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3.9. MEASUREMENT, ANALYSIS, AND INTERPRETATION OF TURBULENT TEMPERATURE FLUCTUATIONS

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ABSTRACT

An analysis of the several radiative components responsible for the heating of an exposed bead thermistor is summarized and the results used to predict the extent of the spurious fluctuations of indicated temperature that arise because of fluctuations in wind speed past the element. For wind speed fluctuations whose average amplitudes are about one fourth of the mean speed, the error amplitudes are less than 10 per cent of the true air temperature fluctuation amplitudes.

The problem of instrument lag is analyzed with respect to the distortion of fluctuation statistics. Certain general relations that must exist between the true and the apparent statistics are derived, and some specific analyses of the magnitude of lag effects are carried out for the case of the bead thermistor.

3.9.1. INTRODUCTION

The record of a rapidly responding temperature-sensing element located at a fixed position near the ground on a clear day reveals that the natural wind is thermally heterogeneous to an extent that would remain wholly unsuspected if only inert instruments such as the conventional mercurial thermometer were used. The investigation of this thermal fine structure of the lower atmosphere involves a number of problems which will form the principal subject of this paper.

Throughout the following discussion, "temperature fluctuations" are to be understood to be those implied in the Eulerian time derivatives and never the Lagrangian. Since the Eulerian "period" is related, through the instantaneous wind velocity, to the Lagrangian "eddy diameter," all of the current ambiguity of the latter term is bequeathed to the former, aggravating a semantic problem with which the writer prefers not to grapple here. If the mean speed of a layer bounded below by the earth is U and the depth of the layer is H, then on the assumption that eddies whose diameters are of the order of H may be embedded in this flow, the Eulerian "periods" of the fluctuations caused by the passage of these eddies past a fixed sensing element will be of order H/U. Considering layers ranging in depth from the surface layer with H of the order of meters up to the whole boundary layer with H of the order of a kilometer, and noting the associated range of U, one forms a crude prediction that he must expect to encounter periods ranging from about 10^{-1} sec to 10^2 sec in micrometeorological fluctuation studies.

3.9.2. PRESENT STATUS OF KNOWLEDGE OF TURBULENT TEMPERATURE FLUCTUATIONS

There certainly does not exist in the literature a definitive study of turbulent temperature fluctuations. Such work as has been reported can frequently not be interpreted clearly due to omission of any information concerning response characteristics of the sensing element employed, especially its lag-time. Hence no attempt will be made to cite all of the literature references wherein temperature fluctuation data have been mentioned.

Giblett [7] and his associates at Cardington made an early contribution to our knowledge of the longer-period eddies and called attention to the negative correlation between wind speed and air temperature. Best [3] drew certain conclusions from a series of measurements of temperatures taken at two separate points and was criticized by Bilham [4] in a paper which seems to represent the only serious effort yet undertaken to assay the statistical distortions which lag can impose upon data obtained from inert instruments (see, however, Hall, [8]). Haude [9] has described surprisingly noncorrelated temperature fluctuations at 1 mm and 1 cm above a point on desert ground. More recent observations of temperature profiles and fluctuations very near the surface (0.01 in. to 1.0 in.) have been reported by Vehrencamp [18]; but the 0.003 in. diameter thermocouples used must have developed appreciable radiation errors (Thornthwaite [16; 17]) which interacted with wind speed variations to add spurious fluctuations. Gerhardt and Gordon [6] have called attention to thermal evidence for the presence of rather long eddies in the airflow near the surface. Schilling *et al.* [13] mention briefly some measurements made with a 0.0005-in. diameter resistance wire, from which they conclude that the air temperature may change by some 10°C in a second or two, notable here as the most extreme oscillation the writer has seen reported in his examination of the literature. Hoehndorf (quoted by Slater [14]) reported "average fluctuations" of 3.9°C at a height of 1 m, 2.4°C at 10 m, and 1.8°C at 20 m above an airport surface near midday on a cloudless summer day. Data such as these, sparse as they are, help point out the nature of the measurement and analysis problems one must expect to face in the investigation of turbulent temperature fluctuations.

In an attempt to fill some of the remaining gaps in our knowledge of these problems, the Geophysics Research Directorate of the Air Force Cambridge Research Center is currently sponsoring a study of temperature fluctuations by the Department of Physics of Iowa State College. Since the present project has been under way only since January 1951, no field observations are available for discussion here. However, some of the preliminary investigations already completed will be described in their relations to the ultimate tasks of measurement and analysis of fluctuation phenomena.

On examining the chief difficulties which stand in the way of fluctuation studies, these seem to fall into the categories of instrumentation, analysis, and interpretation. Working on the theory that the processes of analysis and interpretation can only be carried out in a meaningful way when there exists quite complete information concerning the response characteristics of the instrumentation employed, considerable effort has been expended to study the sensing element tentatively selected for use on our project, the bead thermistor. This paper will be devoted primarily to a summary of our contributions in this direction.

The problems of analysis of turbulent fluctuations are serious. It seems safe to say that the extraordinary effort that has to be made to reduce great lengths of records to usable form has done more than anything else to discourage earlier workers from furthering our knowledge of thermal fluctuations. The development of a rapid automatic electromechanical tabulator which is capable of performing in a few minutes operations which would take days of tedious manual work will be described elsewhere in these proceedings by Kassander. The final step of interpretation of observational results involves problems some of which may be anticipated even before data are available for interpretation, but which will be omitted from consideration here.

3.9.3 RADIATION ERRORS OF A BEAD THERMISTOR

There is very real need for careful determinations of the radiation errors of all of the thermal sensors being used for micrometeorological research. Such information is surprisingly unavailable at present. In this section an analysis of the radiation errors of a bead thermistor will be summarized. This element is currently being used in micrometeorological studies at a number of institutions, but its errors and response properties have not previously been examined critically. One of the first steps taken as part of the fluctuation study in the Department of Physics at Iowa State College was that of investigating both theoretically and experimentally a number of properties of this recent newcomer to the field of meteorological instrumentation.

The first step in the radiation error analysis consisted in calculating expected radiation receipts on a D-176980 bead thermistor, considering direct solar radiation, diffuse sky radiation, ground-reflected radiation, and infrared radiation. A more complete discussion of this analysis is to be found in Report No. 1 of our current project (Kassander [10]). To indicate here the orders of magnitude involved it may be noted that the bead, which has a diameter of about 0.033 cm, intercepts about 0.08 milliwatts of radiant power from the direct solar beam on a clear day at noon the year around (little seasonal variation existing in noon normal-incidence intensity of the solar beam due to opposing influences of path length and atmospheric water vapor content). The incident diffuse sky radiation amounts to about 0.025 milliwatts on a clear day and, for the nearly spherical bead, can be shown to exhibit practically no diurnal variation. Ground-reflected radiation is difficult to evaluate but may amount to another 0.01 to 0.02 milliwatts. Infrared exchange is responsible for a net cooling influence under all conditions except those where the soil surface is so hot as to more than cancel the nocturnal radiation to the celestial hemisphere. It has been estimated that the soil must be hotter than about 40°C for infrared exchange to exert a net warming action on a bead thermistor located within a few meters of the surface.

Knowing incident radiation values, the steady-state radiation error can only be determined when the heat-loss characteristics of the element are known, and in particular, known as a function of ambient wind speed. The experimental determination of these characteristics was carried out by Kassander. By passing 30 microamperes through a bead thermistor it is called upon to dissipate, by transfer to the ambient air, some 0.075 milliwatts of power developed internally as Joule heating. By measuring the temperatures attained by the bead for various ventilation rates and comparing these with corresponding ambient temperatures, the "retentivity error" of the bead for this power level is determined. Since the heat-transfer process is independent of the ultimate source of the dissipated power, the retentivity errors so determined are directly comparable to the radiation errors that would exist for the same level of power dissipation due to radiative heating and for the same ventilation rates. (No attempt has yet been made to evaluate the effects due to differences in spatial distribution of the heating in the radiational and the electrical cases, but since the conductivity of the bead material is about two orders of magnitude larger than the conductivity of the surrounding air, one would expect to find fairly uniform surface temperatures even for the hemispheric heating due to solar radiation.)

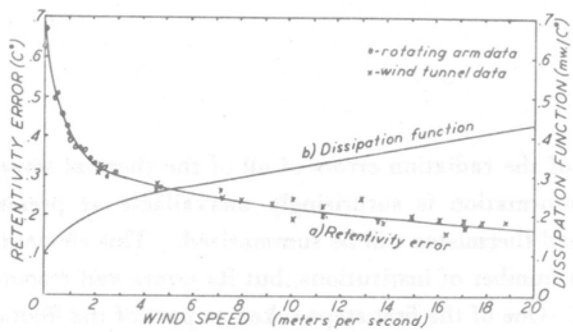

Fig. 3.9.1. (a) Retentivity error of a D-176980 bead thermistor for a power dissipation of 0.075 milliwatts; (b) Dissipation function (mw/°C) for same bead thermistor, (derived from Curvea).

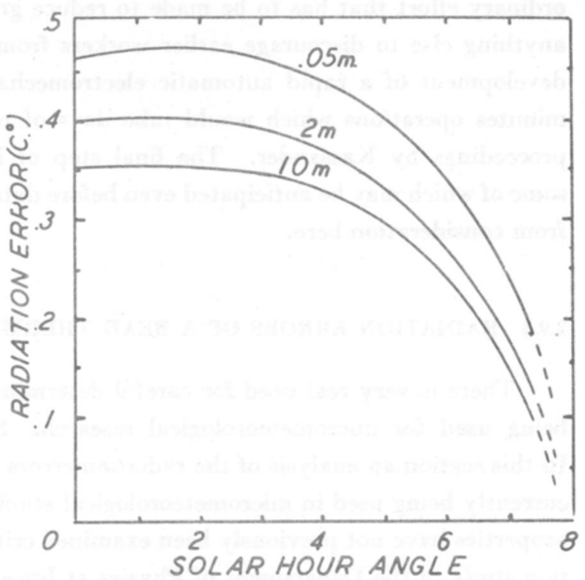

Fig. 3.9.2. Radiation errors of a bead thermistor at three heights above the ground on a clear July day.

In Fig. 3.9.1, the curve labeled "retentivity error" represents the excess of bead temperature over air temperature for a power level of 0.075 milliwatts for a specimen thermistor. The ambient temperatures were determined by passing only 10 microamperes through the same bead, so the differences plotted here as retentivity error are really only about 90 per cent of the full error with respect to true air temperature. The ventilation was obtained in two ways: For low wind speeds a rotating-arm method was used within a cylindrical drum whose walls were close enough to the locus swept out by the bead to suppress Mitwind effects. For higher wind speeds, the thermistors were placed in a thermally-controlled wind tunnel which was originally built to calibrate thermistor anemometers (Sanford [12]).

The indicated errors are themselves applicable only when the total radiant power absorption happens to equal 0.075 milliwatts. To derive from this curve a parameter which is a property of the element's dissipating abilities at the several wind speeds, each ordinate on the retentivity error curve was divided by the power dissipation of 0.075 milliwatts and the quotient plotted as the "dissipation function" of the bead in the same figure. The great value of the dissipation function consists in its providing the information necessary for the calculation of the radiation error for *any* incident radiation intensity and for any ambient wind speed in the range for which the function is determined. Exactly this application has been made and described in Report No. 1 of our current project.

Two examples of the bias introduced into temperature measurements by radiation errors will be given here. First, the vertical bias in temperature profile measurements occasioned by the systematic increase of mean wind speed with height may be noted. For a radiative flux which is the same at all heights, elements mounted near the surface will obtain less adequate ventilation than those at greater heights and so *differences* in radiation error will develop. By using the dissipation function in conjunction with mean wind profiles given by Hellman and reproduced in Geiger [5], the curves of Fig. 3.9.2 have been computed for the case of a clear July day. Both direct and diffuse solar radiation were considered, but no ground reflection or infrared exchange taken into account in these computations. It will be seen that the absolute magnitudes of the radiation errors are appreciable, especially at 0.05 m above the ground, but that the *differences* of radiation error are about one order of magnitude smaller. Most of the bias occurs within the very lowest layers where most of the mean velocity difference occurs. From 2 m to 10 m the difference in error does not exceed 0.05°C throughout the day. Since a fairly representative air temperature difference between 0.05 m and 10 m might be 3°C near midday on a clear summer day, the total relative bias amounts here to about 0.10/3.0, or some 3 per cent.

Turning next to fluctuation measurements one finds that the role played by radiation error is such that a rather different analysis needs to be made. An element which, in calm conditions, develops a radiation error of about 1°C, might in a strong wind possess so large a dissipation rate as to suffer only a 0.1°C error. If, then, appreciable fluctuations of wind speed are occurring simultaneously with strong irradiation of an element, spurious oscillatory fluctuations will be superimposed upon those fluctuations which are due to real variations of ambient temperature. By this process the undesirable effects of wind fluctuations become confounded with the sought-for effects of temperature fluctuations, so the resulting distortion of fluctuation statistics may be termed a "confounding error." The magnitude of this error varies jointly with radiation intensity and with variability of wind speed, but is not the same function of these factors for all wind speeds due to the nonlinearity of the dissipation function. As an indication of the intensity of the spurious background originating in this manner, the conditions prevailing on a clear July noon were assumed (0.10 milliwatts radiant power absorbed) and wind fluctuations of 25 per cent of the current mean used to estimate the fluctuations due to confounding effects. Since the magnitude of the indicated temperature change is greater for wind speed decreases of a given amount than for increases of the same amount, a lull in which the speed drops by 25 per cent of the mean was assumed, conservatively. The resulting dependence of confounding error on mean wind speed is plotted as Fig. 3.9.3. The peculiar S-shape of the curve is a consequence of the curvature of the dissipation function and the arithmetical nature of the computation. It will be seen that for the assumed conditions, the background fluctuation due to confounding effects remains below 0.06°C for all wind speeds likely to be encountered near the surface.

The 25 per cent level of wind fluctuation was employed after a search of the literature for information as to gustiness figures. In most sources, the coefficient of variation of wind speed (standard deviation divided by mean) lay near 20 per cent, the highest being about 35 per cent. Clearly, occasional gusts and lulls must induce confounding effects in excess of those suggested by Fig. 3.9.3, but the coefficient of variation seems pertinent to the estimation of average distortion of fluctuation statistics. Since the peak-to-trough amplitudes of temperature oscillation are of the order of 1°C in the surface layer, confounding effects yield contributions of the order of 10 per cent when bead thermistors are used.

A second type of confounding effect is also to be noted. The structure of the bead, in particular the arrangement of lead wires, is such as to preclude exact spherical symmetry of the element. Early in our work we had assumed that these effects could scarcely have a noticeable effect on so tiny a scale. But in the course of studying the suitability of heated bead thermistors for anemometry work it was discovered that there were quite decided variations in the anemometric response of the beads with variations of orientation (Sanford [12]). To learn the magnitude of this same effect in thermometry applications wherein radiative heating leads to the hot-wire action that is the basis of confounding effects, the dissipation function was determined for a series of different orientations of the bead with respect to the relative wind. This work was completed too recently to be included in graphic form in this paper but it was found that, for the single bead so far examined, angular variations of the relative wind vector within the plane containing the lead wires produced extreme differences of about 0.1°C and this extreme difference was almost independent of wind speed. Fortunately, variations of the wind vector within a plane normal to the principal axis of the beads (the beads resemble rather fat footballs when viewed under a magnifier) yield extreme differences of only about 0.025°C for a power dissipation of 0.075 milliwatts. Having recognized this fact, it will be possible to minimize confounding effects due to fluctuations of wind direction by mounting the beads in such a manner that the horizontal plane is normal to their principal axis. This will not entirely solve the directional confounding problem because fluctuations in the vertical component will act in the most directionally-sensitive planes.

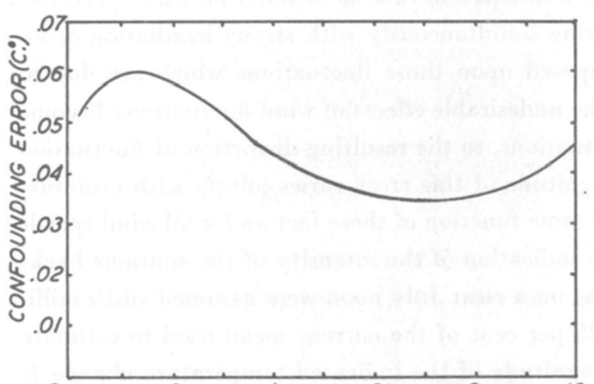

Fig. 3.9.3. Confounding error resulting from a $0.25\bar{V}$ lull at noon on a clear July day for various mean wind speeds.

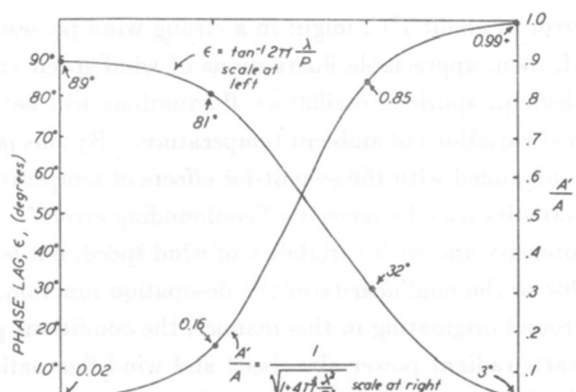

Fig. 3.9.4. Phase lag, ϵ, and amplitude-suppression ratio, A'/A, for the response of a sensing element of lag time λ exposed to sinusoidal temperature oscillations of amplitude A and period P. Dimensionless ratio of period to lag time plotted logarithmically on abscissa.

All of the preceding discussion of radiation errors has been based on a combination of laboratory determinations of the dissipation function plus calculated wind speeds and radiation intensities. Such field observations as have thus far been made in order to check the validity of these analyses are in fair agreement with the results. Further work on this point is being carried out currently. It is to be emphasized, however, that once the dissipation function is accurately determined, a large variety of error analyses may be carried out theoretically in a more clear-cut fashion than is possible experimentally, under field conditions.

The writer would like to urge strongly that analyses similar to the above be carried out for the other sensing elements used in micrometeorological temperature studies. In particular, the thermocouple appears to have received insufficient attention in this respect. The lesson which we have learned concerning bead thermistors is that a sensing element which seems so small as to be incapable of developing those effects which we tend to think of as occurring only in objects of "appreciable" size does indeed exhibit these effects. There seems no good reason to believe that similar results will not appear when some of the other sensors are subjected to similar examination.

3.9.4. LAG EFFECTS

It would appear that Bilham [4] was the first to use the lag equation to discuss the response of a thermal element to a sinusoidally varying environmental temperature. It would also appear that he was the last; for there are, to the writer's knowledge, no other systematic analyses of the role that lag may play in biasing fluctuation statistics. Although Bilham's ideas have not been extended, due to lack of interest in the fluctuations themselves, the solution to the lag equation which he gave is well known and frequently used to deduce response properties of thermal elements after the manner suggested by him. The lag equation,

$$\dot{T} = -\frac{1}{\lambda}(T - T_e), \tag{1}$$

wherein T is the instantaneous indication of the element, T_e is the environmental temperature at the same instant, the dot symbol denotes time differentiation, and λ is the lag time, was examined by Bilham for the case where

$$T_e = \overline{T}_e + A \sin \frac{2\pi t}{P}, \tag{2}$$

in which the bar denotes a time average and A is the amplitude of the environmental temperature fluctuations assumed to occur purely sinusoidally with period P, and was shown by him to have as its solution,

$$T = ce^{-t/\lambda} + \overline{T}_e + A' \sin\left(\frac{2\pi t}{P} - \epsilon\right), \tag{3}$$

where the first term on the right is the transient term with integration constant, c; A' is the response amplitude given by

$$A' = \frac{A}{\sqrt{1 + 4\pi^2 \dfrac{\lambda^2}{P^2}}};$$

and ϵ is the phase lag given by $\tan^{-1}(2\pi\lambda/P)$. For later reference, the amplitude suppression ratio, A'/A and the phase lag, ϵ, are plotted against P/λ in Fig. 3.9.4. It is of importance to note that these two response characteristics are relatively flat for either very large or very small values of P/λ but that in the P/λ range from about 1 to 10 they are quite sensitive to the ratio of period to lag time. Figure 3.9.4 has numerous implications for the design of thermal elements and for the analysis of records of fluctuating temperatures, several of which will be examined below. Before proceeding to this examination, however, the lag differential equation will be discussed from a point of view inverse to that commonly taken and a few general properties deduced therefrom.

3.9.4.1. *The Inverted Lag Equation*

The conventional treatment of Eq. (1) involves solution of the differential equation for a specified $T_e(t)$, as, for example, for a step-function in T_e, linear change of T_e with time, sinusoidal variation of T_e about some given mean value, etc. (see Middleton [11]). In practice the inverse problem arises: one knows $T(t)$ from the record of a lagging instrument and knows the lag time of the instrument, but seeks to determine what forcing function, T_e, must have led to the observed response. Solving Eq. (1) for T_e yields

$$T_e = T + \lambda \dot{T} \tag{4}$$

which will be referred to here as the inverted lag equation. Although the difference between Eq. (1) and Eq. (4) is mathematically trivial, the latter form suggests certain fruitful ideas in a much more obvious way than does the former. Some illustrations will be given.

A. QUALITATIVE ANALYSIS OF A RECORD OF THERMAL FLUCTUATIONS

For any arbitrary $T_e(t)$, Eq. (4) states that the instantaneous error, $T - T_e$, is given exactly by $-\lambda\dot{T}$. Thus, knowing the lag time, λ, of the sensing element used to get the record of $T(t)$, and estimating or measuring the indicated time rate of change, $\dot{T}$, one can determine what the true air temperature, T_e, must have been at any desired time. An examination of a record of thermal fluctuations made under conditions of well-developed turbulence frequently reveals numerous small spurs of very short duration rather densely superimposed upon the longer-period oscillations of much greater apparent amplitude. That this fine "embroidery" must be evidence of true air temperature excursions whose amplitudes often approach those of the longer-period components is apparent when one applies the $\lambda\dot{T}$ correction; for though these spurs on the indicated temperature record never grow very large, they have slopes which are often many times greater than those of the slower oscillations. The same conclusion is, of course, reached by regarding the actual excursion of air temperature as being made up of a number of Fourier components which, according to Eq. (3), suffer attenuation that tends to filter out the high frequencies, with the result that the response function contains only the little spur as a relic of the large pulse in the forcing function. The inverted lag equation tells the story here in a less elegant but more direct manner.

As corollaries to the preceding rule for estimating the instantaneous error of indication of a lagging element, note first that the element is yielding an *exact* indication of air temperature at each instant that $T(t)$ passes through an extremum through which $\dot{T}(t)$ is continuous, and second that any kink where $\dot{T}(t)$ appears (to within the limits imposed by the time scale of the record) to exhibit a discontinuity, must mark the occurrence of a step-function change in true air temperature (to within the approximation implied in the openness of the time scale). If the discontinuity in $\dot{T}(t)$ is such as to involve a change of sign of $\dot{T}$, Eq. (4) shows that the sign of the error, $T - T_e$, must change.

B. A LAG COMPENSATION TECHNIQUE

The error term in Eq. (4) suggests the basis of a means of compensating the output of a lagging element in such a way as to correct for lag effects prior to recording. The method, which becomes feasible only for elements yielding an electrical signal, consists of splitting the signal, differentiating one part with respect to time, multiplying this by λ, and then adding the product to the other part into which the signal was originally split, all operations being performed electrically. Equation (4) shows that when these operations are correctly performed, the final output is the true air temperature, T_e. To take account of variations of λ accompanying varying wind speeds, one might use the output of an anemometer to control the value of the λ-factor in the multiplying stage; or this could be adjusted manually to correspond to the mean wind speed prevailing during a given run.

C. DEDUCTION OF RELATIONSHIPS BETWEEN TRUE FLUCTUATION STATISTICS AND APPARENT FLUCTUATION STATISTICS

The lag equation written as Eq. (4) suggests rather directly how one may derive certain general relationships that necessarily exist between the true fluctuation statistics and the apparent fluctuation statistics obtained from the record of a lagging element.

(a) *Average Value of $T(t)$ for an Arbitrary $T_e(t)$*

For the special case of a purely sinusoidal waveform of $T_e(t)$, comparison of Eq. (3) with Eq. (2) reveals that the response average is identical with the true average after the transient has died out. It is important to know whether this equality exists for an arbitrary $T_e(t)$, since actually observed waveforms are almost never comparable to sine waves. Averaging Eq. (4) over the period of time from t_0 to t_1 yields

$$\overline{T}_e = \overline{T} + \frac{\lambda}{t_1 - t_0}\int_{t_0}^{t_1} \dot{T}\, dt \equiv \overline{T} + \frac{\lambda}{t_1 - t_0}\int_{T(t_0)}^{T(t_1)} dT\,. \tag{5}$$

But

$$\frac{\lambda}{t_1 - t_0}\int_{T(t_0)}^{T(t_1)} dT \leq \lambda\,\frac{T_u - T_l}{t_1 - t_0} \tag{6}$$

where T_u and T_l are the upper and lower bounds on $T(t)$ in the interval $t_0 \leq t \leq t_1$. But since $T(t)$ is, for physical reasons, necessarily of bounded variation in *any* interval of time, the right member of Eq. (6) may be made arbitrarily small by taking the averaging interval $t_1 - t_0$ sufficiently large. Hence in this limit of sufficiently large averaging interval the last term of the last member of Eq. (5) vanishes and

$$\overline{T}_e = \overline{T}, \tag{7}$$

to within an error which for any given case is equal to or less than the value of the right side of Eq. (6). Thus we have the result that for the case of the time average, the apparent statistic approaches equality with the true statistic in the large-sample limit, the approach being oscillatory rather than asymptotic from one side, in general. This result, like most of the results of manipulation of the lag equation written in the form of Eq. (4), is seen, *a posteriori*, to be qualitatively implicit in Eq. (3) since it is the very nature of the response law (Eq. (3)) that the element will continuously "hunt" for the condition of zero error, thereby regressing toward the true mean after each successive disturbance passes by. The result will also be seen to be deducible from the Fourier viewpoint using Eq. (3) and the linearity of Eq. (1).

(b) *Relation of True and Apparent Standard Deviations*

A useful statistic for expressing the degree of variability of $T_e(t)$ is the standard deviation, σ_e. It is indispensable, in any fluctuation study, to have some measure of how biased an estimate of σ_e is given by σ, the apparent standard deviation derived from the record of a lagging sensor. From the definition of the standard deviation and from the inverted lag equation (Eq. (4)),

$$N\sigma_e^2 = \sum_1^N (T_i + \lambda \dot{T}_i - \overline{T}_e)^2 \tag{8}$$

where, for convenience here, the record of $T(t)$ will be regarded as represented by a set of N instantaneous values T_i read off at equal small intervals. On expanding the right side, noting that

$$N\sigma^2 = \sum_1^N T_i^2 - N\overline{T}^2,$$

and making use of Eq. (7), one finds that

$$\sigma_e^2 = \sigma^2 + \lambda^2\overline{(\dot{T})^2} + 2\lambda\overline{T\dot{T}}. \tag{9}$$

But

$$\lambda\overline{T\dot{T}} = \frac{\lambda}{t_1 - t_0}\int_{t_0}^{t_1} T\dot{T}\,dt \equiv \frac{\lambda}{t_1 - t_0}\int_{T(t_0)}^{T(t_1)} T dT \leq \frac{\lambda(T_u^2 - T_l^2)}{2(t_1 - t_0)} \tag{10}$$

where T_u and T_l are upper and lower bounds on $T(t)$ in the interval $t_1 - t_0$. By taking the interval large enough to make the last member of Eq. (10) negligibly small, the last term on the right in Eq. (9) will go to zero and Eq. (7), which has already been used, will be satisfied *a fortiori*; so the desired relation between the true standard deviation (here expressed in terms of the variance) and the apparent standard deviation of the response of a lagging element becomes

$$\sigma^2 = \sigma_e^2 - \lambda^2\overline{(\dot{T})^2}. \tag{11}$$

It is seen from Eq. (11) that the degree to which the apparent variance approximates the true variance depends on the magnitude of the lag time and on the mean-square slope of the response waveform. By tabulating the slopes at equal small intervals of time over a representative period of a given record, squaring, averaging, and inserting the average into Eq. (11), one could estimate the magnitude of the discrepancy between the computed σ^2 and the desired σ_e^2. Such an operation would be quite tedious if performed manually, but could be carried out quite rapidly by the electromechanical tabulator described by Kassander elsewhere in these proceedings.

For waveforms of $T_e(t)$ that are geometrically simple, such as a sine wave or square wave, one can actually calculate the correction term on the right in Eq. (11). For the case where $T_e(t)$ is the sinusoidal fluctuation defined earlier in Eq. (2), the ratio σ/σ_e is simply A'/A, the amplitude-suppression ratio, which is plotted in Fig. 3.9.4. For the case of a square waveform for $T_e(t)$, with equally long plateaus and valleys, the same ratio was found to have somewhat smaller values for the intermediate range of P/λ than was the case for the sinusoidal case. Thus for P/λ equal to 1.0 the ratio is found from Fig. 3.9.4 to be 0.16 for the sinusoidal case, but was only 0.11 for the square wave. In general, it may be concluded from Fig. 3.9.4 that to estimate the true standard deviation of turbulent fluctuations of temperature to within an error of less than 10 per cent requires that the sensing element employed have a lag time which is not more than one tenth as large as the principal periods in the actual fluctuations. And it must be noted that the possibility for a sort of circular deception as to the degree of reliability of a given estimate of σ_e exists here. If one forms his notion as to what are the "principal periods" solely from visual inspection of the record of a lagging instrument, the attenuation of the shorter-period oscillations that has already suppressed the apparent role of these components will mislead one into thinking that the standard deviation estimate is better than it really is. The cycle of deception might then be completed by using this faulty estimate of reliability of the computed σ-value to congratulate oneself on the (apparent) faithfulness with which the response reproduces the forcing function.

3.9.4.2. *Requirements on the Precision of Lag Time Determinations*

Since there can be no debating the fact that the most satisfactory way to solve the analysis and interpretation problems created by lag effects is to use an element having zero lag time, there will be a continual tendency to try to devise elements with lower thermal inertia. As the lag time goes down, however, the experimental difficulties of carrying out an accurate determination of that lag time go up rapidly. It is pertinent to ask how sensitive the various lag bias effects are to variations in lag time so that one may know how closely he must determine λ.

The single quantity which gives the best measure of lag bias would appear to be $1/\sqrt{1 + 4\pi^2\lambda^2/P^2}$, which has been identified as the amplitude-suppression ratio in the solution (Eq. (3)) for the sinusoidal case but which also enters into solutions for other waveforms. Logarithmic differentiation of A'/A with respect to λ yields

$$\frac{d(A'/A)}{(A'/A)} = -\frac{d\lambda/\lambda}{1 + P^2/4\pi^2\lambda^2} \tag{12}$$

as the relation between the relative error of the determination of lag time and the resulting relative error of A'/A. In the limit of very small P/λ, Eq. (12) indicates that there will be the same relative error in A'/A as in λ itself, but that in the limit of very large P/λ the relative error in A'/A associated with a given relative error in λ goes to zero (wide tolerance on λ-determination at this limit). As three indicative magnitudes, it may be noted that Eq. (12) yields $d(\ln A'/A)/d(\ln \lambda)$ equal to -0.9998 for a P/λ of 0.1, -0.97 for a P/λ of 1.0, and -0.283 for a P/λ of 10. Here, as elsewhere, the most serious difficulty attends the study of the highest frequency components in $T_e(t)$, for one can apparently not assess the importance of these components with much more accuracy than he can measure λ in the laboratory. It is most unfortunate that the reverse relationship does not obtain.

3.9.4.3. *Determination of the Lag Times of Bead Thermistors*

The Western Electric D-176980 bead thermistor has a rated lag time of about 0.5 sec in still air. Since the rapid time variations of T_e which form the subject of interest of fluctuation studies are due almost entirely to advective effects rather than to within-parcel (Lagrangian) temperature changes, the practical situation in which lag effects are to be considered is one in which the air is not still, but instead is moving past the sensing element in such a way as to lower the lag time by virtue of ventilation effects. Hence one really does not have full information as to the pertinent response characteristics of his sensing element until he has determined the dependence of lag time on ventilation rate over the range likely to be encountered under field conditions.

Whereas the lag time of a mercurial thermometer may be determined satisfactorily by the standard method of manually transferring the instrument from one medium to another at different temperature to simulate a step-function change of temperature, the lag times of rapidly responding elements cannot be so readily measured since the transit time of changing the element from one medium to the other may be several times longer than the lag time to be determined. For this very reason the literature contains little information on this matter.

By an ingenious side stepping of the difficulties encountered in making very accurate observations of indicated temperature vs time for a single step-function change of temperature, and at the same time incorporating variable ventilation rates into the procedure, Anderson and Heibeck [1] have recently developed a technique whose basic principles will probably be utilized frequently in the future. Anderson and Heibeck employed auxiliary equipment which was capable of measuring lag times down to about 0.05 sec and this minimum can doubtless be lowered by employing different components in the measuring system. A shortcoming inherent in the method consists in its inapplicability to the determination of very short lag times under nearly still-air conditions. The translation of the sensing element from one air stream to the other, which must be accomplished in a time short compared to the lag time, is inevitably accompanied by transient ventilation rates large compared to those whose effects it is desired to evaluate at low airspeeds. There are two reasons why this is not a trivial difficulty: first, the lag time changes very much more rapidly with changing ventilation rate at the very low rates than at higher rates; and second, the micrometeorological domain is one in which rather low wind speeds are the rule rather than the exception.

As part of the preliminary evaluation of bead thermistors, a quite different method of determining the speed of response of the beads under varying ventilation rates (modified from that used by Becker, Green and Pearson [2]) was employed by Kassander. The method consisted in applying a step-function change of voltage across a thermistor and fixed resistor connected in series and then presenting on the face of an oscilloscope a trace of the transient response of the thermistor as its temperature changed due to Joule heating. The persistence time of the fluorescent coating of the cathode tube permitted determination of the lag time by actual measurement (on the curve traced out on the tube face) of the time for the response to undergo e^{-1} of its full change. The voltage step-function was as readily applied with the bead in calm air as in the wind tunnel or on the rotating arm, so the method was quite flexible.

Such a technique immediately raises the question of the comparability of the measured response time and the desired thermal lag time. One aspect of this question has been treated by Smith [15] who derives an expression for the ratio between the thermal lag time and what he calls the "electrical lag time." Examination of his derivation of the ratio reveals that it tacitly assumes uniform temperature throughout the thermistor at all times, though explicitly assuming time variation of this uniform value. Failure to consider transient thermal gradients that may very well accompany the nonuniform spatial distribution of current density and hence of Joule heating within the bead, renders Smith's relationship an incomplete expression for the ratio between thermal lag and what one measures as the electrical lag in the technique under discussion. It is planned to examine this question both theoretically and experimentally soon, and any significant results will be described in future reports of the Iowa State project. For the present it will only be pointed out that all lag times used below are the electrically measured lag times and that Smith's incomplete analysis indicates that our present values may be underestimates of the thermal lag to the extent of about 20 per cent. This, together with the fact that all numerical data used apply to but a single specimen bead, must be noted by any reader who uses these data in connection with his own applications of the bead thermistor.

The results of Kassander's determinations of the electrical lag time of a specimen bead are displayed in Fig. 3.9.5. The still-air value of about 0.7 sec is about 40 per cent larger than the rated still-air lag time, and is about 60 per cent larger than the value which one calculates from the relation given by Becker, Green, and Pearson [2],

$$\lambda = H/c,$$

where H is the thermal capacity and c the dissipation constant. In this calculation a still-air dissipation constant of 0.09 mw/°C was used, and reasonable values of the physical constants determining H. That these two other indications both involve lower values than our electrical determinations may be evidence that the electrically measured lag-time is an over-estimate of the thermal lag time. The most that can be said with assurance is that the orders of magnitude of the electrical and thermal lag times are the same.

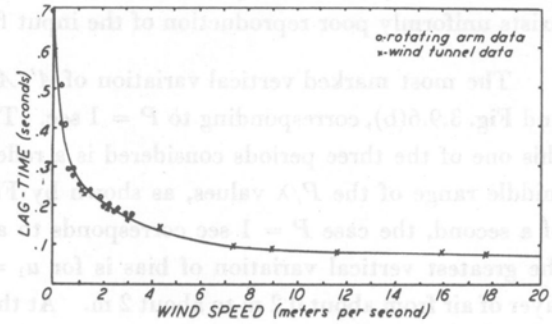

Fig. 3.9.5. Variations of lag time of a D-176980 bead thermistor with variations of ambient wind speed.

For later computational convenience it is useful to have an empirically fitted expression for the dependence of lag time on ventilation rate. Middleton [11] suggests that most elements exhibit a u^{-n} dependence of lag on wind speed, u, so this hypothesis was checked against a logarithmic plot of the experimental points, and the relation

$$\lambda(u) = 2.8/u^{1/2} (\lambda \text{ in sec, } u \text{ in cm/sec}) \tag{13}$$

was found to represent the experimental data with an error of less than 0.05 sec for all speeds in excess of 0.5 m/sec. Clearly Eq. (13) must fail to hold for very low speeds.

Just as the determination of the dissipation function of the bead thermistor made possible the analysis of a number of questions concerning the role played by radiation error in distorting fluctuation statistics, so also does the determination of the lag function, $\lambda(u)$, make possible the analysis of a number of questions concerning a similar role played by lag effects. Some of these will be discussed next.

3.9.4.4. *Effect of $\lambda(u)$ on Fluctuation Statistics*

A. VERTICAL VARIATION OF BIAS IN FLUCTUATION STATISTICS NEAR THE GROUND

The systematic increase of mean wind speed with height, being most pronounced very near the ground, leads to systematic variations in mean lag times of identical elements located at a series of elevations close to the ground, and thereby introduces systematic bias of the apparent fluctuation statistics derived from these elements' records. To obtain a notion of the seriousness of this effect, a power-law wind profile

$$u(z) = u_1 z^p \tag{14}$$

was assumed and two different reference velocities for the 1-m level employed: $u_1 = 1$ m/sec, and $u_1 = 5$ m/sec. To gain some additional information as to possible diurnal variations, two different exponents, p, were considered: $p = 0.1$, representative of daytime conditions, and $p = 0.3$, representative of nighttime conditions. Finally, to assess the relative attenuation of different periods in $T_e(t)$, three different periods of a purely sinusoidal waveform were examined: $P = 0.1$ sec, $P = 1$ sec, and $P = 10$ sec. This yielded twelve cases, and for each the profile of A'/A vs z from the surface to 10 m elevation was computed. The results are presented in Fig. 3.9.6. It may be well to repeat that the $\lambda(u)$ data employed here, though certainly correct as to order of magnitude and in fact to much better than a factor of 2, are subject to amendment pending further study of the thermistor lag characteristics.

Figure 3.9.6 shows that for both rather large (10 sec) and rather small (0.1 sec) periods, a bead thermistor does not impose very serious vertical *variation* of bias; but in the former case this is because there exists uniformly faithful reproduction of the input fluctuations, whereas in the latter it is because there exists uniformly poor reproduction of the input fluctuations at all heights.

The most marked vertical variation of A'/A occurs for the middle pair of curves in both Fig. 3.9.6(a) and Fig. 3.9.6(b), corresponding to $P = 1$ sec. That the strongest variation of bias with height develops for this one of the three periods considered is a reflection of the steepness of the slope of A'/A vs P/λ in the middle range of the P/λ values, as shown by Fig. 3.9.4. With the thermistor lag time of several tenths of a second, the case $P = 1$ sec corresponds to a P/λ of about 2 or 3. Of all twelve curves in Fig. 3.9.6, the greatest vertical variation of bias is for $u_1 = 1$ m/sec, $P = 1$ sec, and $p = 0.3$ and is present in the layer of air from about 0.2 m to about 2 m. At the top of this layer, there is about 50 per cent better registry of the 1-sec periods in $T_e(t)$ than at the bottom of this layer. Mere visual comparison, or uncorrected quantitative comparison, of the records of thermistors at the extremes of such a layer would yield an appreciably distorted impression of the relative importance of thermal fluctuations with periods of the order of 1 sec. It is to be noted, however, that the most intense thermal fluctuations are a daytime phenomenon, represented more closely in Fig. 3.9.6 by the curves for $p = 0.1$, and these are, under all conditions, flatter than those for $p = 0.3$.

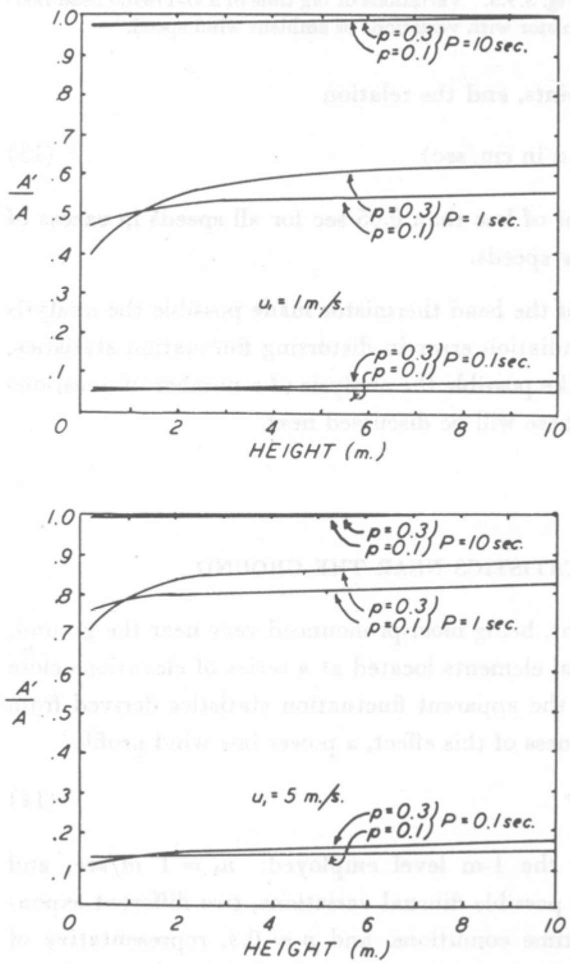

Fig. 3.9.6. Variation of amplitude-suppression ratio, A'/A, with height for three different periods, P, in the presence of a power-law wind profile $u(z) = u_1 z^p$, for $p = 0.1$ (midday) and $p = 0.3$ (nighttime). In the upper plot u_1 is 1 m/sec; in the lower u_1 is 5 m/sec. Curves represent response of bead thermistor type D-176980.

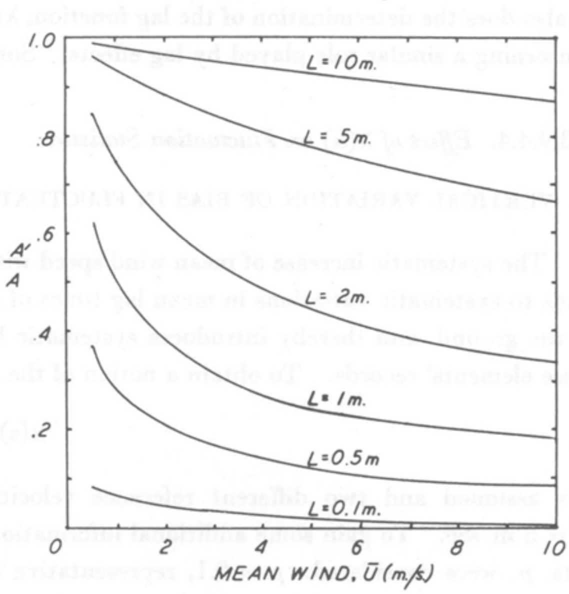

Fig. 3.9.7. Amplitude-suppression ratio, A'/A, for sinusoidal disturbances of wave length L passing a bead thermistor at various mean wind speeds, $\bar{U}$.

B. AMPLITUDE SUPPRESSION AS A FUNCTION OF MEAN WIND SPEED FOR VARIOUS SCALES OF TURBULENCE

As has been previously pointed out above, the "periods" under discussion in this paper are Eulerian periods which really measure the times of transit of thermal inhomogeneities past the fixed sensor. This means that, for a thermal inhomogeneity of a given size, its "period" varies inversely with the speed of the current in which it is embedded. This circumstance, coupled with the inverse dependence of lag time on ventilation rate, leads one to speculate optimistically that perhaps the disadvantageous effect of a wind speed increase on the apparent period may be canceled by the concurrent advantageous effect on the lag time. The issue is determined entirely by the functional form of $\lambda(u)$ of the sensing element employed. This hypothesis was examined for the case of the bead thermistor and found to be wrong, especially for "eddies" whose diameters or wavelengths were of the order of a meter.

Figure 3.9.7 shows the results of the check on this point. Six different wavelengths, L, were considered, where L represents the downwind distance between successive temperature maxima (or minima). The amplitude-suppression ratio, A'/A, is seen to decrease (grow worse) with increasing mean wind speed for all scales of thermal turbulence represented in the figure, and a particularly well-marked variation of the degree of suppression accompanies wind-speed increase when the air stream contains inhomogeneities of wavelength near 1 m. Although the curves are extended only down to a mean wind of 0.5 m/sec (because the $u^{-1/2}$ dependence of lag time on airspeed fails below there) one can see that all of the curves must converge to the point $A'/A = 1.0$, $\bar{u} = 0$, since this point corresponds to infinite periods for all scales of turbulence, but is associated with a finite lag time (about 0.7 sec). Hence Fig. 3.9.7 fails to reveal that the strongest variation of bias really occurs with the very smallest of eddies for almost calm conditions.

C. LAG BIAS IN CORRELATION STUDIES

There is growing interest in the spectrum of turbulence, and one of the observational techniques employed to gain information concerning the spectrum consists in correlating the records of two sensing elements located at two different points in space. If the two sensing elements used have appreciably different lag times, the associated differences in phase lag imposed by each may lead to distortion of correlation coefficients computed from the records. An examination of the sensitivity of the correlation coefficient to such differences in lag times was carried out. The results may be interpreted either as a measure of the sensitivity to lag differences occurring due to manufacturing variations in individual elements, or to differences in lag time imposed by systematic variations of lag time with height implicit in the normal wind profile.

The working method will be to *assume* perfect correlation to exist between two purely sinusoidal waveforms of $T_e(t)$ at the two elements, and to investigate how low the *apparent* correlation can drop due to lag differences. Let the two perfectly correlated waveforms have unit amplitude and let the two sensing elements have phase lag angles ϵ_1 and ϵ_2, where $\epsilon_i = \tan^{-1}\dfrac{2\pi\lambda_i}{P}$, the period necessarily being the same in each case, having assumed actually perfect correlation. Because the standard deviations of the two apparent signals are suppressed in the same ratio as the amplitudes, we have for the ratio of the apparent correlation coefficient, r', to the true value $r(= 1.0)$

$$\frac{r'}{r} = 2\,\frac{\displaystyle\int_0^P \sin\left(\frac{2\pi t}{P} - \epsilon_1\right)\sin\left(\frac{2\pi t}{P} - \epsilon_2\right)dt}{\displaystyle\int_0^P dt}.$$

Changing the variable of integration from t to $\frac{2\pi t}{P} - \epsilon_1$ and putting $\epsilon = \epsilon_2 - \epsilon_1$, and carrying out the integration yields the result

$$r'/r = \cos \epsilon. \tag{15}$$

But $\cos \epsilon$ is so insensitive to differences in the lag times of the two elements that even when the more inert element has a lag fully twice as great as that of the faster, the *apparent* correlation coefficient drops only about 0.1 per cent below the true value. That is, r' appears to be about 0.999 whereas the true value is a perfect 1.000. Apparently correlation studies will never place strenuous demands on accurate matching of lag times of the sensing elements employed.

Recent developments of instrumentation for direct measurement and automatic calculation of the fluxes of the transport properties involve simultaneous measurement of two quantities by means of quite different sensing elements. To the extent that the response behavior of both elements is describable by a differential equation of the functional form of Newton's law of cooling for thermal elements, the above result suggests that one may also be optimistic as to the role unequal phase lags play in these measurements, which are essentially correlation measurements.

D. RESPONSE IN THE PRESENCE OF SIMULTANEOUS FLUCTUATIONS OF TEMPERATURE AND WIND SPEED

If a bead thermistor with lag function of the form given by Eq. (13) is exposed not only to sinusoidal temperature fluctuations of the form of Eq. (2), but concurrently to wind speed fluctuations of the form

$$u(t) = U + u_0 \sin\left(\frac{2\pi t}{P} - \alpha\right) \tag{16}$$

where U is the mean wind speed, u_0 the amplitude of the speed fluctuations, P the same period as that characterizing the thermal fluctuations, and α the phase separation between epochs of highest temperatures and epochs of highest wind speeds, then the problem of deducing the possible distortions of the fluctuation statistics becomes the problem of solving the response equation

$$\frac{dT}{dt} + \frac{u^{1/2}}{\lambda_1} T = \frac{u^{1/2}}{\lambda_1}\left(\overline{T}_e + A \sin\frac{2\pi t}{P}\right) \tag{17}$$

where $u = u(t)$ is the periodic function given by Eq. (16) and λ_1 is a constant. It is possible to reduce the complexity of the differential equation somewhat by expanding $u^{1/2}$ in a binomial series and keeping only the first two terms at the expense of an error which amounts only to about 1 per cent for amplitudes u_0 which are only about 25 per cent of U itself. By this simplification Eq. (17) may be reduced to quadrature by the method of variation of parameters, but the then required integrations can only be carried out numerically. It seems more expedient to integrate Eq. (17) by numerical methods from the beginning and this has been done for a few cases.

Any distortion of the apparent waveform that was due to fluctuations in effective lag time would be expected to be a function of α, the phase lag between temperature and wind fluctuations. Since the wind speed tends to rise in the surface layer during periods when a mass of air descends from the faster moving overlying layers which are also generally cooler than the surface layers, values of α which correspond to this situation should be the ones on which to base the integration of Eq. (17). If the wind were highest just when the temperature was dropping most rapidly, the lowering of the lag time due to augmented ventilation might be expected to cause the element to undershoot a bit, while the large lag time associated with the lulls would act to prevent the element from following the rising ambient temperature up to its full maximum.

Actual calculations revealed that though this last prediction is qualitatively true, the quantitative effects are very small for a bead thermistor and for wind speed fluctuations of 25 per cent of the mean value. For $\alpha = 180°$, the apparent mean temperature was found to be *lowered* only by about 2 per cent of the true amplitude of the temperature fluctuations (mean to maximum). For $\alpha = 90°$, the displacement of the mean is still less. In each case, the nonuniformity of response time does produce distortion of the originally sinusoidal waveform, but by so slight an amount as to be quite negligible by any micrometeorological standards.

As yet no analysis of the problem has been carried out for a case of very large percentual wind speed fluctuations. One could not employ the binomial approximation for $u^{1/2}$ in Eq. (17) for fluctuations much greater than 25 per cent, but in a numerical integration this does not represent a very serious complication. This point will be examined further in the near future, but these preliminary results are fairly strong evidence that one need not be seriously concerned with distortion of the fluctuation statistics by wind speed fluctuations. (Note carefully that in the preceding discussion, the element was tacitly assumed to have no radiation error. The case of fluctuating winds in the presence of radiation error has already been treated in Sec. 3.9.3 above.)

3.9.5. CONCLUSIONS

As soon as the dissipation function and the lag function of a sensing element are known it becomes possible to carry out analyses of a number of errors that affect fluctuation measurements. In the present paper such analyses have been made for the case of a single type of element, the bead thermistor. Without recapitulating the results in detail, it may be concluded that this element is suitable in many respects for the exploration of the thermal fine structure of the atmosphere, but is less than ideal. One would naturally turn at this point to a comparison with other available sensors, but there is a significant lack of comparable performance data for these other elements. It would be particularly helpful if a similar investigation of the errors of thermocouples could be carried out in order that the relative advantages of thermocouples and thermistors might be better assessed. In an attempt to fill this need, the writer is currently searching the extensive literature concerning heat transfer from heated wires for data suitable to the analysis of thermocouple response. Any pertinent results obtained will be described in future reports of the Iowa State studies of turbulent temperature fluctuations. For the present it may be said that from the point of view of sensitivity and ease of recording (especially remote recording), the bead thermistor is definitely superior; from the point of view of radiation error and lag time the finest wire thermocouples would seem to be superior, judging from the scattered comments pertaining thereto. A really sound selection of the instrumentation yielding optimum overall performance will become possible only when more complete investigations have been carried out on all of the principal sensing elements currently available. This paper is presented as a contribution towards that last objective.

3.9.6. ACKNOWLEDGMENTS

All of the experimental work underlying this paper was planned by Dr. A. R. Kassander, and carried out by him and Mr. R. W. Green.

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3.9.7. DISCUSSION edited by O. G. Sutton

MR. P. A. SHEPPARD: I might insert that all of us who have been concerned at all with this problem have probably gone over the sort of ground which Mr. McDonald has so nicely described. My own conclusions are these. We must distinguish between two problems — the measurement of fluctuations and the measurement of gradients. Lower accuracy may be accepted in the former rather than in the latter case. We formed the conclusion that you should certainly get down to about 0.001-in. diameter "bulbs," and possibly 0.002 in. is tolerable; at that level radiation errors are small — at any rate in English weather, with reasonable winds.

MR. C. W. THORNTHWAITE: Those of you who have seen our reports realize we have gone through the same period of exploration that Mr. Sheppard has been having in England. Perhaps our experience with the thermistor is no more valid than his. We have been prejudiced against it because of its lack of stability. In recent months, we have been trying to develop a technique for manufacturing a very fine thermocouple, but I am not sure that it is new or novel. It may be well known to other people in the field, but it was an independent invention of one of the technicians in the laboratory. He has drawn out an extremely fine glass capillary tube and has been pushing the two ends of the wires which need to be joined into the tube so that they are, in effect, butt joined. In that way, there is no overlap which we found to be objectionable. By this method, the thermocouples are very uniform and are surprisingly strong.

MR. H. F. POPPENDIEK: What size is the thermocouple?

MR. C. W. THORNTHWAITE: I don't know. Mr. Halstead, could you give us the information?

MR. M. H. HALSTEAD: I believe it is size 50 B&S. They are 0.0008 in. in diameter.

MR. G. C. GILL: I might say a little bit about this subject of thermocouples. We are using, at the present time, thermocouple wires of 0.0004-in. diameter, made from copper and constantan. Our technique is a little bit different in putting the wires together. These are butt soldered together using a stereoscopic binocular type microscope of 30X. The size of the butt-soldered joint is not over 0.0006 in. in diameter. We expect, next month, to use wires of an even smaller size and to butt solder those. After these junctions were made, for testing we placed them in a box about 18 in. × 18 in. × 18 in. having a small window about 1 in. square covered with very thin celluloid. We put the junctions below the window in the sun's rays. With the window covered, the air in this box was very still (as indicated by the steady reading of a sensitive galvanometer connected to the thermocouple with zero ventilation); when the window was uncovered we got around 0.3°C solar heating with thermojunctions made from wire 0.004 in. in diameter. Then we ventilated the box at an air speed of 100 ft/min and the heating error was reduced to less than 0.1°C.

MR. R. J. TAYLOR (*Communicated*): Mr. McDonald suggested that, owing to instrumental lag, the values of $\overline{(u - u_t)^2}$ given in my earlier paper were too small by some ten-fold at small t. His estimate was based on Bilham's (1935) analysis [4] of the response of an instrument to simple harmonic fluctuations. It is not possible to apply this analysis to wind speed fluctuations without making unjustified assumptions as to the sinusoidal or other character of the variable being recorded. In view of this and of the fact that the measurement of $\overline{(u - u_t)^2}$ is essentially a problem of determining autocorrelations, it is more instructive to consider the false autocorrelation introduced by the instrument, that is, the autocorrelation which would exist in the record if the wind speed varied in random fashion. The computation is most easily made by assuming that the wind suddenly takes on a new random value every second (it can be shown that the choice of another interval leads to the same final result). Then it can be shown that the recorded wind speed u_t at a time t in seconds forms an autocorrelated series generated by the Markov process

$$u_{t+1} = 0.48\, u_t + \epsilon_{t+1}$$

where ϵ is a random variable.

The autocorrelations from a Markov series are well-known (Bartlett, 1946) or can easily be calculated *ab initio* and give the result that the autocorrelations r_t for a time lag of t seconds are: —

$$r_1 = 0.48$$
$$r_2 = 0.23$$
$$r_3 = 0.11$$
$$r_4 = 0.05$$
$$r_5 = 0.02$$

showing that the false autocorrelation is entirely negligible after 2 sec.

My earlier paper did not quote data enabling values of $\overline{(u - u_t)^2}$ to be converted to correlation coefficients. For comparison with those above, correlation coefficients are given below for Run 1.

Time lag (t) (sec)	Correlation coefficient
1	0.97
2	0.93
3	0.90
5	0.83