# The Rate of Descent of Parachutes from Various Altitudes

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SUMMARY

One-hundred and eleven observations were made on the descent times of 200-lb. torso dummies attached to 28-ft. silk, 28-ft. nylon, and 24-ft. nylon parachutes. These observations were made from various launching altitudes up to 40,000 ft. A $C_dS$ value was computed for each descent from the formula $C_dS = 2W/\rho V^2$, where $W$ = weight of the dummy plus parachute and $\rho V^2$ was calculated from the observed atmospheric conditions. The mean $C_dS$ values thus computed were: for 28-ft. silk parachutes, 531; for 28-ft. nylon, 483; and for 24-ft. nylon, 336.

By using the mean $C_dS$ value computed for each type of parachute, a formula has been developed to predict the time of descent from any altitude to any altitude if the type parachute, weight of the man, and certain atmospheric conditions are known.

An improved method of comparing performance of experimental parachutes is presented which negates the necessity for attempting to duplicate a given density altitude on successive flights.

A method is presented for calculating velocity at any instant in parachute descent. Landing velocities at different ground levels and with different types of parachutes and different weight men are discussed.

## INTRODUCTION

A KNOWLEDGE OF THE TIME REQUIRED to reach the ground when a parachute is opened at a specified altitude is sometimes highly useful. Moreover, information concerning the rate of descent is necessary for the calculation of landing impact when the ground is reached. Heretofore, all tables of rates of parachute descent have been based on calculations extrapolated from a few observations of parachute descents from near the ground. It has been presumed, but never proved, that the rate of descent at any given altitude would vary inversely with the square root of the density ratio of the air at that altitude. In connection with tests conducted for the measurement of impact forces of parachute openings at altitude, the occasion was presented to observe experimentally the effect of altitude on parachute descent.

## METHOD

The technique of release of the 200-lb. hard rubber torso dummies was described in a previous report.[1] Briefly, the parachutes were opened by 25-ft. static lines attached to the airplane. Parachutes were released at approximately 7,000, 15,000, 26,000, 33,000, and 40,000 ft. density altitude, and at each release accurate data as to indicated altitude, air speed, and temperature were kept. Flight instruments had been previously calibrated by the Flight Test Branch of the Air Matériel Command, Wright Field. In addition to the usual aircraft altimeter, the altitude of release was recorded by a radio altimeter and a recording barograph.

The timing was done visually by ground observers for the lower altitudes, and for the higher altitudes radio contact between the plane and the ground was maintained and stop watches were started at a signal from the plane. The parachutes were released in succession with sufficient intervals between to allow individual timing. The watch was stopped with the first contact of the dummy with the ground. The ground was a dried desert lake bed, the altitude of which was +2,800 ft. The location was near Kingman, Ariz.; the period of observation was November 30, 1944, to February 10, 1945.

## RESULTS

Since the primary purpose of these parachute drops was to measure and compare opening shock forces of different sizes and kinds of parachutes, a given drop usually included a mixture of parachutes. Thus, the number of observations on a given type of parachute for any one flight varied from two to six.

Some variation existed among altitudes of release with different flights. For instance, the calculated density altitude at release of the silk parachutes at the so-called "26,000-ft." level ranged between 25,850 and 27,050 ft. density altitude. Since there is good reason to presume, as will be shown later, that the rate of descent of a parachute is related to the density of the air, it is misleading to group all the observed descent times from one approximate altitude. Also atmospheric conditions between the altitude of release and the ground will vary from day to day. The observations herein reported were made at the same geographic location but over a period of some 10 weeks. Therefore, it should prove more fruitful to treat each flight individually, taking into consideration the actual observed atmospheric conditions for that day. To this end the Kollsman number, or altimeter setting, at take-

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Received January 10, 1947.

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off and landing and the atmospheric temperature at each 2,000-ft. stratum were recorded for each flight.

Terminal velocity of a freely falling body is reached when its aerodynamic drag is equal to its weight. This relationship is expressed thus:

$$D = W = C_d S(\rho V_t^2/2) \qquad (1)$$

where

$D$ = drag (lbs.)
$W$ = weight (lbs.)
$C_d$ = drag coefficient
$S$ = drag area (sq.ft.)
$\rho$ = air density (slugs per ft.$^3$)
$V_t$ = terminal velocity (ft. per sec.)

If the density of the atmosphere did not vary with altitude, then any falling body, as soon as it had been accelerated to the terminal velocity specified by Eq. (1), would fall with that constant velocity throughout its entire descent, and the problem of predicting total time of descent would be comparatively easy. But in the actual atmosphere, air density is not constant in space, or in time, so the terminal velocity of a body falling through the earth's atmosphere becomes a variable depending not only on its shape and weight but also on the value of the air density at each successive point in its descent. Since a body falling through the atmosphere is continuously moving into regions of greater density, Eq. (1) tells us that this is equivalent to saying that it is always falling from regions of higher terminal velocity into regions of lower terminal velocity; so, strictly speaking, we must conclude that in all but the first few seconds, during which the chute is opening, this steady deceleration implies that the actual velocity of descent must always be slightly greater than the theoretical value determined by Eq. (1). That is, the drag force, $D$, must always be slightly greater than the weight, $W$, with the net effect that the body undergoes a steady deceleration, as is actually observed to occur. The rate of increase of density with decreasing altitude is, however, gradual enough to make this slight disequilibrium and its consequent velocity lag negligible in our calculations. Thus, in summary, Eq. (1) presents the following picture of the variation in rate of descent of any large body (if we agree to equate $D$ and $W$, and thus to neglect any velocity lags): For a body of given weight, $W$, drag area, $S$, and drag coefficient, $C_d$, the velocity at any point in its descent is inversely proportional to the square root of the density. From this basic relationship, formulas expressing the dependence of total time of descent on atmospheric conditions and on parachute type will now be developed.

The weight of our dummies, with parachutes attached, was constant at 225 lbs., plus or minus a few pounds. The $\rho V^2$ values would be expected to vary with the changes in atmospheric conditions from flight to flight. These $\rho V^2$ values were computed from the data of each flight as follows:

Let

$$\rho V^2 = k = 2W/C_d S$$

Then

$$V = \sqrt{k/\rho}$$

also

$$1/V = \sqrt{\rho/k}$$

If, now, we imagine the atmosphere to be divided into a series of strata, each 2,000 ft. in vertical thickness, then the time required to pass through the $i$th stratum will be

$$t_i = 2{,}000/V_i$$

The sum of these 2,000-ft. stratum times represent the total time

$$\Sigma t_i = \sum \frac{2{,}000}{V_i} = 2{,}000 \sum \frac{1}{V_i}$$

But

$$1/V = \sqrt{\rho/k}$$

so

$$\Sigma t_i = 2{,}000 \sum \sqrt{\frac{\rho_i}{k}} = \frac{2{,}000}{\sqrt{k}} \Sigma \sqrt{\rho_i} \qquad (2)$$

Now from the General Gas Law we have

$$P = \rho R T$$

in which

$P$ = pressure in inches of mercury
$\rho$ = the air density expressed in slugs per ft.$^3$
$R$ = the gas constant for dry air (in suitable units)
$T$ = air temperature in degrees Kelvin (°K.)

Then $\rho = P/RT$ and

$$\left.\begin{aligned} \sqrt{\rho} = \sqrt{P/RT} = (1/\sqrt{R})\sqrt{P/T} = \\ \sqrt{\frac{3.38639 \times 10^4 \times 1.951875 \times (P/T)}{2.8886 \times 10^6}} = \\ 0.15126\text{—}\sqrt{P/T} \end{aligned}\right\} \qquad (3)$$

This expression can then be substituted in Eq. (2), yielding

$$\Sigma t_i = \frac{2{,}000 \times 0.15126}{\sqrt{k}} \sum \sqrt{\frac{P_i}{T_i}} = \frac{302.52}{\sqrt{k}} \sum \sqrt{\frac{P_i}{T_i}} \qquad (4)$$

where $P_i/T_i$ represents average pressure and temperature conditions for the $i$th 2,000-ft. stratum.

If the observed time of descent for each parachute is now substituted in Eq. (4), we may rearrange and solve for $k$:

$$k = \left(\frac{302.52\sqrt{P_i/T_i}}{\text{observed time}}\right)^2 \qquad (5)$$

TABLE 1A

Computed $\rho V^2$ and $C_d S$ Values for Each Observed 24-Ft. Nylon Parachute Descent

| Flight No. | Density Altitude | Observed Time, Sec. | $\rho V^2$ | $C_d S$ |
|---|---|---|---|---|
| 12 | 15,550 | 477 | 1.27 | 353.5 |
| | | 464 | 1.35 | 334.6 |
| | | 454 | 1.41 | 320.3 |
| | | 474 | 1.29 | 349.1 |
| | | 477 | 1.27 | 353.5 |
| | | 456 | 1.39 | 323.0 |
| 16 | 15,800 | 455 | 1.43 | 314.0 |
| | | 472 | 1.33 | 337.8 |
| | | 426 | 1.64 | 275.2 |
| | | 468 | 1.35 | 332.4 |
| 23 | 15,650 | 465 | 1.33 | 339.1 |
| | | 440 | 1.48 | 303.6 |
| | | 504 | 1.13 | 398.2 |
| 19 | 26,300 | 828 | 1.24 | 363.8 |
| | | 785 | 1.38 | 326.3 |
| 36 | 26,400 | 783 | 1.42 | 316.2 |
| | | 789 | 1.40 | 321.2 |
| | | 787 | 1.41 | 319.6 |
| | | 815 | 1.31 | 342.7 |
| 34 | 32,740 | 924 | 1.47 | 305.9 |
| | | 925 | 1.47 | 306.8 |
| | | 1,047 | 1.15 | 393.0 |
| 40 | 39,000 | 1,150 | 1.22 | 370.1 |
| | | 1,149 | 1.22 | 369.5 |
| Mean | | | 1.348 | 336.23 |
| Standard Deviation | | | ±0.116 | ±29.1 |

Referring again to Eq. (1), it is seen that $C_d S$ values may be then calculated by the equation

$$C_d S = 2W/\rho V^2 \tag{6}$$

where the value of $\rho V^2$ has been obtained from Eq. (5) and where $W$ is the known weight of dummy plus parachute.

$C_d S$ values have been calculated for 24 observed descents of 24-ft. nylon parachutes, for 46 28-ft. nylon, and 41 28-ft. silk parachutes. These data are presented in Tables 1A, 1B, and 1C.

TABLE 1B

Computed $\rho V^2$ and $C_d S$ Values for Each Observed 28-Ft. Nylon Parachute Descent

| Flight No. | Density Altitude | Observed Time, Sec. | $\rho V^2$ | $C_d S$ |
|---|---|---|---|---|
| 9 | 7,750 | 204 | 1.07 | 419.0 |
| | | 204 | 1.07 | 419.0 |
| | | 214 | 0.98 | 461.1 |
| | | 213 | 0.99 | 456.4 |
| 14 | 7,800 | 212 | 1.00 | 450.9 |
| | | 203 | 1.09 | 413.6 |
| | | 224 | 0.89 | 503.3 |
| | | 219 | 0.94 | 481.3 |
| 4 | 16,000 | 605 | 0.83 | 540.2 |
| | | 540 | 1.05 | 430.2 |
| | | 538 | 1.05 | 426.9 |
| 6 | 15,875 | 557 | 0.93 | 484.4 |
| | | 559 | 0.92 | 488.1 |
| | | 543 | 0.98 | 460.6 |
| 8 | 15,750 | 552 | 0.99 | 454.5 |
| | | 578 | 0.90 | 498.3 |
| | | 563 | 0.95 | 472.7 |
| 13 | 15,850 | 565 | 0.93 | 482.3 |
| | | 612 | 0.80 | 565.3 |
| 21 | 15,950 | 568 | 0.93 | 485.4 |
| | | 574 | 0.91 | 495.6 |
| | | 571 | 0.92 | 490.2 |
| 5 | 27,050 | 927 | 1.02 | 440.3 |
| | | 998 | 0.88 | 510.2 |
| 7 | 26,600 | 949 | 0.94 | 476.2 |
| | | 963 | 0.92 | 490.2 |
| 11 | 25,850 | 999 | 0.86 | 523.9 |
| | | 995 | 0.86 | 520.2 |
| | | 958 | 0.93 | 481.8 |
| 15 | 26,360 | 973 | 0.89 | 503.4 |
| | | 981 | 0.88 | 511.9 |
| | | 968 | 0.90 | 498.3 |
| 19 | 26,300 | 976 | 0.89 | 505.6 |
| | | 954 | 0.93 | 482.8 |
| 36 | 26,400 | 907 | 1.06 | 424.5 |
| | | 943 | 0.98 | 458.7 |
| 10 | 39,400 | 1,241 | 1.10 | 408.3 |
| | | 1,460 | 0.80 | 565.3 |
| | | 1,428 | 0.84 | 540.9 |
| 18 | 40,000 | 1,410 | 0.86 | 525.7 |
| | | 1,294 | 1.02 | 442.9 |
| | | 1,342 | 0.94 | 476.2 |
| | | 1,386 | 0.89 | 507.9 |
| 40 | 39,000 | 1,350 | 0.88 | 510.2 |
| | | 1,352 | 0.88 | 511.4 |
| | | 1,356 | 0.88 | 514.3 |
| Mean | | | 0.938 | 482.83 |
| Standard Deviation | | | ±0.077 | ±38.75 |

### COMPUTATION OF APPROXIMATE DESCENT TIME

Using the above relationship, it is possible to predict with reasonable accuracy the time it will take a man to reach the ground from any altitude if we know his weight, the type of parachute used, and the ground-level elevation.

Transposing Eq. (5),

$$\text{descent time} = \frac{302.52\Sigma\sqrt{P_i/T_i}}{\sqrt{\rho V^2}} \tag{7}$$

If the mean $C_d S$ values for each of the three standard types of parachutes are used, the formulas become:

For 24-ft. nylon parachutes:

$$\text{descent time} = \frac{302.52\Sigma\sqrt{P_i/T_i}}{\sqrt{2w/336}} \tag{8}$$

For 28-ft. nylon parachutes:

$$\text{descent time} = \frac{302.52\Sigma\sqrt{P_i/T_i}}{\sqrt{2w/483}} \tag{9}$$

For 28-ft. silk parachutes:

$$\text{descent time} = \frac{302.52\Sigma\sqrt{P_i/T_i}}{\sqrt{2w/531}} \tag{10}$$

The value for $\Sigma\sqrt{P_i/T_i}$ is a function of the space through which the parachute travels. If the ground

TABLE 1C

Computed $\rho V^2$ and $C_dS$ Values for Each Observed 28-Ft. Silk Parachute Descent

| Flight No. | Density Altitude | Observed Time, Sec. | $\rho V^2$ | $C_dS$ |
|---|---|---|---|---|
| 1 | 7,500 | 224 | 0.87 | 517.8 |
| | | 210 | 0.99 | 455.0 |
| | | 233 | 0.80 | 560.4 |
| 9 | 7,750 | 246 | 0.74 | 608.9 |
| | | 232 | 0.83 | 541.5 |
| 14 | 7,800 | 242 | 0.77 | 587.5 |
| | | 213 | 0.99 | 455.5 |
| | | 240 | 0.78 | 577.7 |
| 4 | 16,000 | 583 | 0.90 | 501.7 |
| | | 539 | 1.05 | 428.6 |
| 6 | 15,875 | 589 | 0.83 | 542.2 |
| | | 572 | 0.88 | 511.4 |
| | | 613 | 0.77 | 586.7 |
| 8 | 15,750 | 562 | 0.95 | 471.2 |
| | | 600 | 0.84 | 537.0 |
| | | 602 | 0.83 | 540.9 |
| | | 615 | 0.80 | 564.6 |
| 13 | 15,850 | 600 | 0.83 | 543.5 |
| | | 558 | 0.96 | 470.2 |
| | | 607 | 0.81 | 556.2 |
| | | 552 | 0.98 | 460.1 |
| 21 | 15,950 | 609 | 0.81 | 557.6 |
| | | 586 | 0.87 | 516.6 |
| 5 | 27,050 | 998 | 0.88 | 510.2 |
| | | 1,115 | 0.71 | 636.5 |
| | | 1,032 | 0.82 | 545.5 |
| | | 1,017 | 0.85 | 530.0 |
| 7 | 27,600 | 970 | 0.91 | 497.2 |
| | | 990 | 0.87 | 517.8 |
| 15 | 26,360 | 1,022 | 0.81 | 555.6 |
| | | 1,032 | 0.80 | 564.6 |
| 19 | 26,300 | 984 | 0.88 | 513.7 |
| | | 964 | 0.91 | 493.4 |
| 10 | 39,400 | 1,585 | 0.67 | 666.7 |
| | | 1,372 | 0.90 | 499.5 |
| 11 | 40,050 | 1,389 | 0.90 | 499.5 |
| | | 1,388 | 0.90 | 499.5 |
| | | 1,389 | 0.90 | 499.5 |
| | | 1,448 | 0.83 | 542.8 |
| 18 | 40,000 | 1,417 | 0.85 | 531.3 |
| | | 1,461 | 0.80 | 564.6 |
| Mean | | | 0.855 | 530.74 |
| Standard Deviation | | | ±0.077 | ±48.63 |

elevation is at sea level, the appropriate $\Sigma\sqrt{P_i/T_i}$ may be read directly from the graph shown in Fig. 1. For ground-level elevations higher than sea level, the appropriate correction value can be obtained by moving up the altitude scale (ordinate) to the altitude at the ground level then moving out along this altitude value until the plotted curve is intersected. The abscissa of this point of intersection is the necessary correction value that must then be subtracted from the value of $\Sigma\sqrt{P/T}$ shown for the launching altitude. Thus, if the ground level is 1,000 ft., the value 0.15 would be subtracted from the value for the launching altitude. If we wish to calculate the descent time from 20,000 to 1,000 ft., we would read the value for 20,000 ft. from the graph, 2.77 and subtract the 0.15 for the 1,000-ft. landing elevation, getting 2.62 as the appropriate value of $\Sigma\sqrt{P/T}$.

The value for $\Sigma\sqrt{P/T}$ shown in Fig. 1 was computed from U.S. Standard Atmosphere tables. Hence, the error of prediction should be in proportion to the variation of the actual atmospheric conditions of the day from the standard atmosphere. However, this error is probably less than the errors introduced by the variation among parachutes. It is seen from the data of Table 1 that identical parachutes dropped in succession from a plane at a constant altitude will vary considerably in the time they take to reach the ground. This variation can be partially accounted for by rising and descending air currents and the amount of pendulum swing; hence, spilling of air. For this reason it is possible to predict only approximately the actual time of descent of a lone parachute. Therefore, the figures calculated by the above method should be taken to represent the mean, or average, descent times of groups of identical parachutes and loads.

## Comparison of Parachute Descent Times

For the purposes of comparing various types of parachutes it is believed that a computation of $C_dS$ values would yield better results than does a simple comparison of observed descent times. Such computations would relieve the necessity for attempting to duplicate pressure altitudes on different flights, as well as

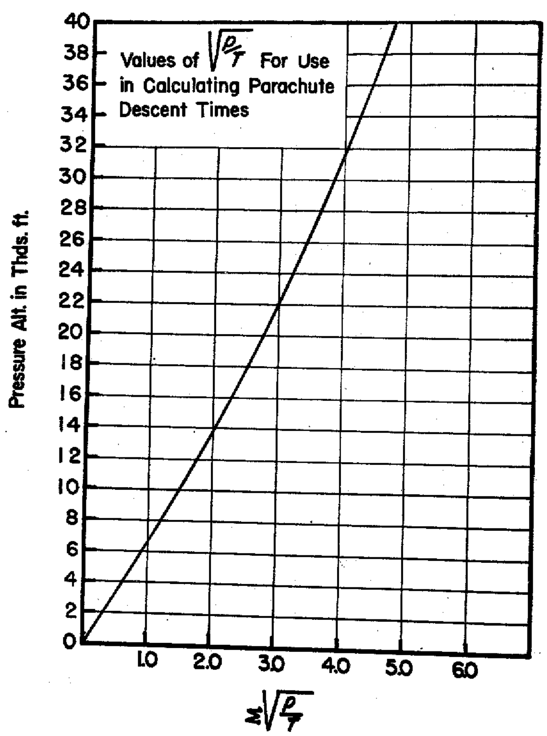


FIG. 1. Graph of $\sqrt{P/T}$ values, computed from U.S. Standard Atmosphere Tables, for use in calculating parachute descent times from the formula

$$\text{descent time} = (302.52\ \Sigma\ \sqrt{P/T})/\sqrt{2W/C_dS}$$

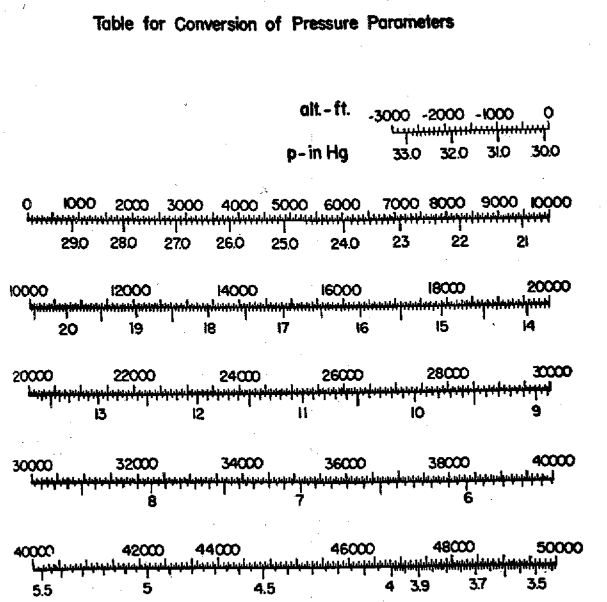


FIG. 2. The actual, or true, pressure at a given altitude is computed by setting one leg of a pair of dividers on zero and the other leg on the appropriate Kollsman number and by applying this correction to each altitude in question. Table adopted from Bellamy.[2]

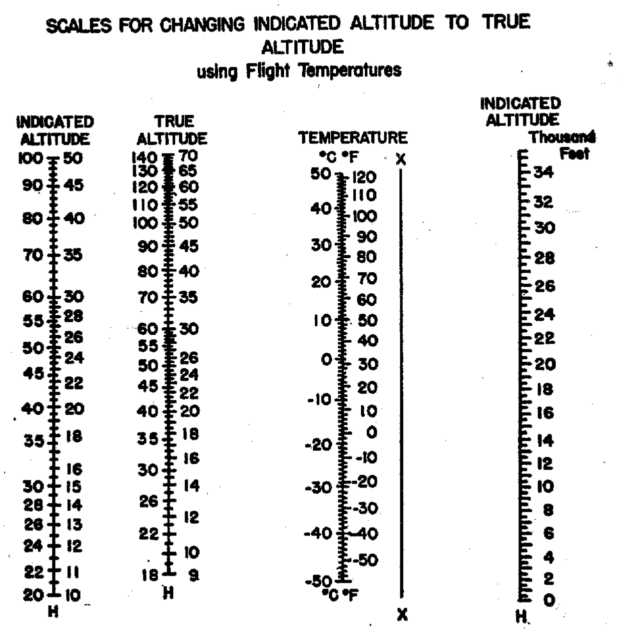


FIG. 3. A straightedge is placed to correct the indicated altitude of the right-hand column with its observed temperature. This locates a point on the line $X$. The straightedge is then rotated about this point so as to connect it with the indicated altitude in the left-hand column. True altitude is then read at the point where this line intersects the "True Altitude" scale.

rule out the effect of changing atmospheric conditions between tests.

For such a calculation the Kollsman number, or altimeter setting of the day, the temperature of the air at each 2,000-ft. level, the exact indicated altitude and temperature at time of drop, and a knowledge of the ground-level elevation are needed for each flight. The altitude of release need not be the same on successive flights. With these data at hand the method of calculation is as follows:

(a) True pressure can be obtained by entering Fig. 2 with a pair of dividers, one leg of which is set on zero altitude (29.92-in. pressure) and the other leg set on the figure for the altimeter setting of the day.[2] Thus, if such settings were greater than 29.92, the top line would be used, and, if it were less than 29.92, the second line would be used. The setting of the dividers so obtained represents a pressure correction that is used for each 2,000-ft. level as follows: Place one leg of the set dividers on the 2,000-ft. level in question and swing the other leg to right or left depending on whether the Kollsman number for the setting was to right or left of the zero point of 29.92 in. Hg. Read the pressure indicated by the second leg. This represents the actual pressure at the given indicated altitude. By doing this for each 2,000-ft. level, true pressures are obtained for each of these levels at which temperature is known. (Temperature readings were taken at every 2,000-ft. level of indicated altitude on each flight.)

(b) Next, the true altitude can be obtained from the nomogram shown in Fig. 3. To use this nomogram, a straightedge is placed on the right-hand portion of the nomogram in such a way as to connect the indicated altitude with its appropriate temperature. This locates a point on the line X. Next, the straightedge is rotated about that point on the line X and made to connect that point with the observed indicated altitude on the scale to the left. Then the true altitude may be read off the "True Altitude" scale at the point where the straightedge crosses that scale.

(c) The value for $\sqrt{P/T}$ can now be calculated for each of the successive levels for which true pressures have been determined in step (a), whose temperatures were recorded during flight, and whose true altitudes have been determined nomographically in step (b). The data up to this point had best be tabulated as shown in Table 2.

(d) The next step involves plotting the true altitudes vs. $\sqrt{P/T}$ values. From this graph the mean $\sqrt{P/T}$ value for each 2,000-ft. stratum may then be read directly. A second table should now be made showing the mean value of $\sqrt{P/T}$ for each complete 2,000-ft. stratum from ground altitude up to altitude of release.

The $\sqrt{P/T}$ values of the above table may now be summed and used in Eq. (5). Finally, Eq. (6) will give the $C_dS$ value for the known weight, $W$, of dummy and parachute.

The above procedure may be illustrated with a hypothetical case in which a 200-lb. dummy with a 24-ft. nylon parachute (total weight 215 lbs.) is dropped from a 9,875-ft. indicated altitude, the time of descent being recorded as 382 sec. Altimeter setting is 30.20 and ground level is +300 ft. The observed temperature and indicated altitudes are shown in Table 2, along with the

TABLE 2
Example of Calculation of $C_dS$ from Flight Data

| Indicated Altitude, Ft. | °C. | Temperature, °A. | True Pressure | True Altitude | $\sqrt{P/T}$ |
|---|---|---|---|---|---|
| +300 | +23 | 296 | 29.86 | 308 | 0.3175 |
| 2,000 | +15 | 288 | 29.09 | 2,020 | 0.312 |
| 4,000 | +13 | 286 | 26.09 | 4,050 | 0.302 |
| 6,000 | +12 | 285 | 24.21 | 6,150 | 0.292 |
| 8,000 | +11 | 284 | 22.43 | 8,250 | 0.2805 |
| 9,875 | + 9 | 282 | 20.89 | 10,220 | 0.2722 |

| Stratum | Mean $\sqrt{P/T}$ |
|---|---|
| 300 to 2,300 | 0.315 |
| 2,300–4,300 | 0.306 |
| 4,300–6,300 | 0.296 |
| 6,300–8,300 | 0.2855 |
| 8,300–9,875 | 0.2163* |
| | Σ 1.4188 |

* 1,575/2,000 of 0.2758 (mean for 8,300 to 10,300 stratum).

From Eq. (5)

$$\rho V_2 = (302.52 \times 1.4188/382)^2 = 1.2625$$

From Eq. (6)

$$C_dS = 430/1.2625 = 340.6$$

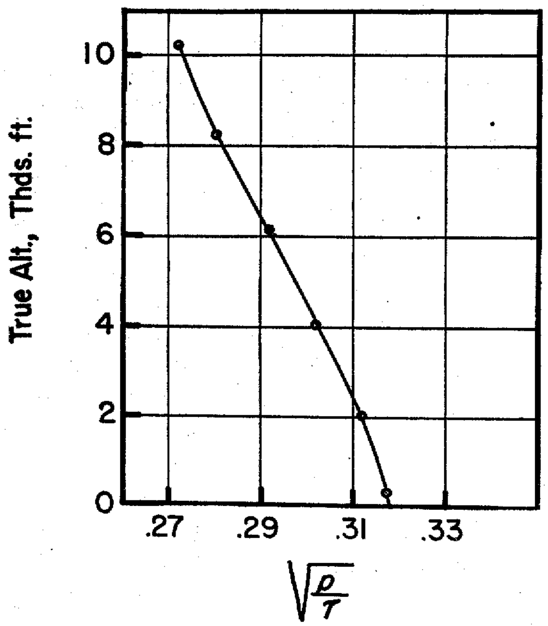


FIG. 4. Graph of true altitude vs. $\sqrt{P/T}$ for a sample calculation of parachute descent time. From this graph the mean $\sqrt{P/T}$ value for each 2,000-ft. stratum may be read directly. These values, together with their sum, are shown in Table 2.

entire calculation. The top stratum will not, in general, be a full 2,000-ft. stratum; therefore, since the coefficient 302.52 in Eq. (5) is based on 2,000-ft. strata, it is clear that the average $\sqrt{P/T}$ value for the top stratum must not be added directly along with the $\sqrt{P/T}$ values for the full 2,000-ft. strata but must be weighted by a fractional factor given by top-stratum-thickness-in-feet per 2,000 ft. An example of this weighting process is included in the sample calculations of Table 2. The graph described in step (d) as plotted for this calculation is shown in Fig. 4.

## LANDING VELOCITY

The velocity with which men of different weight and wearing different sizes and kinds of parachutes hit the ground is sometimes useful information. The effect of carrying heavy equipment, such as paratroops are sometimes required to do, and the effect of high ground elevations can easily be calculated.

The velocity at any altitude can be calculated from the data contained above by using the mean $C_dS$ values determined for each type of parachute. Thus, for 28-ft. nylon parachutes,

$$C_dS = 483$$

But

$$\rho V^2 = 2\,W/C_dS$$

So

$$V = \sqrt{2W/C_dS\rho} = \sqrt{2W/483\rho} \qquad (11)$$

The quantity $\rho V^2$, as expressed above, is seen to be a constant during any given descent (fixed $W$ and parachute type). Therefore, $\rho_1 V_1^2 = \rho_2 V_2^2$, so

$$V_2 = \sqrt{\rho_1/\rho_2}(V_1) \qquad (12)$$

where

$V_1$ = velocity at first altitude
$V_2$ = velocity at second altitude
$\rho_1/\rho_2$ = ratio between the densities of the two altitudes

The validity of the relationship expressed above can be put to test using the observed descent times listed in Table 1. In Fig. 5 the observations on descent times of 28-ft. nylon parachutes have been averaged for each of four approximate altitudes and plotted as launching altitude versus total time of descent to 2,800 ft. (ground level in these tests). Then a theoretical curve has been constructed using Eq. (12), where $V_2$ = velocity at 2,800 ft., $\rho_2$ = density at 2,800 ft., and the resulting curve is plotted to the same axes. The $V_2$ used in this calculation was derived from Eq. (11), using standard $\rho$ for 2,800 ft.

It is evident that the observed mean descent times from the four different altitudes fall very close to this theoretical curve. Since the same relative correlation was found when the 24-ft. nylon and 28-ft. silk parachute descent time data were plotted against such a theoretical curve, it would appear to be safe to accept the validity of Eq. (12).

If we wish to determine the sea-level velocity of men weighing 225 lbs. wearing a 28-ft. nylon parachute, we may refer to Eq. (11) above and use any convenient

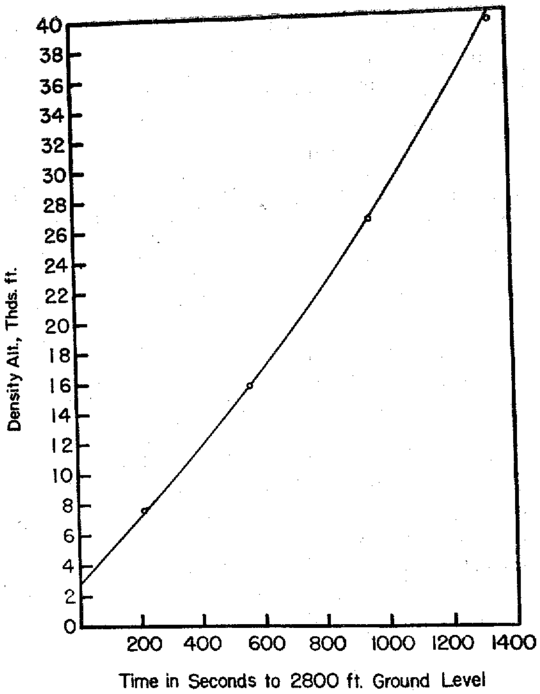


FIG. 5. Descent times of 28-ft. nylon parachutes to 2,800-ft. ground level. The curve is derived from the formula

$$V_\alpha = V_{2,800}/\sqrt{\rho_\alpha/\rho_{2,800}}$$

where $V_\alpha$ = velocity at launching altitude, $V_{2,800}$ = velocity at 2,800 ft., $\rho_\alpha$ = air density at launching altitude and $\rho_{2,800}$ = air density at 2,800 ft. The four points represent means of four groups of observed descent times.

altitude. If we choose 10,000 ft., $\rho_1$ becomes 0.001756 and $V_1$ becomes 23.11. Then, using Eq. (12),

$$23.11 \times 0.859 = 19.85 \text{ ft. per sec. at sea level}$$

where 0.859 is the square root of the density ratio for 10,000 ft. The comparable sea-level velocities for 24-ft. nylon and 28-ft. silk parachutes are 23.78 and 18.96, respectively.

The precise effect of weight change on the rate of descent of parachutes cannot be shown by our data, since we used only one weight dummy. In general, we know that the heavier the weight the faster the rate of descent and, hence, the greater the landing impact; but one fundamental question remains unanswered: Does the weight of the incorporated air mass combine with the effect of dummy weight to determine the

TABLE 3

Calculated Effect of Varying Weights of Men on the Sea-Level Velocity of a Standard 24-Ft. Nylon Parachute

| Weight, Lbs. | Velocity, Sea Level, Ft. per Sec. |
|---|---|
| 150 | 19.38 |
| 175 | 20.93 |
| 200 | 22.37 |
| 225 | 23.78 |
| 250 | 25.02 |
| 275 | 26.27 |
| 300 | 27.40 |

actual descent rates? If it does not, then the effect of the weight of the man on descent rate is easily calculated as follows:

For 24-ft. nylon parachutes with man-shaped dummies, the $C_dS$ value is 336. Then,

$$\rho V^2 = 2W/C_dS = 2W/336$$

Next, applying the above in Eq. (11), using the appropriate value of $\rho$ in slugs per cu.ft., the velocity may be calculated. Table 3 shows the results of such a calculation for sea-level velocities. If the weight of the air mass does enter into the problem, it would have a damping effect on the change in velocity, since the incorporated air mass should remain relatively constant with changes in weight of men. Hence, the percentage change in weight of the whole system would be less and the total spread in velocity shown in Table 3 would be too great. Thus, for the larger weights in Table 3, the velocities may involve small errors, but it is to be noted that any errors are on the safe side, since, at worst, the landing velocities in Table 3 are overestimated.

In this treatise no account is taken of the effect of the so-called "ground reflection" of air on the landing velocity of the parachute. This phenomenon, said by Stasevich[3] to give a retardation of 10 to 20 per cent, is a variable factor and has not, to our knowledge, been subjected to scientific measurement. This effect is presumed to be due to two factors: (1) rising air currents near the ground due to ground heat and (2) the striking of the ground of the air mass being pushed ahead of the canopy.

Practical experience in both the United States and British airborne services and air forces has shown that an increase in ground altitude of 5,000 ft. has been sufficient to increase greatly the number of landing injuries beyond those encountered in descents to near sea level.[4] It is generally assumed that this rise of injury rate is due to a rise in landing velocity occasioned by the reduced density of the air at the ground level in elevated areas. To show more clearly the effect of altitude on

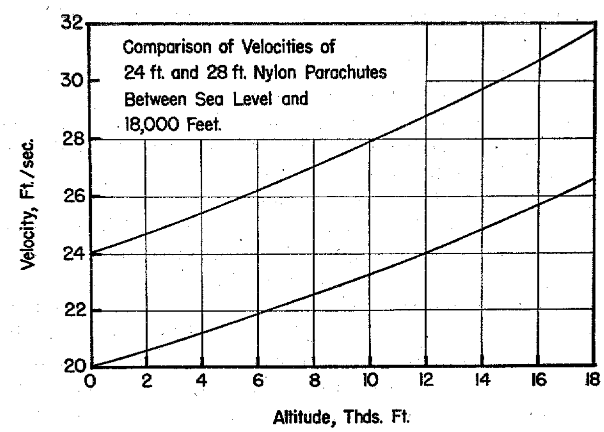


FIG. 6. The curves are derived from the formula $V_\alpha = V_0 \div \sqrt{\rho_\alpha/\rho_0}$, where $V_\alpha$ = velocity at any altitude, $V_0$ = velocity at sea level, and $\rho_\alpha/\rho_0$ = the density ratio.

landing velocities, Fig. 6 represents the theoretical increase in landing velocities of 24 and 20 ft. per sec., respectively.[5]

It may seem surprising, since the landing velocity at 5,000 ft. in a 24-ft. nylon parachute is less than 2 ft. per sec. faster, that the injury rate should be appreciably higher. However, it must be recalled that the kinetic energy of a falling body increases as the square of the velocity. This relationship is given by the formula

$$E_k = {}^1/_2 M V^2$$

Thus, calculation shows that the kinetic energy to be dissipated upon striking the ground at 5,000 ft. is some 17 per cent greater than at sea level for the same weight of man and same type of parachute. On the other hand, it may be calculated from the same formula that the kinetic energy associated with the 28-ft. parachutes is considerably less than for the 24-ft. parachutes at the same ground level. In the case of the 28-ft. nylon, for a 200-lb. man, it is some 43 per cent less, and in the case of the 28-ft. silk, it is 57 per cent less at sea level.

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# Letter to the Editor

Dear Sir:

Mr. Krzywoblocki's Letter to the Editor (Journal of the Aeronautical Sciences, pp. 23–24, January, 1947) is so significant and representative of the opinion of a certain group of students of the problem of jet and rocket propulsion that it seems desirable to discuss further the definition of propulsive efficiency.

The formula for propulsive efficiency was established correctly many decades ago. It reads, using Mr. Krzywoblocki's notation:

$$\eta_p = \frac{2[(W_a + W_f)v_j - W_a V]\, V}{[(W_a + W_f)v_j^2 - W_a V^2]} \qquad (1)$$

Let us remember a few basic laws and principles of elementary physics:

(1) Velocity and force are vectors; the same is true for momentum, while kinetic energy is a scalar. Therefore, the direction of velocity vector is of importance in computing momentum, while it is immaterial in computing kinetic energy.

(2) The principle of relativity, a few centuries old, still holds true in classic mechanics.

(3) Principle of conservation of energy and principle of conservation of momentum invariably hold true.

(4) Every problem of mechanics must be placed in a certain given system of reference before it is theoretically treated. Of course, there is no difference as to the system chosen—i.e., one system of reference is as good as another. But once chosen, it must be respected until the whole computation of either energy or momentum balance is accomplished. Leaping from one system to another during the computation is inadmissible.

(5) In the very nature of the notion of efficiency lies the fact that its 100 per cent value is a limit that is either asymptotic in function of at least one of the physical parameters involved or occurs for zero value of at least one other physical parameter involved, which is essentially positive (i.e., absolute temperature, mass, etc.). This limit is, of course, not actually attainable, but it must be logically admissible. This means that practically attainable values of efficiency may be imagined as close to 100 per cent as the technical facilities now available permit, and there is no limit in improving these facilities so as to approach more closely the 100 per cent value in the future. Any formula for efficiency giving a theoretical maximum value of efficiency less than 100 per cent is erroneous. To illustrate the above, let us take the theoretical efficiency of the Otto cycle, which has an asymptotic limit of 100 per cent for the volumetric compression ratio infinitely large. Another example is the efficiency of Carnot cycle

$$\eta = \frac{(T_1 - T_2)}{T_1}$$

which has either an asymptotic limit of 100 per cent for the absolute temperature $T_1$ infinitely large or a limit of 100 per cent for the absolute temperature $T_2$ equaling zero; of course, the absolute temperature cannot be negative.

Mr. Krzywoblocki's assumption (2) is erroneous. Taking the propelled unit as a system of reference, air enters the engine at the rate of $W_a$ lbs. per sec. and with the speed $V$; the input momentum is, therefore, $(W_a/g)\,V$ and the input kinetic energy is $(W_a/2g)\,V^2$; this is an actual fact. Ascribing this latter energy to the thermal efficiency would only displace its asymptotic limit from the 100 per cent value, which is wrong. The energy to be produced by the engine is, therefore, only

$$[(W_a + W_f)/2g]\, v_j^2 - (W_a/2g)\, V^2$$

Now, Mr. Krzywoblocki changes to another system of reference. Energy $L_1$ given by his Eq. (2) was computed relative to the propelled unit. Now he adds the kinetic energy $L_2$, given by his Eq. (3), relative to the earth. This energy is, of course, nil in the former system of reference. A further step along the path chosen by Mr. Krzywoblocki may be made. The earth as a system of reference is no better than any other system. Let us take the sun as a system of reference. The velocity $V$ in Mr. Krzywoblocki's Eq. (3) for $L_2$ will become about 90,000 ft. per sec. The "most favorable conditions" in which the maximum energy can be imparted to the jet unit become still more favorable. Any other fixed star or even the Milky Way may also be chosen as a