accumulations must vary still less due to differences in assumed pressure law. To check this latter inference, columnar totals were computed directly by the method described above. The results were as follows: linear, 117 ly per day; square root, 118 ly per day; 0.3-power, 121 ly per day; and no pressure correction, 124 ly per day. Thus, the highest of the four values is a mere six per cent greater than the least.

The percentual variation in solar-heating rates as altitude varies is relatively slight, as may be seen from fig. 2. The physical explanation for this has not been noted by previous writers but is readily understood with reference to equation (4). Within the brackets of (4), one may distinguish two opposing factors; one might call  $(p/p_0)w$  the "absorptive factor" and  $u'^{-0.70}$  the "screening factor" associated with the level where the effective precipitable water vapor is  $u'$ . As one proceeds downwards into the atmosphere, the absorptive factor almost invariably rises monotonically, tending thereby to increase the magnitude of the solar heating rates, but concurrently the screening factor invariably decreases in magnitude due to steadily rising optical mass overhead as one descends. The two factors so interact as to suppress greatly the height-variations in heating rate. They do not entirely eliminate such variations, however, and all of the pressure laws tested in fig. 2 yield profiles with a broad maximum centered near the 600-mb level. If one gives attention not to the relatively small percentual height variation but to the presence of this midtropospheric maximum, he is immediately reminded of the basic physical processes responsible for formation of a simple Chapman layer in the ionosphere, and indeed quite analogous physical phenomena are operative in the two instances.

It should be pointed out that the relatively insensitive response of heating rates to variation in assumed pressure-correction law found here is also

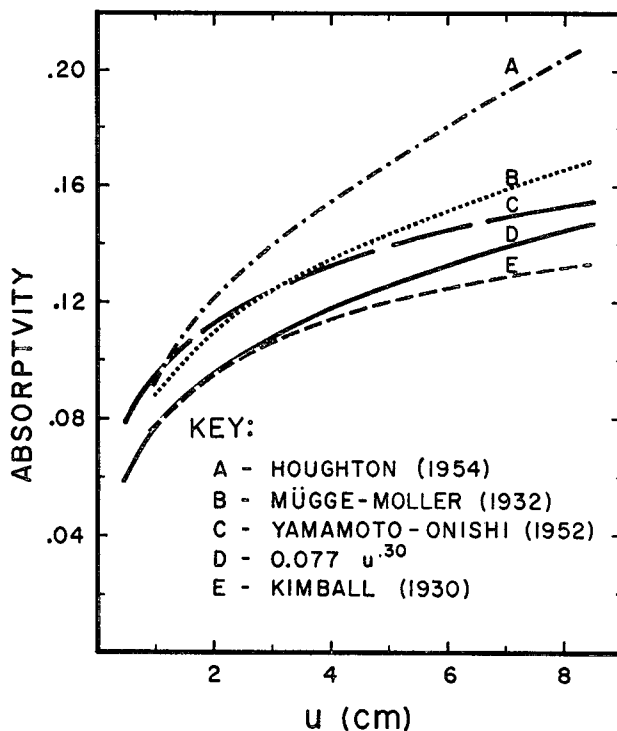


FIG. 3. Comparison of five curves of insolational absorptivity versus precipitable water-vapor  $u$ .

attributable to opposing influences of the absorptive factor and the screening factor. For example, when a strongly pressure-dependent law is used (*e.g.*,  $p/p_0$ ), this yields quite low absorptive factors at high altitudes but concurrently holds the screening factor to low values aloft, *etc.*

Finally, it should be pointed out that low sensitivity of total columnar absorption to form of pressure-correction is *not* due to any such opposing influence of the absorptive and the screening factors. The latter enter only into the formulation of heating rate at a specified level, which is an essentially differential phenomenon in contrast to the integral character of the total columnar absorption. The fact that varying the pressure exponent from zero to unity produced only a six-per-cent change in total absorption in the sample calculation results solely from the operation of the  $u^{0.30}$  factor itself. In the preceding sample case, the effective value of columnar total  $u'$  varied from 3.46 cm for a zero pressure exponent (*i.e.*, no pressure-correction) to 2.57 cm for unity pressure exponent (linear law), an excess of larger over smaller amounting to about 35 per cent, but the 0.30 powers of these total  $u'$ 's are found to differ by merely six per cent. This effect, to repeat, is quite unrelated to the opposition of the absorptive and screening factors.

## 6. Comparison with previous results

In fig. 3 comparative curves are displayed of insolational absorptivity versus precipitable water vapor as

deduced by five different investigators. The spread between these results, which involves an almost 40 per cent excess of highest (Houghton) over lowest (Kimball) at  $u = 8$  cm, is disconcertingly large. A sixth investigator, Karandikar (1946), obtained absorptivities considerably higher than those of Houghton, but Karandikar's are not shown because, as Yamamoto and Onishi (1952) have pointed out, he made assumptions that clearly limit applicability of his results to extremely low vapor contents such as exist only above the tropopause.

Houghton did not discuss the mechanical steps taken in arriving at his absorptivity curve so it was not initially possible to compare his method with that used here to obtain the absorptivities plotted as curve  $D$  in fig. 3.<sup>5</sup> Kimball's curve  $E$  (copied from List,

<sup>5</sup> After completion of the present manuscript, I received from Dr. Houghton a personal communication that explains how it was possible for his absorption values to be so much higher than all other published values, even though drawn from the data of the Smithsonian Astrophysical Observatory, of which Fowle was a member. The present analysis and all others except Houghton's 1954 analysis have been based on those of Fowle's publications that dealt with absorption *per se*. Houghton, however, had examined those particular publications and had rejected them when he discovered that they were not consistent with much more voluminous data published by Fowle and his Smithsonian co-workers in the course of their solar-constant studies. By methods which Houghton indicates could only be made clear through a very long discussion, he deduced from analysis of the Smithsonian solar-constant work a set of water-vapor absorption values that were internally consistent with the extensive body of "long method" observations. Apparently no one else has examined the latter data and their involved history in any such detail as Houghton has. Regrettably, this basic inconsistency between Fowle's absorption work and the absorption values implicit in his and Abbott's solar-constant data was never discussed or even noted in any publications of Fowle, Abbott, and associates, according to Houghton. As Houghton concludes, there appears to be nothing that can now alleviate this difficulty other than a whole new attack on this very fundamental problem of atmospheric depletion of the solar beam.

How, then, must the present results be regarded, in view of this clarification of the basis of the Houghton absorptivities? At best, the present results can now only be viewed as providing a fairly carefully checked and easily applied absorption equation and related methodology based upon the more familiar one of two apparently incompatible sets of Smithsonian absorption values—namely, the one yielding the *lower* values of absorption. This is a weak enough position to assume in submitting the present contribution that withdrawal was considered. However, because I have tried to discuss carefully a number of points of importance to an understanding of the present state of knowledge concerning insolational absorption by water vapor, points not to be found in any other published discussions, it is hoped that this paper may provide a useful summing up of a really quite confused situation in a problem of great conceptual importance in meteorology.

It may be noted that Houghton's study of this problem also led him to the conclusion, reached above in section 2, that Fowle's Mt. Wilson absorption data were automatically corrected to sea level (to at least first approximation). Houghton (1954) discusses Kimball's failure to make adequate pressure correction to the Smithsonian *scattering* data, but, in his personal communication, Houghton stresses that this is not to be interpreted as casting doubt on Kimball's handling of the pressure problem in using the Fowle *absorptivities*.

For the reader who wishes to use the calculational methods described above, but who seeks a probable *upper* limiting estimate of absorption rates, I should point out that the Houghton absorptions were found roughly to fit the following equation of the Mücke-Möller type, analogous to (1) above:

$$a(u) = 0.093u^{0.37}.$$

1951, p. 437, curve 16) was obtained by Kimball by methods not clarified by that investigator. This curve was published in Kimball's 1930 paper with reference made to his 1927 paper for details of method, but the 1927 paper gives essentially none, and it also contains a similar curve that does not even match the 1930 curve, so one is left doubly in doubt about Kimball's methods. There is not, however, any basis for doubting Kimball's results on grounds that he omitted a pressure correction, since none is required in reducing Fowle's data to a sea-level absorptivity curve.

Mücke and Möller (1932) also omitted details of their method of deducing curve  $B$  from the Fowle data, though they made a somewhat obscure reference to an "ultraviolet and infrared correction which Fowle and Abbott have given," which opens room for speculation. If they had *added* to the direct observed solar flux some too-large corrections for the unmeasurable ultraviolet and the infrared, they might have obtained a too-large solar constant, and this would push curve  $B$  above  $D$ . But  $B$  is some 15 per cent above  $D$  at  $u = 8$  cm, which demands a solar constant 15 per cent greater than 1.94 ly per min or about 2.2 ly per min, which is certainly far too large to have been acceptable to Mücke and Möller, even three decades ago. It may also be noted that, if Mücke and Möller had failed to detect the (typographical?) errors in the originally published band-limits (Fowle, 1915), the sense of the resulting error would have been such as to put their curve below, not above,  $D$ , which also fails to account for the discrepancy. The fact that the Mücke-Möller empirical function analogous to (1) has the same exponent as that found here (namely, 0.30) is a very strong indication that they proceeded by the same basic method, but I must conclude that their introduction of the solar constant involved some error whose nature cannot be deduced at this point.

The most recently published curve, curve  $C$ , was obtained by Yamamoto and Onishi (1952) by a somewhat circuitous method. To derive Elsasser-type generalized absorption coefficients from Fowle's relatively simple observational data, introduce these into the elaborate formalism employed by Yamamoto and Onishi, and finally work back to absorptivities (really the starting point when one turns to Fowle's work), is, I believe, dubious procedure. However, another even more simple difficulty comes to light on intercomparing their method with that used here. Yamamoto and Onishi state that they "used the energy curve of a black body at 6000K and a solar constant of 1.94 ly per min" which I take to mean that they assumed the *relative* distribution of total flux within each wave-

Both this relation and (1) are written with two digits in coefficient and exponent, but the user must be reminded that only one-digit significance can be expected.



length interval associated with Fowle's work to be that corresponding to Planck's function for 6000K but applied these fractional amounts per interval to a total flux of 1.94 ly per min. Since the sun's actual spectral emission is not identical with that of a 6000K black body, this opens the question of how such a method should tend to make curve *B* depart from my curve *D*. The simplest means of answering this is to insert this pair of assumptions of Yamamoto and Onishi into the method used above.

To do this, I first computed the *fraction F* of total blackbody flux lying in each of the six band intervals already cited, assuming the emission temperature to be 6000K and employing tables presented by List (1951, p. 412). Table 4 shows the resulting *F* values.

TABLE 4. Test of the Yamamoto-Onishi solar-energy distribution assumptions.

Absorption band	Wave-length interval ( $\mu$ )	<i>F</i>	Available flux in band = 1.94 <i>F</i> (ly/min)	Absorbed flux for $u = 1$ cm (ly/min)
<i>a</i>	0.70–0.74	0.0389	0.0755	0.0023
0.8 $\mu$	0.79–0.84	0.0394	0.0764	0.0031
$\rho$	0.86–0.99	0.0792	0.1534	0.0200
$\Phi$	1.03–1.23	0.0758	0.1470	0.0206
$\Psi$	1.24–1.53	0.0633	0.1228	0.0515
$\Omega$	1.53–2.10	0.0562	0.1090	0.0348
Sum = 0.1323 ly/min				

Multiplying these into 1.94 ly per min gave the band-interval flux amounts displayed in the fourth column of table 4. The last column of the table gives absorbed flux values computed for a single value of  $u$ , using band absorptivities from table 1. The insolational absorptivity can be computed by dividing the sum of the fluxes over all six bands, 0.132 ly per min, by the solar constant, yielding 0.068 as the  $u = 1$  cm insolational absorptivity compatible with the Yamamoto-Onishi solar-energy-distribution assumption. This value is even *smaller* than my table-1 value of 0.076, and it is much smaller than Yamamoto's and Onishi's own value of about 0.095 for  $u = 1$  cm. That is, one finds by the foregoing simple check that the errors inherent in Yamamoto's and Onishi's approximations concerning the solar flux should have tended to yield not higher, but even *lower*, absorptivities than I have obtained. This is because the sun is, as has been well established in recent rocket measurements, an emitter with effective emission temperature well under 6000K in the blue and ultraviolet end of the spectrum but well over 6000K in the infrared. Hence, Yamamoto's and Onishi's assumptions were certain to lead to underestimates of absorptivities as far as this one point is concerned. (The solar profile employed in the present analysis is compatible with the rocket findings, it should be noted.) This leaves unexplained an even

greater discrepancy between their results and mine. Since the only Fowle papers cited by Yamamoto and Onishi (Fowle, 1915, 1917) contain no results more relevant to the present problem than those I have used, I can only conclude that, somewhere in their involved and indirect manipulation of Fowle's data, Yamamoto and Onishi have erred and that such errors were partly, but not completely, cancelled by the other errors inherent in their use of inadequate solar-energy-distribution assumptions.

## 7. Summary and conclusions

The Fowle absorptivity data, though derived in part from mountain-observatory measurements, do not require any pressure-reduction to sea level. Fowle's techniques were such as to insure that all of his absorptivity *versus* precipitable water-vapor relationships were automatically reduced to sea-level pressure.

Combining Fowle's data with recent data on the solar-energy distribution yields absorptivities which can be fitted by a simple exponential relation with accuracy of about one per cent from about 0.3 to 8 cm and with accuracy believed to be better than ten per cent from 0.1 (or perhaps a bit less) up to 0.3 cm of precipitable water vapor. These accuracy estimates are expressed relative to the Fowle 1915 absorptivities only, it should be emphasized.

Analysis of a specific example suggests that present uncertainty as to the exact type of pressure correction to be used in applying this empirical absorptivity function to aerological data is surprisingly unimportant, for variation of the pressure exponent from zero to unity produced only a six-per-cent change in total columnar insolation absorption between 250 and 950 mb. Uncertainties relating to pressure effects become more noticeable in the upper troposphere where, for a variety of reasons, the entire method gives lowered accuracy. Up to about the 400-mb level, the present method is believed to yield results that are accurate, relative to the Fowle absorption data, to about ten per cent.

Other previously developed methods cannot, for the most part, be conclusively compared with the present method, due to lack of published details as to the exact techniques employed in their development. However, the basic Fowle absorption data are of such inherent simplicity in the only finished form in which Fowle appears to have published them that it is difficult to see how any other than the present results can be obtained from that one source.

Finally, however, recent clarification (footnote 5) of the quite different approach employed by Houghton in arriving at absorptivities as much as thirty per cent greater than those treated here renders quite uncer-

tain the validity of reliance on the Fowle's data specifically concerned with absorption itself—i.e., the data used here. One can only conclude that insolation absorption predictions cannot be regarded as yielding *absolute* accuracies of better than thirty per cent at present. Only a major observational program can eliminate the confusion resulting from failure on the part of Fowle, Abbott, and co-workers to reconcile the two categories of Smithsonian solar-absorption results.

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