A Note on Anomalous Adiabatic Cooling Rates in Clouds[1]

JAMES E. McDONALD

Institute of Atmospheric Physics, The University of Arizona, Tucson, Ariz.

(Manuscript received 17 February 1962, in revised form 7 April 1962)

ABSTRACT

During the brief period of extremely rapid condensation accompanying the activation phase of cloud-droplet growth, latent-heat release produces adiabatic cooling rates well below the conventional saturated adiabatic value. Using results of drop-growth calculations made by Mordy, the main features of this anomalous thermodynamic process are examined quantitatively. For a convective updraft of the not excessive speed of 1 m sec^{-1}, it is found that an isothermal expansion process prevails throughout a depth of approximately 10 m just above cloud-base. It seems almost certain that the even more extreme case of expansion in which the parcel temperature actually *rises* slightly during ascent will be found to occur when computer studies are extended to models with updraft speeds closer to the maximum observed cumulus-base updraft speeds.

1. General

It is the purpose of this note to call attention to the following point of conceptual interest in meteorological thermodynamics: air ascending into an active convective cloud characteristically undergoes a peculiar type of adiabatic expansion in which the temperature may remain almost constant, or even *increase*, with height.

Moist air ascending to and through the base of a convective cloud is conventionally described as cooling at the dry adiabatic rate γ_d until the base is reached, and then cooling at the saturated adiabatic rate γ_m above that level. Although the latter cooling rate holds to high degree of approximation throughout almost all the vertical depth of the cloud, it does *not* apply in the first few tens of meters above the base of the cloud. As was first clearly demonstrated by Howell (1949), the relative humidity H, after passing through a value of 100 per cent at the nominal cloud-base level, continues to rise until peak values generally below 101 per cent are attained, whereupon H subsides monotonically to values of about 100.05 per cent throughout the remainder of the in-cloud ascent. Inasmuch as H does not remain at precisely 100 per cent in this portion of the cloud, it is clear that the actual cooling rate, which will be denoted here as γ_a (to denote the "activation adiabatic cooling rate"), cannot be equal to γ_m. As will be shown below, marked departure of γ_a from γ_m occurs in a stratum of the cloud where quite small differences of temperature (and hence of droplet vapor tension) are important to the subsequent history of the cloud, hence the principal features of the γ_a-process warrant special attention.

If the parcel temperature at the cloud base is T_0, dynamic cooling at rate γ_d will have tended to *cool* the parcel by an amount $z\gamma_d$ by the time it reaches a height z above the base; but initial droplet-growth will there have yielded some small liquid-water mixing ratio W, and the condensation of this amount of water will have tended to *warm* the parcel by an amount WL/c_p, where L is the latent heat of condensation and may be taken here as constant and equal to its value at T_0, while c_p is the specific heat of air at constant pressure. Hence temperatures along the "activation adiabat" are specified, for z in the range of present interest, by

$$T \doteq T_0 - z\gamma_d + WL/c_p. \quad (1)$$

If, for all positive z, it were precisely true that $H=100$ per cent, (1) would simply describe the initial portion of the saturated adiabat through T_0 and p_0 the cloud-base pressure. But initially, droplets (most of whose radii will still be on the small-radius side of their activation peaks) possess so small a total surface area that diffusional deposition cannot remove vapor fast enough for W to rise significantly above zero, with the result that the last term in (1) is then still negligible and γ_a starts out with a value not differing sensibly from γ_d.

However, as Howell showed, continuing increases of H become self-limiting as the activation phenomenon brings into play larger and larger numbers of the total nuclei population, so H passes through some maximum for the given updraft conditions and nucleus spectrum, and starts its decline. In the middle of this activation phase, the extremely rapid growth of drops (growth

[1] Work supported by the Office of Naval Research.

down the unstable large-radius sides of the activation peaks) makes up for the preceding deficiency in condensation rate, with the result (of prime importance to the point of this note) that W rises so rapidly with respect to time and height that the last term in (1) is increasing rapidly, indeed *much more rapidly than in the* γ_m*-process*. Thus we see that the γ_a-adiabat must start out with the value γ_d, then decrease in absolute magnitude to equality with γ_m, fall to absolute values even *less* than γ_m, and finally, as the rising parcel's humidity subsides and the *rate* of increase of W correspondingly subsides, γ_a must again *increase* in absolute magnitude asymptotically towards γ_m.

Curiously, in Howell's analysis which first revealed the critical importance of the activation phase and which brought out the controlling importance of diffusional gradients that depend upon extremely small differences in temperature-sensitive terms, Howell did not recognize this fact that $\gamma_a \neq \gamma_m$, and hence computed all of his air temperatures in the activation region on the erroneous assumption that T was falling at the rate γ_m. When the writer attempted a number of years ago to assess quantitatively the consequences of Howell's error, the effort had to be abandoned because the IBM 650 computer employed was not fast enough to handle the extremely short time-steps required to obviate the computational instability characteristic of the growth equation for small droplets. The point was called to the attention of Neiburger and Chien (1960) too late for them to incorporate it into their IBM 709 drop-growth calculations. However, as the writer only recently noted, the correct adiabatic equation was built by Mordy (1959) into his work with the BESK computer in Stockholm, and from results presented by him, it becomes possible to examine quantitatively some interesting properties of the γ_a-process.[2]

2. The γ_a-process

It is clear that the peculiarities of the γ_a-process will be most strongly developed in updrafts of high velocity w, so we consider Mordy's data for his nucleus distribution II and for $w=1$ m sec^{-1}, the largest ascent velocity he used in his computations. Using his values of $W(z)$ for that case in Eq (1) yields the calculated adiabatic path labelled γ_a in Fig. 1, where the dry and saturated adiabats through the cloud-base point are labelled γ_d and γ_m, respectively. It will be seen that this activation adiabat exhibits all of the properties outlined above. The activation adiabat starts up along the dry adiabat through the cloud-base point, veers suddenly to values well below the γ_m-rate, and finally recurves to approach, asymptotically at large Z, the saturated adiabat through the cloud-base point. The point A marks the level (about 25 m above cloud-base in this instance) at which γ_a first equals γ_m. Point A falls almost at the level of peak supersaturation for this case ($H=101.0$ per cent at $z=27$ m). On general grounds, it seems to the writer quite likely that these two levels will always lie very close together as they do in this one of Mordy's models. Everywhere above A, γ_a will be less than γ_m, although already near $z=60$ m, the activation adiabat can be seen to differ only very slightly from γ_m in the present example.

The striking feature brought out by Fig. 1 is that within the layer B-C from about 30 to 40 m above the base, W is growing so rapidly that latent heat release is able to provide essentially *all* of the energy required to perform the p-v expansion work of the ascending cloud parcel so that the parcel's stock of *internal energy* does not have to be drawn upon at all during this brief period. The result is that the segment BC is not only an adiabat but also (at least in its central portion) an *isotherm!* Furthermore, this particular case need not be regarded as an upper limit, inasmuch as $w=1$ m sec^{-1} is not a particularly large updraft speed for cumuli, and inasmuch as small variations in nuclei distribution characteristics could easily make the rate of release of latent heat just above the level of peak supersaturation distinctly higher than for this illustrative case.

The latter points argue, in fact, the strong probability that the γ_a-curve must, within at least short height intervals in the bases of very active convective clouds, decrease even *beyond* the isothermal value and become *negative*, as suggested by the hypothetical dashed segment shown near C in Fig. 1. It must be stressed that there is no thermodynamic limit standing in the way of such an extreme latent-heating effect, so investigators carrying out future computer studies of diffusional drop-growth might well search for such a case in models having w rather larger than the upper limit of 1 m sec^{-1} available in Mordy's integrations.

Since both Mordy (1959) and Neiburger and Chien (1960) drew special attention to the characteristic way in which the mode of the cloud liquid water content distribution shifts, almost explosively, from an initial location at the large-radius limit of the nucleus spectrum to a final location in the smallest (and hence most numerous) nucleus size-class after the parcel passes

[2] Unfortunately, it is not possible to go still further and exploit Mordy's calculational results to shed light on the quantitative effect of Howell's error in predicting air temperatures in the activation region from γ_m rather than γ_a, and this despite the fact that Mordy did numerical integrations for two models that had parameters identical with those of two of Howell's. This impossibility stems from the fact that Mordy's working equations differ from Howell's not only in this one respect but also with respect to correction of Howell's omission of the van't Hoff i-factor and with respect to Mordy's use of much smaller compensated diffusion coefficients. The interactions of various factors in the diffusion equations are so complex that there is no other way to assess the effect of any *one* given error or simplification than to do comparison integrations varying that one condition or factor and *no* others. The writer would submit that need for just such numerical experimentation still exists, though the requisite computer-time may be forbidding.

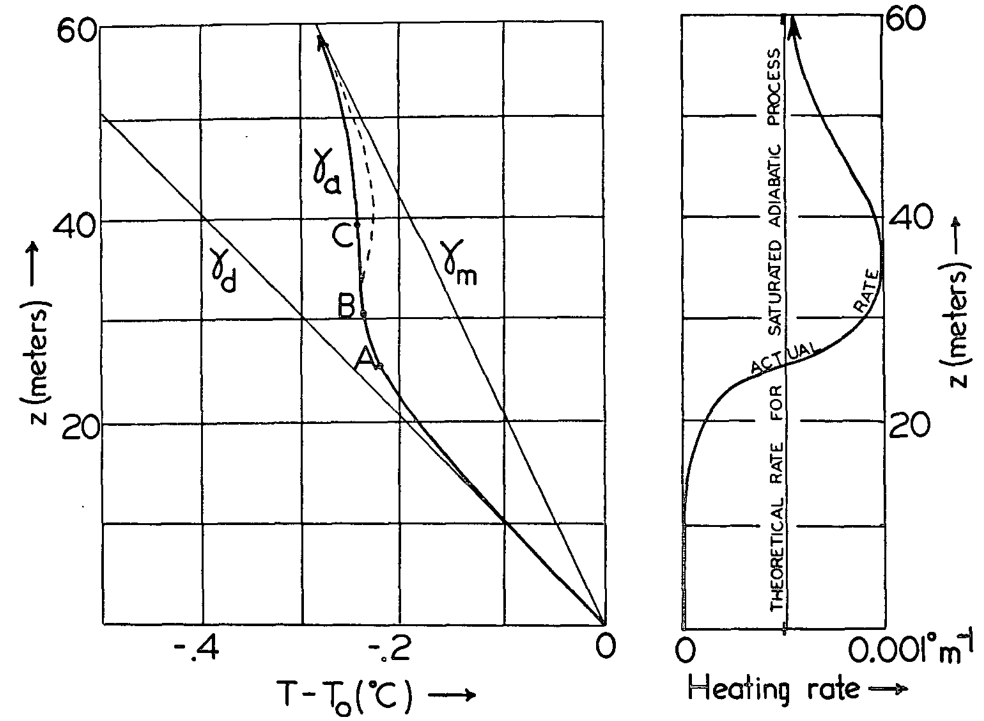

FIG. 1. Activation adiabat γ_a calculated for Mordy's Type II nucleus distribution and updraft speed 1 m sec^{-1}. See text for explanation of hypothetical dashed portion. Right panel, height-variation of condensational heating rate.

through the level of maximum supersaturation, the writer examined the data used in constructing the γ_a-curve of Fig. 1 to see if it was also the *smallest* activated size-class that dominated the latent heat release in the isothermal-adiabatic process. It was not. Rather, the next-to-smallest size-class plays by far the most important role in latent heat release in the isothermal-adiabatic expansion BC, contributing about six times as much heat as does the more numerous smallest size-class of the model in question. The reason for this relationship appears to the writer to be such that it will probably hold in general: Despite their being almost twice as numerous (about 500/cm^3 *versus* about 350/cm^3 as read from Mordy's nuclei plots), the smallest class had, by the time the parcel reached the 30-m level, been too recently activated to have grown very much beyond nuclear size; they had radii of only 0.27 μ. By contrast, the next larger class had by that level attained radii of about 2.0 μ. Since the rate of latent heat release, for a given vapor-pressure gradient, varies as the *square* of the radius, that tended to make the second-smallest class some 55 times more important than the smallest. That the actually computed growth increments indicated only a sixfold excess was due partly to the smallest class of activated nuclei being almost twice as numerous, but even more to the fact that the vapor-pressure gradients are, of course, *not* identical for droplets of different sizes even when exposed to the same ambient conditions, being larger over the smaller droplets whose Raoult effect is larger. As stated above, it seems likely that these trends will typically make the next-to-smallest size-class dominate the latent heat release responsible for isothermalcy of the γ_a-adiabats just above the peak-H levels.

As a final point, it is interesting to note that the isothermal adiabatic process described here is strongly reminiscent of the "hail stage" of older cloud physics theories. In the latter case, as also here, the isothermalcy is imposed by release of latent heat (there latent heat of *fusion*) as a result of an accumulation process (there accumulation of liquid water in a *reversible* adiabatic ascent). However, the classically assumed well-behaved freezing of liquid water at 0C without any undercooling and at a rate fast enough to keep pace with the dynamic cooling tendencies due to expansion, which led Hertz in 1884 to postulate the "hail stage," is an idealization now known to be quite unrealistic. On the other hand, the isothermal adiabatic process described here is not only a very real one, but one which attends the most crucially important step in the diffusional-growth phases of cloud formation, that in which the nuclei are suddenly being activated under influence of the small but decisively important supersaturations. In this activation phase, we have *the anomaly of Boyle's law holding for an adiabatic expansion.* And if the suspicions cited above prove correct, future numerical studies may show the even more extreme anomaly of parcel-*heating* during adiabatic expansion.

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