ABSORPTION OF ATMOSPHERIC RADIATION BY WATER FILMS AND WATER CLOUDS [1]

James E. McDonald

University of Arizona

(Manuscript received 20 July 1959)

ABSTRACT

Spectrally averaged infrared absorptivities of thin water films are computed using recent spectrometric results. A water film 5 μ thick irradiated at normal incidence with 0C blackbody radiation is found to have a total absorptivity of 0.49, while, for thicknesses of 10, 50, and 100 μ, the absorptivities are 0.67, 0.92, and 0.95, respectively. Spectrally averaged reflectivity at normal incidence for bulk water is 0.04. Change in the shape of the blackbody emission curve with a change in temperature from 0C to 40C produces negligibly small effect on the spectrally averaged absorptivity and reflectivity. A simple bulk-water method yields infrared cloud absorption half-depths of 17 m for 0.25 g m^{-3} (and 5 m for 1 g m^{-3}) liquid water content. Only 0.5 per cent is transmitted deeper than 400 m for 0.25 g m^{-3} or deeper than 100 m for 1 g m^{-3}. Evidence for concluding that Mie-theory effects will tend to reduce these absorption depths to still lower values is presented.

1. Introduction

The relatively strong absorptivity of liquid water at wavelengths corresponding to atmospheric radiation plays an important role in many aerological, micrometeorological, and cloud-physical problems. Despite this importance, the literature contains only few estimates useful in answering such questions as the one recently asked by Gergen (1958): "Just exactly how thick must a cloud be to be considered 'black'?" As far as I am aware, we now depend chiefly upon a rough type of estimate rather casually made by Brunt (1944, p. 122). Brunt noted that a layer of bulk water 0.1 mm thick absorbs, according to the work of German investigators near the turn of the century, 99 per cent of the incident radiations even in the wavelength interval (near 10 μ) of weakest absorption; so a layer of cloud about 50 m thick having a liquid water content of 2 g m^{-3} might be expected to have an absorptivity rather greater thn 0.99.

The objective of the following discussion is to refine, in several ways, the type of estimate Brunt has given. First, recently reported infrared absorption data that differ from the older data, especially in the very important "window" region from about 7 to 10 μ, will be utilized. Second, account will be taken of both the spectral variation of absorptivity and the spectral variation of intensity of incident (blackbody) radiation in order to obtain spectrally averaged absorptivities for water. Third, some results of Mie-theory calculations for infrared irradiated water drops will be used to set approximate upper bounds on the possible anomalies that might arise from the circumstance that drop diameters are of the same order of magnitude as wavelengths in the problem of cloud absorption of atmospheric radiation.

2. Liquid-water absorptivities

Brunt used the water-film absorption data of Rubens and Ladenberg and of Aschkinass (for references, *cf.* Brunt, 1944, p. 120 or, more completely, Dorsey, 1940, pp. 326–329). Because radiometric techniques have improved in a variety of ways in the half-century since those data were reported, it seemed desirable to search current optics literature for recent work on the infrared absorptivity of liquid water. From this search, two particularly useful studies were found. Plyler and Acquista (1954), using absorption cells with silver chloride and thallium bromide-iodide windows, have obtained transmission data out to 42 μ. Since the earlier German work gave absorptivities only to 18 μ and since this limit leaves over 30 per cent of the total blackbody emission at atmospheric temperatures unaccounted for, the extension to 42 μ is a significant improvement (leaving only about 6 per cent of the longer wavelength emission unspecified). In the second study, Adams and Katz (1956), using silver-chloride windows, report transmission data only to about 15 μ, but their work is important in that it has led (see below) to a cross-check and revision of the Plyler and Acquista data. In order to extend the absorption spectrum beyond 42 μ, data of Cartwright (1935) have been used here.

Adams and Katz, noting disagreements between their data and those of Plyler and Acquista at the fundamentals near 3 and 6 μ, suggested that Plyler and Acquista's nominal 5-micron water-cell thickness must actually have been about 9 microns. In a sub-

[1] Work supported by the Office of Naval Research under Contract NR 082–164.

sequent check,[2] Plyler and Acquista repeated their experiment and found that their 5-micron cell thickness was in error; however, it should be taken not as 9 microns but as 6.5 microns, with a final estimated error of about 0.1 micron. In the present discussion, all absorptivities out to 42 μ have been computed from the Plyler and Acquista transmission data for their nominal 5-micron cell, but the revised cell-thickness of 6.5 microns has been used in computations involving their data. It is to be noted that the very difficulty of having to work with such extremely thin cells attests to the high opacity of liquid water at wavelengths of interest in atmospheric radiation problems. Even with a water cell of only 6.5-micron thickness, absorptive saturation near the 3-μ fundamental precludes accurate calculation of the absorptivity in that region of exceedingly strong absorption.

For convenience of computation, absorptivities have here been expressed in the form of the *decimal* coefficient of absorption, α' (Brunt, 1944, p. 113), the unit being cm^{-1}. In fig. 1, the computed absorption coefficients are shown for the region from about 5 to 90 μ. Below 5 μ, the values oscillate extremely rapidly and are not given; but this is of little concern here since only about 0.5 per cent of total blackbody emission at atmospheric temperatures is contained in wavelengths below 5 μ. The absorption beyond 42 μ is perhaps only roughly specified by the mere three observational points derived from Cartwright's work, but, since only 6 per cent of total emission from a blackbody at atmospheric temperatures is contained in the region beyond 42 μ, uncertainties at the long-wavelength tail are not serious here. It should be noted that, at 52 μ, fig. 1 is based upon $\alpha' = 467$ cm^{-1}, whereas Dorsey (1940, p. 328) gives a Naperian absorption coefficient of 1160 cm^{-1}, corresponding to $\alpha' = 503$ cm^{-1} for this wavelength. Dorsey's 52 μ value was apparently miscalculated from Cartwright's original data.

The peak emission intensities for blackbodies at atmospheric temperatures are found near 10 μ, a region that happens nearly to coincide with the important "window" separating the vibration-rotation bands of the fundamentals near 3 and 6 μ from the pure-rotation bands beginning just above 10 μ. (For liquid water, one must, of course, speak of hindered rotational modes due to near-neighbor interactions.) This region is, relatively speaking, a window for both water vapor and liquid water, but the window is comparatively much more opaque for the condensed phase, a fact of very great meteorological importance since it accounts, for example, for the large reduction in nocturnal radiation produced by cloud overcasts. In the window region, the absorption coefficient of a given optical mass of liquid water is three orders of magnitude higher than that of the same mass of water vapor (Albrecht, 1947).[3]

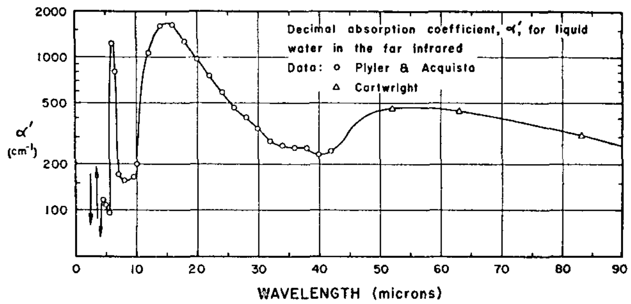

FIG. 1. Absorption spectrum of liquid water in the near and far infrared. Ordinate is the decimal absorption coefficient α' expressed in units of cm^{-1}. Arrows below 5 μ indicate rapid oscillation of α' near the 3-μ fundamental band (unimportant in problems of atmospheric radiation).

The absorption coefficients derived from the Plyler and Acquista data for the window just below 10 μ are significantly smaller than those reported by Rubens and Ladenberg or Aschkinass, but this is the only interval in which the new data seriously disagree with the earlier data. (Near 5 μ, the data of fig. 1 also lie below the older data, but so little infrared flux is found below 5 μ that this disagreement is meteorologically very much less serious than the disagreement in the window.) At 8.5 μ, the window center, the older data give α' values near 300 cm^{-1}, whereas Plyler and Acquista's measurements give about 150 cm^{-1}. Pending further spectrometric work on water, I believe that the latter should be accepted since they agree fairly closely with the window absorptivities derived from Adams and Katz (180 cm^{-1} near 8.5 μ). Furthermore, these lower values of fig. 1 are also well substantiated by the work of Sohm (1940). It may be noted that the infrared absorption data tabulated by Centeno (1941) are chiefly taken from the cited German studies of earlier years and thus constitute no independent check. For reference use by other workers, the numerical values underlying fig. 1 are listed in table 1.

3. Thin-film absorption

The spectrally averaged absorptivity $a(T)$ of a water film of specified thickness s when irradiated at normal incidence by blackbody radiation corresponding to a given temperature is given by

$$a(T) = 1 - r(T) - \tau(T) \tag{1}$$

[2] Private communication from Dr. E. K. Plyler. One absorption cell was accurately shimmed at 20 microns, and the other cell thicknesses were then calculated by applying Beer's law at 4.7 μ.

[3] It would be interesting to know whether this meteorologically very important intensification of absorption per unit mass of water substance is due to creation of *new* energy levels as a result of the greatly enhanced molecular interactions accompanying condensation or whether it is to be regarded as essentially a type of pressure-broadening effect, but I have been unsuccessful in seeking an answer to this question.

TABLE 1. Values of the decimal coefficient of absorption for liquid water. (From 4 to 42 μ, data from Plyler and Acquista; beyond 42 μ, data from Cartwright.)

Wavelength (μ)	α' (cm^{-1})	Wavelength (μ)	α' (cm^{-1})
4.0	20	20.0	982
4.5	116	22.0	762
5.0	109	24.0	593
5.5	96	26.0	477
6.0	1220	28.0	412
6.5	814	30.0	341
7.0	175	32.0	288
7.5	166	34.0	267
8.0	158	36.0	258
8.5	156	38.0	258
9.0	158	40.0	239
9.5	166	42.0	248
10.0	201	52.0	467
12.0	1010	63.0	442
14.0	1610	83.0	301
16.0	1610		
18.0	1270		

where $\tau(T)$ is the spectrally averaged transmissivity given by

$$\tau(T) = (F_{\infty,T})^{-1} \int_0^\infty 10^{-\alpha_\lambda' s} dF_{\lambda,T} \qquad (2)$$

in which α_λ' is the wavelength-dependent decimal absorption coefficient and $F_{\lambda,T}$ is the radiant flux emitted by a blackbody at temperature T in the wavelength interval from 0 to λ. The spectrally averaged reflectivity $r(T)$ is defined analogously to (2).

Use of the blackbody spectrum as the spectrum of the incident radiation is, of course, only a convenient approximation. For example, the spectrum of *downcoming* atmospheric radiation from cloudless skies, particularly from the zenith sky, is deficient in those wavelengths near the 10-μ "window" in the water-vapor spectrum. On the other hand, there is a number of significant problems wherein assumption of the blackbody distribution will approximate fairly closely to reality. Downcoming radiation with a low or middle cloud deck is such a case, and so, in the case of drop-absorption, is the radiation arriving from below at the base of a low or middle cloud deck.

Limitation to normal incidence is a further restriction of the scope of this discussion. A fairly complete treatment of the non-normal case is underway and will be published in the near future.

The integral in (2) has been evaluated numerically here, using 1-micron increments of λ from 4 to 40 μ, 2-micron increments from 40 to 50 μ, and 5-micron increments from 50 to 70 μ. Values of α' were read from an enlarged plot of fig. 1 at midpoints of these λ-intervals, and corresponding values of $dF_{\lambda,T}$ were obtained from tables of the Planckian function (List, 1951, p. 412). The analogous integral involved in computing $r(T)$ was obtained by plotting normal-incidence reflectivity data given by Dorsey (1940, p. 298), reading off values at the mid-point of each λ-increment and combining with the same Planckian function increments used in (2) prior to performing the summation over wavelength.

In the present study and in other studies in cloud infrared exchanges, it is of interest to know whether $a(T)$ does, in fact, really vary significantly with T due to change in shape of the Planckian emission curve as T varies. To determine this, the numerical integrations were performed at 0C for four different film thicknesses and were then repeated at 40C for two of these four thicknesses.

TABLE 2. Spectrally averaged reflectivity $r(T)$, transmissivity $\tau(T)$, and absorptivity $a(T)$ for four water-film thicknesses s and two blackbody temperatures T.

T (°K)	$r(T)$		$s(\mu)$ 5	10	50	100
273	0.045	$\tau(T)$	0.466	0.282	0.031	0.004
		$a(T)$	0.489	0.673	0.924	0.951
313	0.040	$\tau(T)$	0.491	—	0.040	—
		$a(T)$	0.469	—	0.920	—

The results are displayed in table 2. The values of $r(T)$ are assumed here to be independent of s, though this will be only approximately true for film thicknesses of the order of a wavelength and less. It will be seen that the spectrally averaged absorptivity is only slightly temperature dependent (*e.g.*, 0.49 at 0C *vs.* 0.47 at 40C for a 5-micron film). This dependence is weak enough to be ignored in typical meteorological studies. This permits a simplification which should prove useful in future more detailed analyses of cloud absorption in the infrared. Table 2 shows, on a more adequate quantitative basis than has previously been given, that exceedingly thin water films absorb a very large fraction of total incident blackbody radiation at atmospheric temperatures. For example, a film 100 microns thick absorbs about 95 per cent of the total normally incident flux, and, of the remaining 5 per cent, some 4.5 per cent is reflected, leaving only about 0.5 per cent transmitted.

An interesting characteristic of the transmission through the window in the water-absorption spectrum is brought out by computing for each of the four s-values the fraction of total transmission contributed by flux transmitted in the 4-micron interval from 7 to 11 μ. The results are as follows: for s = 5 μ, 0.37; for s = 10 μ, 0.48; for s = 50 μ, 0.78; and for s = 100 μ, 0.84. Thus, the window transmission becomes *relatively* more important for the larger thicknesses, for which total transmission is least. This trend results from the fact that as the optical thickness increases, the moderate to strong absorption regions approach saturation, contributing therefore less and less to total transmission and leaving relatively more and more of the burden of transmission to the weakly absorbing window region. Essentially the same characteristic is found from calculating an "effective" or "spectrally averaged" decimal absorption coefficient from the data of table 2. In order of increasing film thickness, the four s-values give effective α' values of 664, 550, 299, and 228 cm^{-1}. The variation of these latter figures may also be interpreted as illustrating the failure of Beer's law when applied to polychromatic as distinguished from monochromatic radiation.

Two other observations may be made with respect to implications of table 2: (1) That the spectrally averaged normal-incidence reflectivity of liquid-water films is only about 4 per cent for atmospheric radiation provides at least a rough rebuttal to what seems a very persistent misconception on the part of students of elementary meteorology—namely, the misconception that cloud decks reduce nocturnal radiation by *reflecting* infrared flux emitted by the ground. To the rough extent that film reflectivities may be identified with total cloud reflectivities, table 2 provides quantitative basis for disposing of this fallacy. (2) In working with radiometers, one often desires that all or part of certain thermally sensing surfaces be highly reflective in the infrared (*e.g.*, the polished metallic undersurfaces of the plates sometimes used to convert net radiometers into total radiometers). The rapid rise of absorptivity with increase of water-film thickness together with the low surface reflectivity of water films point to a possible difficulty caused by the very appreciable decrease in effective plate-reflectivity that would result from *invisibly thin* deposits of water on such surfaces. Even a 5-micron film of dew might lower the reflectivity of a metal surface from near unity to about one-half, judging from table 2.

4. Cloud absorption

We may use the results of table 2 to secure an estimate of cloud absorptivity by ignoring the actual spherical nature of the cloud drops with their complex optical properties and assuming, following Brunt, that the drops in a given cloud layer absorb infrared radiation exactly as would a horizontal layer of water of the same vertically projected water content. This method will be referred to in the following as the bulk-water method of estimating cloud absorption.

In table 3, specimen results of a few such calculations are given. Even in a cloud whose liquid-water content is as low as 0.25 g m^{-3}, the half-depth for absorption (plus about 4 per cent reflection) is only 17 m, and even the 90-per-cent depth is only 100 m.

TABLE 3. Infrared absorption depths based on the bulk-water approximation. (Roughly 4-per-cent reflection is here counted as absorption.)

Liquid water content (g m^{-3})	50% depth (m)	90% depth (m)	99.5% depth (m)
0.25	17	100	400
1.0	5	26	100

The depth beyond which only 0.5 per cent would be transmitted is 400 m in such a cloud and only 100 m in a cloud of 1 g m^{-3} liquid-water content. One finds that about one quarter of the atmospheric radiation incident upon the edge of a cloud of liquid-water content 1 g m^{-3} is absorbed within the first three meters of the cloud-edge.

However, it is very important to note that though these cloud-absorption data are based on several improvements upon Brunt's rough estimate, they are still subject to the objection that a cloud is not obviously optically equivalent to a water-film of the same optical mass per unit horizontal area. It is, therefore, important to ask whether the above data should be expected to constitute underestimates or overestimates of cloud absorption-depths. Brunt (1944) notes that the effects of multiple reflections at the many drop surfaces encountered by a beam penetrating a cloud will tend to reduce transmission below his simple bulk-water estimate. In addition, the fact that one almost always deals with diffuse rather than plane-parallel normally incident atmospheric radiation tends further to lower the absorption depths to values less than suggested by table 3. But neither of these circumstances is decisive in the face of uncertainty concerning a third factor—the possibly anomalous absorption effects that might arise because the drop diameters are of the same magnitude as the wavelengths in question. In scattering theory for the non-absorptive case, it is known from the work of Mie that, when the ratio of drop diameter to wavelength is of order unity, very large oscillations of scattering cross-section can arise. Before it can be concluded that table 3 presents upper bounds to absorption depths, it is indispensable to have some notion of how Mie-theory effects may influence cloud opacity in the infrared. Unfortunately, despite extensive recent electronic-computer investigations of Mie theory for the non-absorptive case, almost no work has been done for the *absorptive* case of present interest. I am indebted to Dr. J. C. Johnson for calling my attention to what appears to be the only investigation of Mie absorption in water droplet clouds yet completed, a study by Haugen (1953).

By utilizing preliminary computational data derived from work by Johnson and Terrell (1955), Haugen has calculated both scattering and absorption

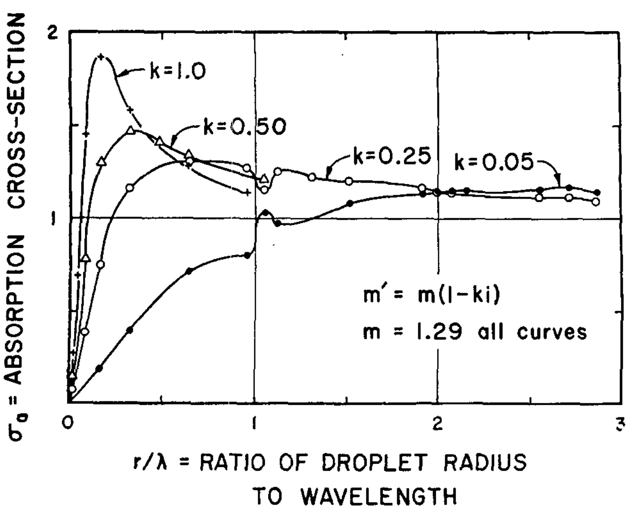

FIG. 2. Mie-theory water-droplet absorption cross-section coefficient σ_a (ratio of effective absorption cross-section to geometric cross-section) as a function of the ratio of droplet radius to incident wavelength, after Haugen (1953). All curves for real refractive index $m = 1.29$. Each curve labelled with the value of the absorption parameter k for which it was computed.

cross-sections for a small number of cases applicable to the cloud infrared exchange problem. In fig. 2, there are reproduced four of Haugen's computed curves for a single refractive index m of 1.29 combined with four values of the absorption parameter k so defined that the complex index of refraction m' satisfies the relation

$$m' = m(1 - ki).$$

The relation between k and α' is

$$mk = \lambda\alpha'/(4\pi M) \tag{3}$$

where λ is the wavelength and M is the modulus 0.4343 for conversion from Naperian to common logarithms.

In order to see, at least roughly, the extent to which Haugen's Mie-absorption curves of fig. 2 apply to the present problem, values of m taken from Dorsey (1940) and of α' taken from fig. 1 have been used in (3) to compute for selected wavelengths the values of k shown in table 4. At 13, 21, 26, and 40 μ, m was obtained by interpolation in Dorsey's data so these are less reliable cases. The index of refraction of 1.29 used in all of Haugen's work is found to fit experimental data closely only near 8 μ, 14 μ, 32 μ, and 45 μ, with rather higher m values over most of the spectral range of interest here (chief exception is a sharp minimum near 11 μ, where $m = 1.15$). A more serious limitation from the present viewpoint is the fact that Haugen's lowest value of k was 0.05, whereas, in the window region from 7 to 11 μ, where the most significant net exchange occurs between cloud and ground, or between cloud and free space if we consider cloud tops, k falls below 0.02. Thus, fig. 2 can only give us a general notion of Mie effects in cloud infrared exchange; nevertheless, these data will permit us to infer that the bulk-water method *overestimates* rather than underestimate cloud absorption depths, as will now be shown.

As a first comparison between Mie-theory and the bulk-water estimates of cloud infrared absorption, consider a case for which $k = 0.25$. From a plot of the data in table 4, one finds that this k value is satisfied near a wavelength of 13 μ and again near 20 μ and 48 μ. The refractive index is 1.27 at 13 μ, so this case fits the Haugen data very well. We are free to pick any ratio of drop radius r to wavelength λ. As one case of interest, let us choose that for which the absorption cross-section σ_a is 1.0—namely, $r/\lambda = 0.22$. This demands $r = 2.9\ \mu$, a radius that happens to be quite representative of freshly formed drops near bases of convective clouds. By arbitrarily setting the droplet concentration at 400 cm^{-3} as a plausible cloud-base value (which puts the liquid-water content at 0.04 g m^{-3} if we regard the cloud as monodisperse), we find that a horizontal layer 100 m thick contains sufficient drops per unit horizontal area to yield just 1 cm^2 total drop cross-section per cm^2 of projected area. For reasons of randomicity of droplet location, a certain fraction of the total cross-sectional area will be wasted in vertical overlap, but this effect will be ignored in the present rough treatment. Thus, since the Mie cross-section per drop equals the true geometric cross-section per drop here, and since we have taken a depth of cloud just sufficient to provide total drop area equal to the basal area of the cloud in vertical projection, we may say (approximately) that all infrared radiation of 13-μ wavelength will be absorbed within a layer 100 m thick, according to Mie theory. From the liquid-water content of 0.04 g m^{-3}, we next find that a 100-m layer has total projected bulk-water thickness of 4 microns. Since $\alpha' = 1400\ cm^{-1}$ for 13 μ, we find that the absorptivity of the layer on the bulk-water basis is 0.72

TABLE 4. Values of real index of refraction m, decimal absorption coefficient α', and absorption parameter k for selected wavelengths λ.

λ (u)	5	6.2	7.0	8.0	10	12	13	15	18	21	26	40	50
m	1.33	1.36	1.33	1.29	1.20	1.19	1.27	1.33	1.50	(1.54)	1.41	(1.25)	1.36
α' (cm^{-1})	110	1300	180	155	200	1005	1400	1600	1360	860	480	235	450
k	0.008	0.109	0.018	0.018	0.013	0.186	0.258	0.331	0.299	0.215	0.163	0.137	0.304

(*vs.* 1.0 on the Mie basis). Thus, in this first case, the bulk-water method underestimates the absorption.

As a second comparison, we may again use $k = 0.25$, but here we use $\lambda = 20\,\mu$, in a region of relatively strong hindered rotational absorption. It is of interest to select a radius-wavelength ratio yielding maximal Mie-absorption cross-section, so we may use $r/\lambda = 1.0$, for which the absorption cross-section σ_a is 1.2. The above implies $r = 20\,\mu$, typical of well-developed cumuli at heights well above the base where number concentrations are of the order of 100 cm^{-3}. Using the latter value, we find that, in view of the 1.2 ratio of Mie-absorption cross-section to geometric cross-section, we require only 6.6 m of cloud depth to yield what we are here regarding as total absorption (ignoring randomicity of spatial drop distribution). We find that this depth of our second cloud model (whose liquid-water content is 3.3 g m^{-3}) gives an equivalent bulk-water film-thickness of 22 μ. Since $\alpha' = 1000$ cm^{-1} for $\lambda = 20\,\mu$, we obtain 0.99 as the bulk-water estimate of the absorption for this 6.6-m layer whose Mie absorptivity is 1.0. In this case, the two methods give essentially the same results, though taking account of spatial randomicity would push the Mie absorptivity a bit below the bulk-water value.

For a third comparison, we turn to the lowest absorption parameter Haugen used, $k = 0.05$, which fits actual water data for only a single wavelength near 9.5 μ, where $m = 1.23$ is in fair accord with Haugen's value. By arbitrarily selecting the case $r/\lambda = 0.4$, for which $\sigma_a = 0.5$, we find that this requires $r = 3.8$ microns, again a value that might be associated with a convective cloud base, for which 400 cm^{-3} may again be taken as a reasonable droplet concentration. Computation shows that 110 m of cloud will give total Mie absorption, ignoring randomicity. This cloud's liquid-water content of 0.09 g m^{-3}, quite plausible at the cloud base, is such that a 110-m layer will yield an equivalent bulk-water film thickness of 10 microns. Since $\alpha' = 166$ for $\lambda = 9.5\,\mu$, we obtain a bulk-water layer absorptivity of only 0.34, roughly a third as great as the Mie absorptivity of unity. Thus, here the bulk-water absorption-depth estimate quite significantly overestimates the Mie depth.

As a fourth and last comparison that may be drawn from the limited number of realistic combinations afforded by fig. 2, consider $k = 0.05$ and $\lambda = 9.5\,\mu$ again, but now with $r/\lambda = 1.1$ so that $\sigma_a = 1.0$, whence $r = 10.5\,\mu$. By taking 200 cm^{-3} as a plausible drop concentration for such a drop size (liquid-water content 1.0 g m^{-3}), we find complete absorption within a layer of 14.5 m depth on the Mie basis, whereas the bulk-water result is found to be 0.41 for this same 14.5-m layer, again well below the Mie absorptivity.

Although four comparisons are not conclusive, they suggest strongly that *the bulk-water basis of estimating cloud absorptivities will, as far as errors due to neglect of Mie-theory effects are concerned, tend generally to yield too low absorptivities* or *too high values of absorption depths.* Inasmuch as errors of the same sign result from neglect of effects of multiple reflection and large-angle scattering, and inasmuch as errors of the same sign also result from neglect of the diffuseness of the incident atmospheric radiation in actual cloud infrared exchange problems, there seems little doubt that even though bulk-water absorption depths such as those given above in table 3 are very small, *they are not small enough.* This seems a useful result, for it suggests that in most problems involving cloud absorption (and hence in most problems involving cloud emission) the well-established custom of treating clouds as opaque bodies with respect to infrared wavelengths is not seriously in error. Only in very thin clouds will this assumption fail; and even there, the above arguments suggest that one can always obtain a reliable *upper* bound to the absorption depth by a simple bulk-water computation.

To confirm these inferences with a larger number of cases applicable to the liquid-water spectrum would require a far larger set of Mie-absorption results than were computed by Haugen, who used desk-calculator methods. A very extensive set of such computations is now being carried out by B. M. Herman at the University of Arizona using an IBM 650 computer. These will permit a much more comprehensive treatment of the problem of this section.

5. The absorption paradox

One of the distinctive and now familiar features of Mie scattering theory for the non-absorptive case is the large-amplitude oscillation of the scattering cross-section coefficients. Values as large as about four occur due to interference phenomena occurring "downstream" from the drop where the refracted wave fronts encounter the diffracted wave fronts. A somewhat similar but rather more surprising Mie effect characterizes the absorptive case. The curves of fig. 2 show that, for large k and small r/λ ratios, σ_a rises to values approaching two. A few other similar cases for absorbers other than water are given by van de Hulst (1957), showing that strongly absorbing spheres of radius rather less than a wavelength succeed in removing from the incident plane-parallel beam and converting into heat radiant flux in excess of that which one would naively expect to impinge upon their geometric cross-sections. That is, "upstream" of the drop, interference phenomena set up by the secondary radiation from the excited drop cause an inward bending of the rays that, we may say, sends down into the drop photons not originally approaching on a collision path. This "upstream-bending absorption paradox" is not discussed in the literature, but in a private communication van de Hulst has pointed out that this is electromagnetically identical to the phenomenon whereby a small dipole radio antenna combined with a suitably matched resistance removes energy from an area wider than its own.[4] The values of k for water drops in an atmospheric radiation field, table 4, do not become large enough for the most extreme form of this absorption paradox to occur in cloud-exchange problems, but fig. 2 shows that mild effects of this type will be found in certain cases of meteorological interest.

In contrast to the non-absorptive case, the absorptive case exhibits a rapid damping of the absorption cross-section as one goes to larger radius-wavelength ratios. However, out to the limit of Haugen's data, σ_a remains slightly above unity, suggesting that the upstream bending effect is still occurring but is becoming percentually less significant as the drops grow to several multiples of the wavelength.

6. Conclusions

Using recently reported infrared absorption data and taking account of spectral variations of intensity of blackbody radiation at atmospheric temperatures, values of the spectrally averaged absorptivities of thin films of liquid water have been calculated. The normal incidence absorptivity is found to be about 0.5 for as thin a film as 5 μ and reaches 0.95 for a 100-μ film. The spectrally averaged normal incidence reflectivity of water is about 0.04. Over the normal range of atmospheric temperatures, these absorptivities and reflectivities do not vary significantly as a consequence of change in shape of the Planckian emission curve.

Using these thin-film total absorptivities, the bulk-water method gives cloud absorption half-depths of 17 m and 5 m for liquid-water contents of 0.25 g m^{-3} and 1.0 g m^{-3}, respectively. Only about 0.5 per cent of incident radiation penetrates beyond depths of 400 m and 100 m in these same two cases, according to the bulk-water estimates.

From Mie-absorption cross-sections given by Haugen, a small number of cases of cloud-radiation interest can be discussed. These indicate that bulk-water estimates need not be suspected of underestimating true absorption depths. By taking into account Mie effects, multiple reflections and large-angle scattering effects, and diffuseness of atmospheric radiation, true absorption depths will almost certainly be smaller than those derived by the bulk-water method. There remains need to assemble much more complete Mie-absorption data and to apply it, if possible, to the problem of diffusely incident infrared flux in order to describe more accurately the spatial distribution of heat sources and sinks within clouds.

Acknowledgments. The author wishes to express his indebtedness to Drs. E. K. Plyler, J. C. Johnson, and H. C. van de Hulst for their assistance in parts of this work. Mrs. Nell Hansen's aid in some of the computational work is also appreciated, and the support of the Office of Naval Research is gratefully acknowledged.

[4] The same analogy was independently called to the author's attention by Dr. H. G. Houghton.

REFERENCES

Adams, R. M., and J. J. Katz, 1957: New variable thickness infrared cell and the infrared spectra of HF, DF, H_2O, and D_2O. *J. opt. Soc. Amer.*, **46**, 895–898.

Albrecht, F., 1947: Eine einheitliche Darstellung des Absorptions-spektrums von Wasserdampfhaltiger Luft und flüssigem Wasser. *Z. Meteor.*, **1**, 194–196.

Brunt, D., 1944: *Physical and dynamical meteorology.* Cambridge, Cambridge Univ. Press, 428 pp.

Cartwright, C. H., 1935: Extreme infrared investigations of hindered rotation in water. *Nature*, **135**, 872.

Centeno, M., 1941: The refractive index of liquid water in the near infrared spectrum. *J. opt. Soc. Amer.*, **31**, 244–47.

Dorsey, E. N., 1940: *Properties of ordinary water-substance.* New York, Reinhold Pub. Co., 673 pp.

Gergen, J. L., 1958: Black-ball observations and the radiation chart. *J. Meteor.*, **15**, 350–368.

Haugen, D. A., 1953: *On the scattering of light by spherical absorbers.* Unpubl. S.M. thesis, Mass. Inst. Tech.

List, R. J., 1951: *Smithsonian meteorological tables*, 6th rev. ed. Washington, Smithsonian Inst., 527 pp.

Johnson, J. O., and J. R. Terrell, 1955: Transmission cross sections for water spheres illuminated by infrared radiation. *J. opt. Soc. Amer.*, **45**, 451–454.

Plyler, E. K., and N. Acquista, 1954: Infrared absorption of liquid water from 2 to 42 microns. *J. opt. Soc. Amer.*, **44**, 505.

Sohn, M., 1940: Über die absorption von flüssigem H_2O and D_2O in ulttraroten Spektralgebiet zwischen 5 μ and 27 μ. *Z. Phys.*, **116**, 34–46.

van de Hulst, H. C., 1957: *Light scattering by small particles.* New York, Wiley and Sons, 470 pp.