

CONCERNING THE FORCES AND MOTIONS OF BODIES

BOOK ONE.

This book considers solid bodies, and is divided into two parts, the first of which concerned with the equilibrium of bodies under the action of gravity and other relevant forces here, and the second part shall attend to motions, which arise from forces of this kind acting, and which can be changed as it pleases.

SECTION I.

Concerning the diversity of actions applied to bodies in equilibrium and the resulting directions.

26. Since thus the various disturbing forces applied to bodies may be brought to act together amongst themselves, so that none of these can predominate over the rest, these forces themselves are said to remain in a state of equilibrium. Disturbing forces of this kind were signified by the name of *mechanical forces* by the ancients, which nomenclature we will retain at times also. Therefore in the first part we will investigate, what equilibria are concerned with the resultant directions and other related matters according to the actions of these disturbing or mechanical forces, whether the bodies shall be rigid or flexible, which pertain to such forces.

CHAPTER I.

Concerning the proportion between the forces of gravity acting, or the weights of the bodies, compared with the masses of the same bodies.

An elegant property of bodies is discussed in the philosophical works of the geometers, there stating at once that the weights of bodies increase in the same ratio as their masses or quantities of matter ; or the same in other words preferred to be said by geometers : *so that the weights of bodies or the weights of the same masses shall be in proportion.* The illustrious Newton, from the most accurate of pendulum experiments, always verified that property of the weight being tested : page 305. *Princ.Nat.Math.* 1st ed., also he had elicited that elegantly from Cor. I. Prop. XXIV. Book II. The most noble Huygens not only grants this property, but also it is deduced from the laws of motion on page 140 [of Huygens' *Optics*] *Discourse on the Cause of Weight.* And because this property of bodies is of the utmost importance in the whole of natural philosophy, I will attempt to deduce another demonstration of this here, which is allowed to follow from a single hypothesis.

HYPOTHESIS.

27. *With the amount of matter remaining constant, and with the direction of gravity present being along parallel lines, or converging to a centre at some indefinite distance, the weights of the bodies do not change, with the position of these varied with respect to the horizontal and the shape of the body.*

That is, any body retains the same weight, in whatever manner that shall be placed with respect to the horizontal, nor shall its weight be agreed to change, evidently if its figure shall be changed into any other figure whatever, into a sphere, cylinder, a cone, etc., provided the quantity of matter or mass shall have remained the same.

PROPOSITION I. LEMMA.

28. *The cause of gravity, whatever it shall be, not only acts on the exterior parts of bodies, but also on all the interior parts.*

For if the cause of gravity acts on the exterior parts of bodies only, it may act on these surfaces of the body only, which are turned away from the horizontal, and thus the weights of bodies shall be proportional to these surfaces and, for the varied position and situation of the body, the surface turned away from the horizontal sometime may be greater or sometimes less; therefore, if the impressed force of gravity were proportional to these, it would follow the weight of the body to be changing, for all different positions of the body, contrary to the hypothesis.

Then, if gravity acts only on the exterior parts of bodies, and truly not in the interior parts it may follow, as the shape of the body may be changed, but with the same amount of matter remaining, successive weights of the body may arise ; since with the figures varied and likewise the positions and quantities of particles of the surface turned away from the horizontal may be varied, and thus the weights of bodies may be varied with the same amount of matter remaining. Which all are contrary to the hypothesis stated in article 27.

COROLLARY.

29. Therefore the weight of a body can remain the same in any position and shape, provided the individual elements receive equal forces due to their weight. With which in place the following will be proven easily.

PROPOSITION II. THEOREM.

30. *The weights of bodies are proportional to the quantities of matter or to the masses of these.*

In the first place there shall be two commensurable bodies C and c, one element of these shall be e, the number of elements in body C, N and the number in the other c, may be called n, and finally the masses of the bodies, or the quantities of matter, may be

called respectively M , m . With which in place, because (due to §.29.) the individual elements of each body experience an equal force from gravity, if one impressed force of this kind may be called i , the weight of the body C or the whole action from gravity, by which the body may be urged to fall (§.23.) is the sum of all the actions, by which the individual elements are pressed upon, and since the directions of the actions of these are parallel by hypothesis, these actions or the forces of gravity are acting together; and thus the action of the whole of gravity on the body C will be one action or impression i multiplied by the number of elements N , which are held in this body, this is the weight of C itself which I will indicate by pC becoming $= N.i$ and $p.c = n.i$. Truly there is $N.i : n.i = N.e : n.e$; therefore because $N.e$ is the sum of the particles e in the body C , or the mass of which body M , & $n.e$ the mass of the other body, c or m , there will be generally $pC : pc = M : m$ that is the weights of the body C shall be proportional to the masses of these. Q.E.D.

COROLLARY I.

31. If the mass of the body, which will be designated from now on by the same letter, as the body itself is assigned, is multiplied by the force impressed by gravity (for these two here and elsewhere signify one and the same thing), by which one element of the body is acted on, the absolute weight of the body will be established always by the product made, so that if the body were C , and the action of gravity, by which its individual elements may be affected, may be called G , the quantity or the product $C.G$ establishes the absolute weight of the body C , on understanding by this letter C to be the mass of this body.

COROLLARY II.

32. Hence if, as above, the volume and density of this body are called V and D , and the weight pC , the weight pC will be as $G.D.V$. That is, the weight of any body is as the product from the action of gravity G , by which its individual elements e may be acted on, and with the density D by the volume V . For (§.32) pC is as CG , and (§. 28) C , which here indicates the same as M there, clearly the mass of the body, is as $D.V$, therefore pC or CG is just as $G.D.V$.

COROLLARY III.

33. And because the specific gravity of bodies are consistent with the proportion of the absolute weights of bodies, within equal volumes, thus the specific gravities will be in proportion to the densities, since (§.26.) the masses of bodies equal in volume shall be proportional to the densities and by the present proposition to the weights of the bodies, of these masses. Therefore if the specific gravity of our body C may be called S , its absolute weight or pC will be as $S.V$., since S is as $D.G$.

SCHOLIUM.

34. This proposition itself at this point can be demonstrated otherwise from the theory of the motions of pendulums, as performed by the celebrated Newton, now indicated above in pride of place at the first part of this chapter. We will elicit a similar proposition from principles requiring to be derived in the easiest of ways also, for which we have used the properties of the accelerated motion of gravity.

CHAPTER II

With regard to the actions which may be impressed on rigid or inflexible bodies, and the resultant directions of these forces.

DEFINITIONS.

I.

35. Since any forces, and as a consequence also the actions shall be from the generation of a quantity along right lines and from these actions right lines can be put in place in proportion. Therefore if the points A, C, E on the right line AE are directed to move by actions along the directions AB, CD, EF, these actions or forces may be expressed by the right lines AB, CD, EF, with these themselves taken in the same homologous directions as the forces, thus so that these lines likewise designate the action arising from forces as well as the directions of these, and this property will not always be remarked on to any extent in the following.

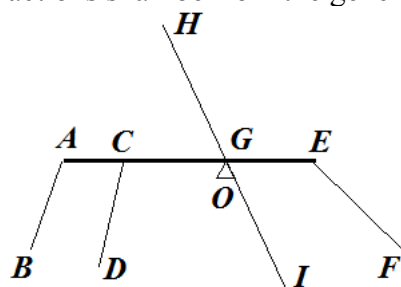


Fig. 1

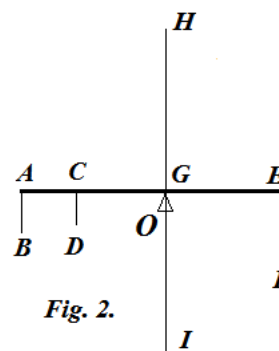
II.

36. If several of the actions or forces may be set up in equilibrium, some put in place shall be for one part of the body to which they relate, truly the rest or the others will be for the opposite part; The actions, which are on one direction, may be said to be equivalent to the rest in the opposite direction. So that if for the right line AE from the actions or forces of some other body on it, the applied lines AB, CD, EF shall be from one direction, truly the force GH is acting from G at H in the opposite direction, and this with the remainder opposed to itself stand in equilibrium. The single force GH, is said to be equivalent to all the remaining AB, CD, EF. Besides the beginner is required to be warned, that the force or action [called a *solicitation* by Hermann], which is said to be equivalent to the others, itself cannot be agreed to be equal to the rest; and indeed *equivalent* and *to be equal* are not synonymous phrases in Mechanics. For one is the case, in which the force GH is equivalent to the opposite forces AB, CD, EF at once taken equal to these, then clearly, since these are acting together (§.22.), and with the other GH (§.23) itself acting in the contrary direction.

[Here Hermann summarizes the many experimental details provided by Varignon in his *Mechanics*, where known forces are applied by strings via pulleys at various angles to rigid bodies, or simply to a central knot.]

III.

37. With GH produced to I, the continuation GI of that will henceforth be called the *resultant direction* or the *axis of equilibrium* of the forces AB, CD, EF, Fig. 2, & finally the point G, at which it crosses the resultant direction of the body, and to which the impediment O placed under sustains the forces placed together for equilibrium, which impediment or small support is usually otherwise called the fulcrum of the lever, hereafter to be called the centre of equilibrium.



AXIOM.

38. The force which arises from other forces acting together along agreeing directions, will be equivalent to all of these, and will have a direction agreeing equally with the directions from all of these themselves.

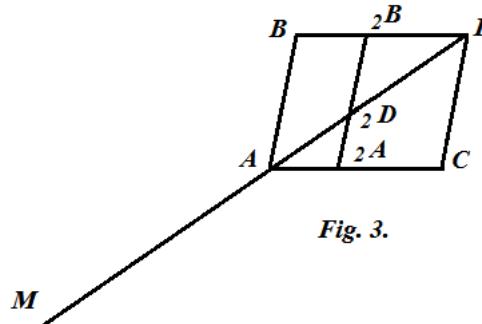
Truly an action resulting from directly opposed forces, will be equivalent to the excess by which the opposing stronger forces overcome the weaker.

PROPOSITION III. THEOREM.

39. An action arising from two forces acting neither in the same direction, nor directly opposite to each other, gives rise to a certain force on a mobile body, which is equivalent to the diagonal of the parallelogram, which results from the motion of each of these, the sides of which parallelogram establish the directions of these forces, according to the length of the side

A moveable body A may be set in motion by some forces acting AB, AC : the directions of which may contain some angle CAB, described by the parallelogram ABCD, the diagonal of which AD sets out the direction & magnitude of the force, which results from the sides AC & AB.

Demonst. The mobile point A may be thought to be adhering to the moveable line AB; but yet so that thus it can move freely on that from A towards B, truly just as it can be carried on the line AB, it can be carried by a motion parallel to itself on AC to the line CD, then the right moveable line AB urges the body A to be carried itself towards CD by the force AC : truly with the mobile point A to be urged along the line AB by the force AB ; and it will be evident the force A_2A , by which the line of the mobile point AB is trying to



approach the parallel line CD itself, being urged along by the force AC, finds itself by the effort ${}_2A_2D$ from the force AB acting on the body A to be moved along this line, so that AC finds itself at CD ; now because of the hypotheses, ${}_2A_2B$ itself is parallel to CD, and just as we have seen $A_2A : {}_2A_2D = AC : CD$, the point ${}_2D$ will be on the diagonal AD of the parallelogram BC. Again, because the mobile point A partakes in the motion A_2A , or rather endeavors to move along the moving line AB, on which it can be understood to be moving, and the proper motion, or rather the attempt at this motion to be ${}_2A_2D$, and because, if the line of the mobile point and the body at the same time were able to describe minute distances, however small A_2A and ${}_2A_2D$, likewise that would be as if the body A had traversed through the incremental distance A_2D ; certainly it is clear, from the efforts of the mobile line and of the body, A_2A and ${}_2A_2D$ attempt to produce A_2D along the direction AD, which finds itself either as A_2A to ${}_2A_2D$ just as AC to DC or to AB ; therefore from the actions AC and AB the action AD takes place in the direction A_2D . Q.E.D.

SCHOLIUM.

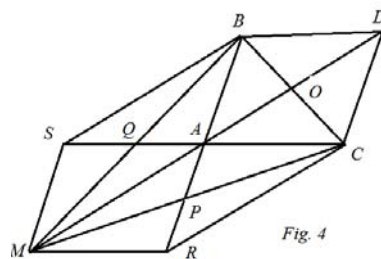
40. The forces, which are set out by the sides AC, AB of the parallelogram BC, thus can be said to be the lateral forces acting. From which, in the common language of writing used in mechanics, the motion is said to be composed either from the action along the diagonal AD, or from the lateral actions of the components AC and AB. But because I see there is no lack of authors, who seem to be in a dream, I know not what spectres of difficulties there may be arising in this matter, judging from the mistakes [made by them] from composition and resolution, addition or subtraction, of motions or forces requiring to be understood from components; I have considered these more carefully, lest it might make a business of this kind ambiguous for beginners, the motions or forces are said customarily to be composed from others, and those arising from these components or called the resultants.

COROLLARY I.

41. With the diagonal of the parallelogram produced to M, and making $AM = AD$, the force AM in the direction AM will be equivalent to the remaining forces AC and AB. For the action AM is equivalent equally to the former and in the opposite direction to AD, and AD arises from the sides AC and AB; neither can it excel more nor less than the actions of these two side forces together, and therefore likewise the moving point A may be urged by a single force AD, or by the two forces side by side AC and AD, therefore AM is equivalent to the opposite of AD and equal to the sides AC and AB entirely.

COROLLARY II.

42. Hence if three forces AM, AB, AC impressed on the same moveable point remain in equilibrium, any one of these will be to either taken from the others, that is AM to AB, as the sine of the angle



BAC, to the sine of the angle MAC from the force AM and from the other AC formed from the others. For (§.42.) $AM = AD$, and thus $AM : AB$ thus as AD to AB , truly it is agreed from trigonometry, to be AD to AB , as the sine of the angle ABD to the sine of the angle BCA , therefore also AM is to AB as the sine of the angle ABD , or of its supplement to two right angles BAC to the sine of the angle BDA , or of its equal alternative DAC , or also of that to the supplement for two right angles MAC . And thus from the external forces acting, for the equilibrium of the forces acting together, any two are between each other as the sine of the opposite angles to themselves, that is

$$AB : AC = \sin. MAC : \sin. MAB, \text{ \& } AM : AC = \sin. BAC : \sin. MAB.$$

And thus generally.

This elegant theorem was first demonstrated, as far as I know, by the Cel. Pierre Varignon, which laid the foundations of nearly all his new *Mechanics*, publ. in the year 1687. Paris edition.

COROLLARY III.

43. And from these the account of an elegant theorem of Roberval stands out, which formerly was proposed by Torricelli, but without demonstration, as is to be seen in the *Works of the Royal Academy of the Sciences of Paris*, p. 302. Truly the theorem needs to be expressed according to our manner, in this way : If through the ends of the right lines AM , AB , AC of the three forces applied to the same point A , of the representation, the right lines MB , BC and MC are drawn, the point A , to which the actions are impressed standing in equilibrium, will be present at the centre of gravity of the triangle MBC . Around AM , AB and AM , AC the parallelograms $AMSB$, $AMRC$ shall be described: with which described, and because the three forces AM , AB , AC are in equilibrium, according to the hypothesis, AC will be equivalent (§.36.) to AM & AB themselves and thus (§.42.) $AC = AS$, and these two will be put in a direction, so that CA produced will cut the side of the triangle MB at the mid-point Q , similarly AB produced will cut the side of the triangle MC in its mid-point P ; now from statics the centre of gravity of the triangle MBC is agreed to be on the individual lines CQ , BP drawn from the angles C & B of any triangle to the points Q & P at the mid-points of the opposite sides BM and CM to these angles, therefore the centre of gravity of the triangle BMC , is at the common intersection A of these lines CQ and BP .

PROPOSITION IV. LEMMA.

44. *If individual bodies A, B, C, D &c. may be drawn with each placed at a given distance from the given plane aPd [with gravity acting vertically into the page], the sum of the products of all the weights by their distances, will be equal to the sum of the weights of all the bodies, at the centre of gravity of these E, at the distance Ee from the given plane, and in which case all the bodies will be placed at the same distance from the plane aPd.*

But if indeed some bodies A, B &c. were placed on one side of the [vertical] plane PQ, and the rest C, D &c. were placed on the other side; The excess, by which the

products made from the weights to the centre of gravity of all the parts to the position of the centre of gravity, E, at their homologous distances from the plane PQ [i.e. on the left of PQ], will surpass the product from the remaining bodies at their distances from the same plane [i.e. on the right of PQ], and [the moments] will be equal to the product of the weights of all the bodies, of all the weights taken together at the distance eP, of the common centre of gravity E, from the plane PQ. [Thus, the moments of the weights are considered about any point.]

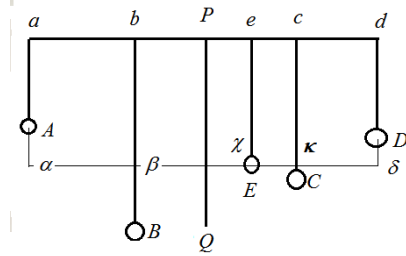


Fig. 5.

In the first case it must be shown, that, by putting $A + B + C + D + \&c. = M$, there shall be $A.Aa + B.Bb + C.Cc + D.Dd = M.Ee$. And in the second case, to be $C.cP + D.dP - A.aP - B.bP = M.eP$, with cP, dP, aP, bP designating the distances of the bodies C, D, A, B &c. from the plane PQ.

Demonst. I. The bodies may be cut by the plane $\alpha\delta$ through the centre of gravity E of all the bodies parallel to the plane ae , and eE parallel to the plane PQ at right angles to the other line ad , and $\alpha\delta$ can be considered as the axis of equilibrium of the weights A, B, C, D, and thus on account of the case of equilibrium, there will be :

$A.A\alpha + D.D\delta = B.B\beta + C.C\kappa$, or because it returns the same :
 $B.B\beta + C.C\kappa - A.A\alpha - D.D\delta = 0$. Likewise $A.ae + B.be = C.ce + D.de$, or
 $C.ce + D.de - A.ae - B.be = 0$ with respect to the axis of equilibrium eE .

Demonst. II. Because

$Aa = \alpha a - \alpha A = Ee - Aa$, $Bb = Ee + B\beta$, $Cc = Ee + C\kappa$, & $Dd = Ee - D\delta$,
there becomes :

$$\begin{aligned} A.Aa + B.Bb + C.Cc + D.Dd &= A.Ee + B.Ee \\ &+ C.Ee + D.Ee + B.B\beta + C.C\kappa - A.A\alpha - D.D\delta = M.Ee, \end{aligned}$$

because

$A + B + C + D = M$, & (from 1 of this) $B.B\beta + C.C\kappa - A.A\alpha - D.D\delta = 0$.

Which is the first part.

III. Because $aP = ae - eP$, $bP = be - eP$, $cP = ce + eP$; & $dP = de + eP$, there will be
 $C.cP + D.dP - A.aP - B.bP = C.eP + D.eP + A.eP + B.eP + C.ce + D.de - A.ae - B.be = M.eP$,
because (from 2 of this) there is $C.ce + D.de - A.ae - B.be = 0$, & $C + D + A + B = M$
as before. Which were all required to be demonstrated.

COROLLARY I.

45. Fig. 6. If the bodies rise directly along the right lines Aa , Bb , Cc , Dd &c. on the perpendiculars to the horizontal, and the centre of gravity of these E on the right line Ee parallel to these likewise, there will be

$$A.Aa + B.Bb + C.Cc + D.Dd = M.Ee.$$

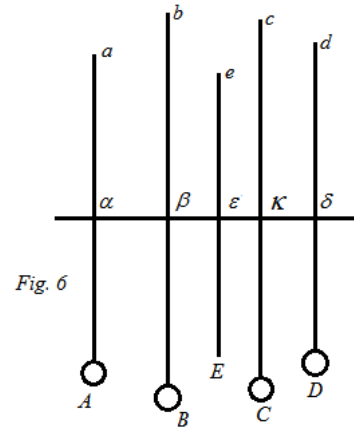
For with the right line $\alpha\delta$ drawn as it pleases, perpendicular to Aa , Bb &c., and there will be (§.44.)

$$A.A\alpha + B.B\beta + C.C\kappa + D.D\delta = M.E\varepsilon,$$

and

$$A.\alpha a + B.\beta b + C.\kappa c + D.\delta d = M.\varepsilon e.$$

therefore $A.Aa + B.Bb + C.Cc + D.Dd = M.Ee$.



COROLLARY II.

46. If all the bodies A , B , C , D &c. shall be equal in weight, all the distances of these from the plane aPd , see Fig. 5, will be a multiple of the common distance of the centre of gravity E of these from the same plane, according to the number of weights. And where the excess of the distances of those from the plane PQ in the second case, which are placed according to the centre of gravity of all the sides, exceed the distances of those, which are on the opposite side, it is a multiple of the distance of the centre of gravity E of all from the same plane PQ , according to the number of the bodies A , B , C , D &c.

SCHOLIUM I.

47. Just as now Guldin's rule can be deduced easily from these, as well as being indicated so well by Pappus, so that I need consider nothing about these matters in the future, if I may give this a demonstration here, although that has been shown now by many.

Guldin's Rule may thus be set out : *The [volume of] the figure arising from the rotation of any magnitude [area] by the path of its centre of gravity.* The generating figure may be called F , the distance of its centre of gravity from the axis of rotation D , the ordinate of the figure y to the axis of rotation, dx an element of the axis, the volume arising from the rotation of the figure F about the axis x may be called S , its element dS ; with which in place, from the present proposition there is :

$$D.F = \text{sum of the moments of the elements of the magnitudes } F = \int \frac{1}{2} y y dx,$$

as the element of F is $y dx$, and its moment $\frac{1}{2} y y dx$, the circumference p of the circle of rotation for which the radius is 1, [recall that π had not at this time its dedicated value,

and $p = 2\pi$] and the preceding equation may be multiplied by p , so that there becomes

$pD.F = \int \frac{1}{2} ppydx$. Now pD is the circumference of the radius D , and py the circumference of the radius y , and thus $\frac{1}{2} ppydx$ is the area of the circle of the same radius y , and as a consequence $\frac{1}{2} ppydx$ expresses the cylinder for the volume S , or rather its element dS ; therefore $pD.F = \int dS = S$. Which was required to be shown for round solids; the same is shown for other shapes.

[The factor $\frac{1}{2}$ arises above as $2D.dF$ is the moment of the two equidistant elements from the axis of rotation.]

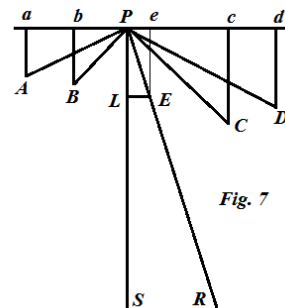
SCHOLIUM II.

48. Although in this proposition the weight shall be assumed the same or uniform at the individual bodies A, B, C, D , thus so that the weights of these shall be proportional to the absolute masses, yet because the weights can be different, and this lemma also is true generally, evidently then also, when the forces of gravity of the masses of the bodies themselves A, B, C, D actually are not present in proportion, nevertheless that can be shown here also. For if by A, B, C &c., as in the proposition, are no longer to be understood as the masses of these bodies, but the absolute weights of these, the rule of the proposition at this point will be found. In truth it might be that by A, B &c., as in the proposition, rather the masses of these bodies to be understood, and alongside in article 32, the weights P, Q, R, S of these to be designated by the products of the masses by the forces of gravity acting, which the individual masses A, B, C, D may be acted on respectively, that is by $A.P, B.Q, C.R, \& D.S$ &c. Which we will observe in other applications.

PROPOSITION V. THEOREM.

49. *The resultant direction PE of any forces PA, PB, PC, PD &c. impressed on the movable point P is a right line joining the centre of gravity of the moveable point P and the centre of gravity E of all the points A, B, C, D &c. from which the representations of the forces acting may be described correctly; and the action resulting from all the impressed forces on the body must be set out as a multiple of the right line PE, according to the number of points or impressed actions.*

The plane PS is understood to be drawn through the centre of gravity of the mobile point P and ad in turn to be a right line to that, and through the ends of the right lines PA, PB &c. perpendiculars Aa, Bb, Cc, Dd , & may be sent to the



plane ad from the centre of gravity E of the points A, B, C &c. of the normal Ee . The number of forces acting may be called N , & in PE produced there may be taken $PR = N \cdot PE$. I say that PR expresses the forces PA, PB, PC, PD &c. arising from all the impressed forces on the mobile point P .

I. Through E and R , EL and RS may be acting equidistant in the plane aPd , on account of the similar triangles PLE and PSR , and because $PR = N \cdot PE$, there will be $PS = N \cdot PL$ and $SR = N \cdot LE$.

II. Because E is the centre of gravity of the points A, B, C, D &c. the number of which according to the hypothesis is N , there will be (§. 46.):

$$aA + bB + cC + dD + \&c. = N \cdot eE, \text{ and } Pc + Pd + \&c. - Pa - Pb - \&c. = N \cdot Pe.$$

III. Because the actions from the individual oblique actions (§. 39.) are equivalent to the actions of these sides perpendicular and parallel to the plane aPd , evidently PA itself is equivalent to the sides aA, Pa , truly PB itself to the sides bB and Pb and thus for the rest, with all the oblique actions equivalent to all the components [sides in the text] of these acting, that is aA, bB, cC, dD , acting together from one direction Pa, Pb and the opposing Pc and Pd from the other side of the plane PS ; and from the actions together an action comes about from all these (§.38.) that is equal to the action

$aA + bB + cC + dD + \&c. (n.11.) = N \cdot eE (n.1.) = PS$, and from all the opposites (§.38.) an action arises equal to the excess by which the stronger exceed the weaker, that is itself $Pc + Pd + \&c. - Pa - Pb - \&c. (num.11.) = N \cdot Pe (num.1.) = SR$, therefore to two components PS & SR will be equivalent to all the oblique forces PA, PB &c., and from these, by (§. 39.) the single force PR denotes the resultant force arising from PA, PB, PC, PD &c., or what amounts to the same, the equivalent force from these.

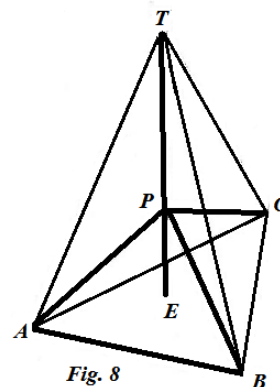
Which was to be demonstrated.

This elegant theorem taken from the illustrious Leibniz has been introduced, as which was set out in some of his letters, now previously given by Wallis, without demonstration. See Book III. Works of Wallis, p. 687.

COROLLARY I.

50. Hence the point P , on which the four forces PA, PB, PC, PT indeed are impressed acting together for equilibrium; but not present in the same plane, will be at the centre of gravity of the pyramid TAC , at the vertices of which the lines representing the forces are terminated. E shall be the centre of gravity of the points A, B, C and PE is joined, which will be the resultant direction of the forces PA, PB and PC , and from these $3 \cdot PE$ will be the equivalent force; hence because all are in equilibrium, there will be $PT = 3 \cdot PE$; and thus there will be

$PT : PE = 3 : 1 = A + B + C : T$ by considering these points as equally small weights, and therefore P is the centre of



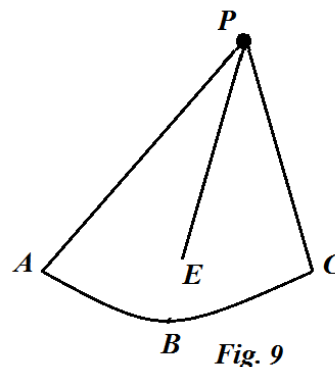
gravity of the four points of the pyramid A, B, C, T. But the same is the centre of gravity of the pyramid, because as they are the vertices of its angles. Therefore the assertion is clear.

This is another theorem Torricelli had proposed at one time in the above place, which Roberval has now indicated (§.43.).

COROLLARY II.

51. Hence also, if the mobile point P may be drawn towards the individual points of the surface of the curve ABC expressed by the distances of the points from the centre to the mobile point ; the resultant direction of all the forces will be the right line PE, joining the centres of gravity of the surface of the curve ABC and the mobile point P, and the action arising is equivalent to all of these forces [i.e. tensions in the curve], made from the surface ABC at the distance EP of its centre of gravity from the centre of the body P. For the number of points or of the forces arises from that surface ABC itself, the centre of gravity of which is the centre of all the points, by which the right lines representative of the forces are terminated.

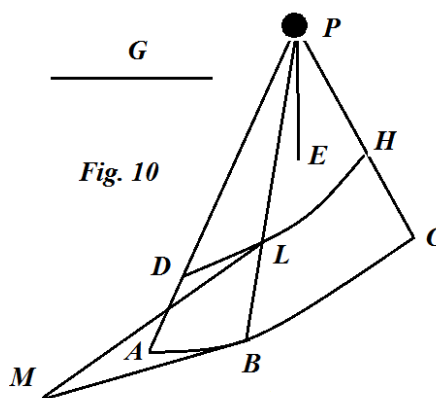
[This corollary is a generalization of the previous one with discrete weights, and expresses the position of the centre of gravity of a heavy surface or hanging curve in terms of the resultant direction of the tension forces integrated over an area or length, determining the position of the centre of gravity of the surface or curve.]



COROLLARY III.

52. For indeed, if the same mobile point P may be urged towards the individual points of the curve ABC, but not by forces at the distances PB themselves from the mobile point, but taken at the distances LP themselves on the individual lines BP, and proportional to the ends of some curve DLH.

Now also PE will be the resultant direction of all the forces PL with the right line joining the mobile point P with the centre of gravity of the curve DLH, and the resultant force from all these forces, made from the weight of the curve DLH shall act along the right line EP; but the weights of these two curves is understood not to be the same, but to be dissimilar [it is unclear if this disparity is due to gravity acting according to the inverse square law, or is constant but with different line densities of the curves; in either case, the tension is less for the outer curve]; thus so that with the tangents BM and LM drawn through the homologous parts of each curve AB and DLH at the points B and L, with the tangents concurring at the point M, the weight of the point B of the curve ABC

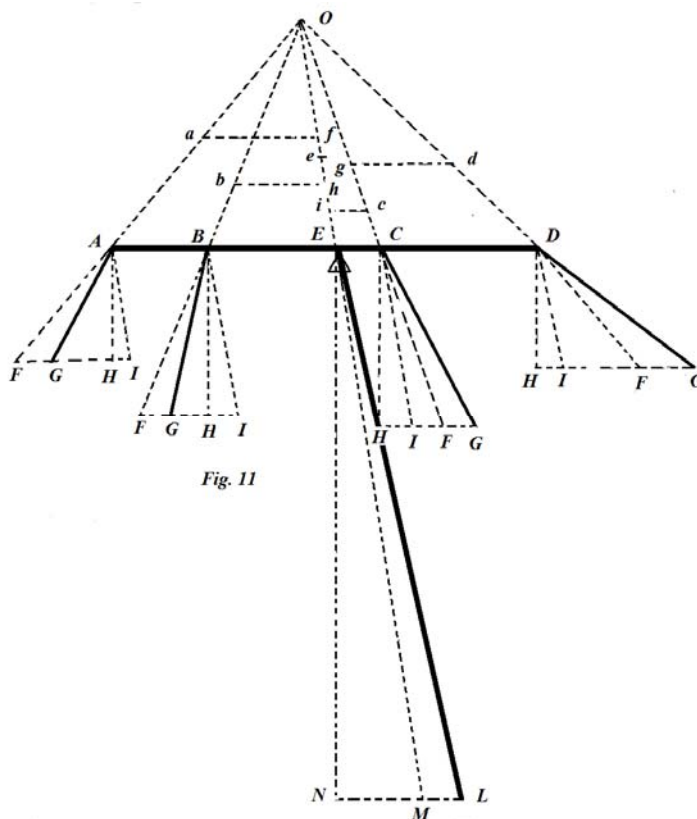


uniformly heavy, shall be everywhere to the weight of the homologous point L of the curve DLH, as the rectangle MBP to the rectangle MLP.

[Thus the resultant tensions due to homologous parts of the two heavy curves act along the tangents MB and ML of differing magnitudes, and the corresponding weights of the parts of the curves will be proportional to these lengths, from the last corollary, while the turning effects of each curve or moment is the same if $MB.BP=ML.LP$.]

PROPOSITION VI. PROBLEM.

53. To find the resultant direction of any number of forces acting AG, BG, CG, DG by which the points A, B, C, D may be pressed to the rigid line AD.



The forces are acting on the line AD from any point O outside the line through the individual points of the line A, B, C, D by which the line is urged along the adjoining directions AG, BG &c. , and with the indefinite lines OAF, OBF, OCF, ODF drawn through the individual points in which the lines of action, or representing the applied forces, are terminated by the indefinite line FG parallel to the line AD, and for the lines OA, OB, OC, OD &c., there is a need to be produced as far as crossing at the point F. For the individual lines AF, BF, CF, DF &c. acting along their directions may be transferred to their point of concurrence O, respectively to equal Oa, Ob, Oc, Od &c and the centre of gravity e of the points a, b, c, d may be found, and EM may be taken on the line OM through this centre e and the point O, and crossing the line AD at E,

which shall be to Oe as the number of points a, b, c, d to unity, [it appears that the masses of the points A, B , etc. and a, b , etc. are all the same or unity] ; and with the right line NL drawn through the point M equidistant to AD itself, it is assumed this ML to be equal to all the intercepts FG between AF, BG , and BF, BG and in the ∞ case, in which all the GF are acting together, or all taken next to the order of the letter from F , towards G inclined in the same direction, for then ML is to be taken in the same direction and equal to all the FG ; but if truly some of the FG point in one direction, the remainder truly FG in the opposite direction, in that case ML must be taken equal to the excess by which the stronger FG taken together, exceed the remaining FG acting in the direction opposite to the first, and it shall be required to be drawn in that direction along which the stronger FG act : the line EL joining the points E and L will give the resultant direction of all the actions AG, BG, CG, DG &c. and its length EL sets out the force for all of the equivalent forces applied in this resultant direction EL , or also the load which the inverted V support is subjected to sustain at the point E from the same applied forces AG, BG, CG &c. Which was required to be found.

[Note here that the forces are not resolved along xy axis, but rather a little in the manner of polar coordinates, with one component going towards a fixed point O , and the other along a convenient direction, the horizontal weightless rod AD ; the mean direction or resultant force, as we have translated the term here, is then the equivalent force due to all the points through the fixed point.]

Demonstration. Any action or force applied AG is equivalent to the components [*i.e.* sides of the triangle of forces] (§. 39) AF and FG ; and thus all AG, BG, CG &c. will be equivalent to all the AF, BF, CF, DF converging to the point O according to the hypothesis, and with all of these with the sides along FG . Now the rod or line AD may be considered, to which the applied forces or actions AG, BG &c. are impressed, to be clearly without weight, and likewise that line to be fixed, thus so that the line may be unable to move [*i.e.* vertically], but there is no reason why indeed likewise the force may not be moved, so that the action AF may have the same force by being moved in the plane, at whatever point in the plane itself it may be applied in the direction OA , either at the point A , or at some other point a, O &c. and that by itself is evident enough, for if the action AF applied to the right line AD at A in the direction OA were a greater moving force in the plane $AODF$ than this same action AF and in the same direction OA , but applied to the point O of the plane AOD , it follows the force AF to be equal with all the others, applied in the same direction to the point O , but in the opposite direction from a towards O , or rather arising beyond on the line AO may act on the point O in the plane, not to be in equilibrium, but itself to prevail, or that to be present stronger there ; and thus the two equal forces AF & aO acting on the same line together in opposite directions AO cannot mutually maintain their equilibrium, which is absurd. Therefore the force AF applied to the right line AD sets up the same effect and a force Oa itself equal to AF , and in the same direction OA applied to the point of the plane O , which also is understood for the remaining BF, CF &c. and hence the resultant direction of all the forces AF, BF, CF, DF , is the same which arises equally from these $Oa, Ob, Oc, & Od$ &c. and of these the resultant direction OeM (§. 49.) is the line joining the point O and the centre of gravity e of the points a, b, c, d &c. and the force from all applied at the point O , or being the equivalent of the right line at the points A, B, C, D &c. is a multiple of Oe according

to the number of points a, b, c, d &c., and thus is expressed by the right line EM, which (constructed) is to Oe as the number of points a, b, c &c. to unity. Therefore the force EM is equivalent to all the forces AF, BF, CF & DF, truly the action arises equal to all these from the lines FG parallel to AD, if they were acting together, or equal to the excess, by which the stronger FG exceed the weaker and opposites FG, and hence in each case is displayed by the right line ML, which from the variety of circumstances (according to the construction) is equal to the sum of all FG regarding the same direction, or of the difference of these which are directed in the opposite directions, so that with all AF, BF, CF, DF & with all FG the two sides EM & ML shall be equivalent & from these two (§. 39) with a single EL, generally it is apparent this EL also to be equivalent to all AG, BG, CG, & DG, which are applied to the line AD. Therefore EL signifies the resultant direction of all the applied forces and is equivalent to all the actions themselves in this resultant direction. Q.E.D.

COROLLARY I.

54. If the lines AH, BH, CH, DH are acting perpendicular to the line AD crossing FG themselves at the points H, these perpendiculars will represent just as many forces applied to the line AD, and the moments of all of these, which are from one side of the point E, will be equal to the moments of these, which are from the other side, that is, $AH.AE + BH.BE = CH.CE + DH.DE$. Indeed with af, bh, dg, ci drawn parallel to the right line AD, and AI, BI, CI, DI &c. drawn parallel to OM, and on account of the centre of gravity e , $af + bh = dg + ci$, or with the common height OE adopted the rectangles will be made $OE.af + OE.bh = OE.dg + OE.ci$, or

$Of.AE + Oh.BE = Og.DE + Oi.CE$, because $OE.af = Of.AE$, on account of the triangles OAE and Oaf; likewise $OE.bh = Oh.BE$, on account of the similar triangles OBE and OBh, and because on account of the remaining triangles the similar ratios, $OE.dg, OE.ci$ are equal respectively to $Og.DE, Oi.CE$. Again on account of the parallels AI, BI, CI &c. (constructed) to the right line OM, and the individual Oa, Ob, Oc, Od equal to the individual AF, BF, CF, DF, also Of, Oh, Oi, Og will be themselves equal to AI, BI, CI, DI, because any triangle such as Oaf is similar and equal to AFI, therefore with the rectangle substituted into the equations :

$Of.AE + Oh.BE = Og.DE + Oi.CE$, in place of the right lines Of, Oh, Og & Oi, equal to AE, BI, DE and CI with the same order taken, there becomes

$Al.AE + Bi.BE = Di.DE + Ci.CE$. Or at this stage, with the other right lines AH, BH, CH, & DH substituted in place of AI, BI, CI, DI, which themselves on account of the similar triangles AHI, BHI, CHI, DHI are analogous, that is, in proportion; finally there will be $AH.AE + BH.BE = DH.DE + CH.CE$. Again, just as

$EM = Of + Oh + Og + Oi = AI + BI + DI + CI$, thus also EN is perpendicular to the right line ML, and there will be $EN = AH + BH + CH + DH$.

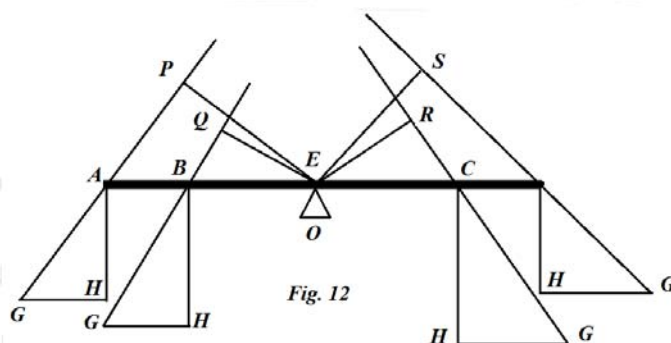
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COROLLARY II.

55. With the perpendiculars EP, EQ, ER & ES sent from the centre of equilibrium E to the directions of the forces AG, BG, CG, DG produced, the moments of these oblique forces from one direction, will be equal to the moments of the forces from the other direction.

That is, $AG \cdot EP + BG \cdot EQ = DG \cdot ES + CG \cdot ER$. For since any of the triangles PAE & HAG equally shall be similar, the rectangles becoming AH.AE, BH.BE, DH.DE and CH.CE will be equal to the homologous rectangles AG.EP, BG.EQ, DG.ES and CG.ER, from which (§.54) $AH \cdot AE + BH \cdot BE = DH \cdot DE + CH \cdot CE$, and also there will be:
 $AG \cdot EP + BG \cdot EQ = DG \cdot ES + CG \cdot ER$.

SCHOLIUM.

56. The case of this second corollary is obtained not only then, when the line AD is right, to which the oblique forces AG, BG &c. may be applied; but also in the case, in which having the forces applied to the line itself is a curve, indeed also in the wheels and with other kinds of mechanical devices of this kind. In a word, if the forces or actions are in equilibrium around some point, the moments of the forces, which act in one direction, are equal to the moments of the forces, which act in the opposite direction; and thus unexpectedly we fall upon a direct and immediate demonstration of the principle of Archimedes concerning the equality of the moments, in the case of the equilibrium of forces brought together amongst



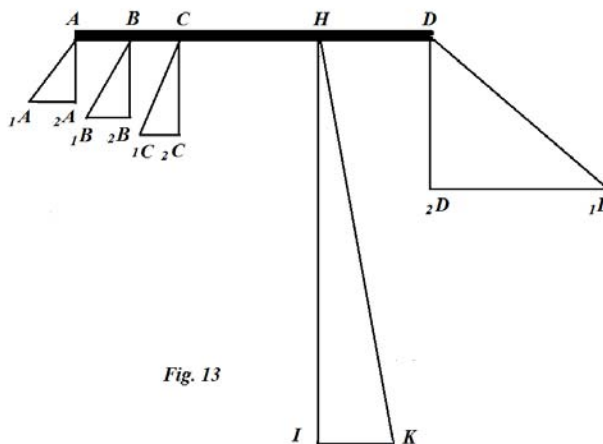
themselves, which a number of people have tried to demonstrate in a number of ways. It follows with the aid of this first, the problem of this proposition can be solved much more simply, indeed many more will be established than before, as can be seen easily from the following.

PROPOSITION VII. THEOREM.

57. If any number of actions or forces A_2A , B_2B , C_2C , D_2D are applied to an inflexible and weightless line AD (such as also has been considered in the preceding Proposition), and all these forces may be resolved into forces with their sides perpendicular and parallel to the line AD, there will be, with the line HI itself drawn through the centre of equilibrium H to be equal and parallel to all of the forces of the perpendiculars to the line AD: A_2A , B_2B , C_2C , D_2D , and through the point I the right line IK to be parallel to AD and equal to the excess, by which the force of the equidistant lines, which are for one direction of the line HI as $_2D_1D$, exceed these, which are opposing on the opposite part $_2A_1A$, $_2B_1B$, $_2C_1C$, the right line joining the points H, K will be the resultant

direction of the oblique forces applied to the rod or line AD, and the magnitude of which may establish the load, by which the pivot, or the sustaining point of the rod H is subject to be urged by all the forces, near to the resultant direction HK.

Since any action or oblique force shall be able to be considered A_2A (§.39.), arising as it were from the actions A_2A and ${}_2A_2A$ of [the sides of its triangle] parallel and perpendicular to the AD, all the oblique forces can be considered as arising from all the perpendiculars A_2A , B_2B , C_2C , D_2D , and from all the parallels ${}_2D_1D$, ${}_2C_1C$, ${}_2B_1B$, and ${}_2A_1A$. And (§ 54) from all the perpendiculars the action or force likewise from all these equal and parallel, passing through the centre of equilibrium H of these, that is (by the construction) the action $HI=$

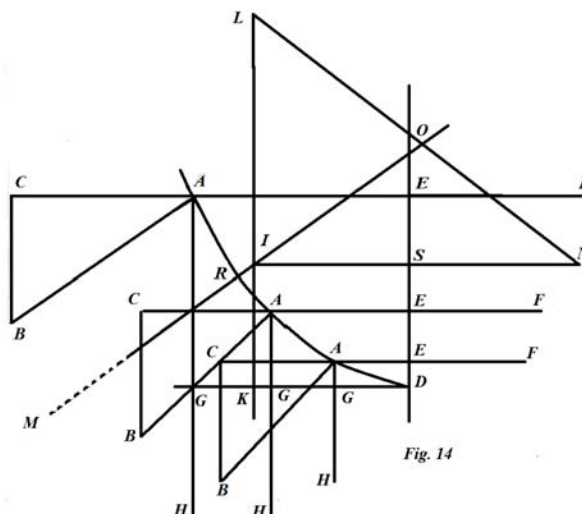


$A_2A + B_2B + C_2C + D_2D$; from the parallels the action arises equal to the excess (§. 38.) to the excess, by which the stronger exceed the weaker from these contrary ones, from which since (by construction) there shall be $IK = {}_2D_1D - {}_2A_1A - {}_2B_1B - {}_2C_1C$, this IK sets out the action arising from the parallel forces on the rod; and thus since all the lateral forces are understood to arise from the perpendiculars and parallels, then likewise the oblique forces A_1A , B_1B &c. as well as also the two sides HI & IK, and from these (§.39) the single HK, it is apparent this force HK to be the resultant direction from all the oblique equivalent forces and its direction to be the resultant direction of these, and thus the point H of the rod to be urged by the force in that mean direction or by the load, which is expressed by the magnitude of the right line HK. Q.E.D.

SCHOLIUM.

58. By the same method, with all necessary changes made, the resultant direction of all the forces applied to any desired curve DAA is found. [These might be, for example, the force acting on a cylindrical sail of some cross-section due to the wind, etc. ; if the final resultant force is expressed as the force by some distance, then we can assume Hermann had in mind initially the force per unit length, or the line pressure.]

For if the forces or actions applied are represented by the lines AB, each one of which can be resolved into its two lateral forces AC, perpendicular to the

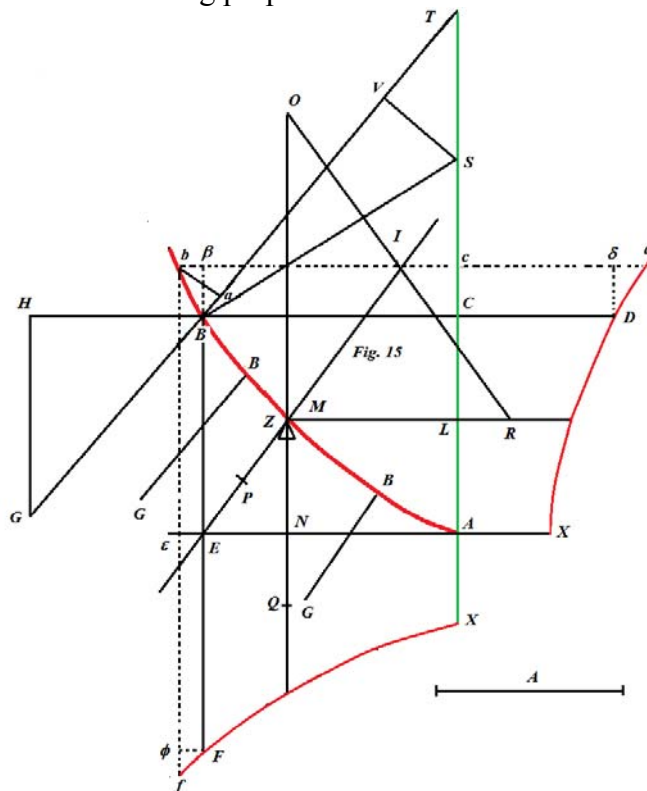


axis of the curve DE, & CB equidistant to the same, and with CA produced beyond the axis DE, so that the individual forces EF shall be made equal to the individual forces AC, the force or action from all the equivalent EF will be the forces IN drawn through the same centre of equilibrium and perpendicular to the axis (§.54) with the individual EF likewise taken equal; and from which since these individual EF shall be equal and opposite in direction to the individual AC, all the EF shall remain in equilibrium with all the AC, therefore also the single force IN is equivalent to all the forces EF with the same AC agreed to be in equilibrium with these; equally with DG drawn normal to the axis DE, and with the indefinite line AH parallel to the same axis, and with the GH made equal on these to the respective CB, the force IL will be equal and parallel to the whole force GH, passing through the centre of equilibrium K of these forces GH, in equilibrium with the same GH (§. 54.), therefore the two lateral forces IN and IL with the whole AC, and with the whole CB acting in the direction AG, that is, it is agreed to be in equilibrium with the whole GH, and thus for all the oblique forces, but truly the force arises from the forces IN & IL (§.49.), the direction of which IO passes through the point I and the centre of gravity O of the points L and N, and which may be represented as a multiple of the right line IO according to the values of the points L & N, that is by 2.OI [*i.e.* the diagonal of the resultant rectangle] ; therefore it will be only with such a force AB being applied at equilibrium to the whole curve DAA, therefore (§37.) the right line OI produced to M, gives the resultant direction IM of all the forces AB, and applied to the fulcrum R of the curve DAA will sustain a load equal to 2.OI from all the applied forces AB. Which was required to be found. Otherwise even if the point I, which may seem to be common to the intersection of the right lines KL and SN on the curve DA, yet it must not lie on that, and therefore the points I and R are always different.

For the construction of this scholium is not much different from that found by the most illustrious Varignon. Truly in this construction not all the points of the curve are acted on by the applied forces, for the case in which the forces are applied to all its individual points, depends on the quadratures and centres of gravity of certain curvilinear figures, as may be determined from the following proposition.

PROPOSITION VII.
THEOREM.

59. *If the curved lever ABB, of which the axis AT meets the ordinates BC at right angles, may be acted upon at individual points by some oblique forces or actions BG, BG &c., with GB moved forwards to T, and with the right line BS through B drawn normal to the curve ABB, and*



with BE parallel to the axis AT, and with SV sent from S normal to BT above, CD is taken on the ordinate BC produced, which shall be representative of the force BG applied to the curve at the point B, as BV to BT, and on the right line produced EF may be taken, which shall be to the same BG, as the rectangle BV.CT to the rectangle SC.BT, and this is understood to happen with respect of any other point of the curve B, and thence the mixed curvilinear figures will be produced AXDC, AXFE, the centres of gravity of which shall be at R and Q; through which there will act the right lines MR equidistant to AN, and QO equidistant to the axis AT, and intersecting at M; if the rectangle under MO and with the given ordinate A should be made equal to the area AXFE, and the rectangle under the right line MR and with the same given A should be made equal to the area AXDC, the right line IM through the point of concurrence M of the right lines QO and LR, and through the middle I of the right line OR drawn joining the points O and R and drawn forwards towards P, gives the resultant direction MP, of all the forces BG [i.e. force per unit length] applied to the curve AB, and the load which the pivot Z will sustain from all the forces, will be expressed by the rectangle 2MI.A within twice MI and the given A.

Thus Bb shall be an element of the curve AB, through the end b of which bd may be drawn parallel to the right line BD and bf parallel to the line BF, then also through the end G of the force BG with the right line GH parallel to the axis, of which the ordinate CB produced crosses at H, and finally with the perpendicular ba sent to GT, with which in place :

I. Because (by construction) $CD : BG = BV : BT$, and because the triangles BHG and BCT are similar, there is $BG : BH = BT : BC$, also from the equation there will be $CD : BH = BV : CB$. Again because the triangles BVS and Bba, on account of the right angle, bBa and VBS are equal to a right angle, and thus VBS and VSB themselves (which taken together are equal to a right angle) are equal to a right angle, thus on taking away the common VBS, the equal angles remain bBa and VSB, and on that account the angles for a and V right; above we will have $BV : BS = ba : Bb$, truly the similar triangles bBβ and BCS produce $BS : BC = Bb : Bβ$, therefore from the equation anew : $BV : BC = ba : Bβ$. But at the beginning of this also there was found : $BV : BC = CD : BH$, therefore $CD : BH = ba : Bβ$, and thus $BH.ba = CD.Bβ = \text{rectangle } C\delta \text{ of the inscribed area AXDC.}$

II. Because (by construction) $EF : BG = BV.CT : SC.BT$, & $BG : GH = BT : CT = SC.BT : SC.CT$, therefore from the equation there will be $EF : GH = BV.CT : SC.CT = BV : SC$. Besides, as number 1 has shown, there is also $BV : BS = ba : Bb$, & $BS : SC = Bb : bβ$, therefore again from the equation $BV : SC = ba : bβ$, but just as we had also $BV : SC = EF : GH$, therefore $EF : HG = ba : bβ$, and thus $HG.ba = EF.bβ = \text{rectangle } E\phi \text{ of the area AXFE inscribed.}$

III. Now because the force of the curve BG applied at the point B (§.39.) results from the sides BH perpendicular to the axis AT, and HG parallel to the same, and the individual points of the element Bb are acted on by the same oblique force BG, the force

by which the whole element is affected, is as the rectangle BG.ba sets out, the height of which ba sets out the multitude of the oblique forces applied from all the points of the element Bb, the rectangle BH.ba sets out the force perpendicular to the axis, and HG.ba the force parallel to the same, which are derived from the oblique BG.ba for the related element of the curve Bb, and from which this results (§.40.).

From which, because (from I. & II.) it is shown :

BH.ba = elemental rectangle $C\delta$ & HG.ba = elemental rectangle $E\phi$,

all the forces BG.ba will result from all the elemental rectangle $C\delta$ of the inscribed area AXDC, and similarly from all the $E\phi$ of the inscribed area AXFE, with the individual BG.ba arising from the individual $C\delta$ and $E\phi$. And from all the forces $C\delta$ (§. 54.) the force A.MR is the result since this rectangle (by construction) shall be equal to all the $C\delta$, or to the area AXDC, to which they are inscribed, and for the force A.MR the direction must pass through the centre of gravity AXDC ; and from all the forces $E\phi$ force results A.OM, because (by construction) here the rectangle A.MO = AXFE and its direction OQ passes through the centre of gravity Q of the same area AXFE. Therefore the forces A.MO and A.MR. remain in equilibrium along their directions MO, MR with all the oblique forces BG, BG, etc. of the curve AB at the individual points of application. But from the forces A.MO and A.MR. the force results (§.49.) 2.MI.A the direction of which MI passes through the centre of gravity I of the points O and R, of which the weight is imagined to be A. Therefore also this force 2.MI.A will be in equilibrium with all the BG, &c., and thus its direction MI produced in the opposite direction P to MP (§.37.) will be the resultant direction of all the forces BG, BG &c. and 2.MI.A will express the load which the pivot Z in the resultant direction MP will sustain from the right forces. Q.E.D. In the figure the points Z and M may be badly confused, of which the former is not a point on the curve AB, but must be placed within the figure ABC.

COROLLARY I.

60. If all the forces BG shall be perpendicular to the curve AB, the angles TBS will vanish, and there will be $CD = EF = BG$ everywhere. For with BG and BS coinciding, the right lines BV and BT shall become equal to BS itself, and $CT = CS$; from which since (by construction) there shall be $CD : BG = BV : BT$ generally there becomes $CD = BG$. Thus also because $EF : BG = BV.CT : SC. BT$, and in this case $BV = BS$, $CT = CS$, and $BT = BS$, the ratio BV.CT to SC.BT will become one of equality, and thus $EF = BG$.

COROLLARY II.

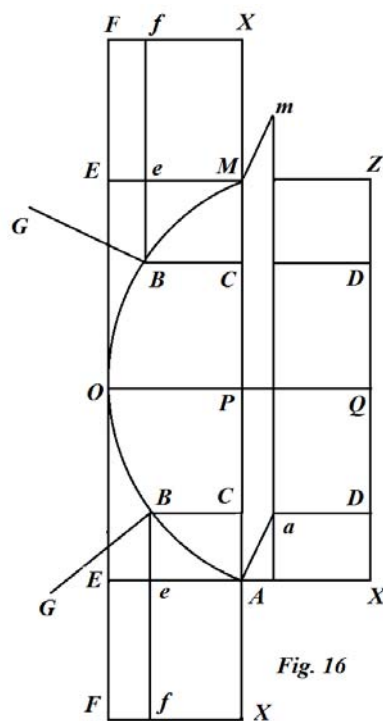
62. And if besides all the BG were equal to each other, the figures of mixed curvature and right lines AXCD and AXFE will be changed into the rectangles CAX & EAX.

COROLLARY III.

62. Fig.16. From which, if the forces shall be perpendicular to the individual points of the curve AOM, and all the applied forces BG shall be equal to each other ; the total force by which the curve AOM will be pressed upon, will be as the same total force, by which the base of the curve AM may be pressed upon, if also the forces themselves of its individual points shall be equal to BG, and also to be applied

perpendicular to the base AM, and thus it can be expressed by the rectangle MAXZ. For with the rectangle ME.EA described around the curve, the side of its base EE shall be a tangent to the curve at O, and to the sides ME, AE for the rectangles MXFE, AXFE, the base MA may be considered as acting as the axis of the curve AOM, and because following the hypothesis, all the forces BG are normal to the curve and equal to each other, there will be (§.61.) $ef = CD = BG$ and from these, equal homologous lines are present with respect to any other point B of the curve ; and thus the normal forces to the part ABO of the curve, will be equivalent to the two forces, of which one is placed perpendicular to the right axis AXQP, and the other is placed parallel to the rectangle AXFE. Thus also all the forces of the curve OBM are equivalent to the two, of which one is perpendicular to the axis and the other parallel to the same, and of which the latter shall be the rectangle MXFE, the former truly

PQZM. And on account of the equal rectangles MEFX and AEFX, the forces parallel to the axis AM and directly opposite belonging to the parts of the curve ABO, MBO, as equal to each other, cancel each other mutually, and only the forces perpendicular to the axis come to be considered; evidently these, which are expressed by the rectangles XAPQ, & ZMPQ ; therefore the force, by which the whole curve AOM may be pressed on by all of these applied forces BG, is put equal to the rectangle AXZM. But the same force of the base AM will enter also, if at any of its individual points anywhere it may be pressed or pulled by equal perpendicular forces of this kind. For the forces, by which the whole of the base is affected, is, as this base multiplied by the force, by which its individual points may be pressed, that is as the rectangle AMXZ.



[Again we may surmise here that initially the forces BG are per unit length, or per unit area if the shape is cylindrical.]

COROLLARY IV.

63. Equally, if in place of the curve AOM the surface of a right cylinder may be understood, of which the height shall be Mm ; and equal and perpendicular forces are considered to be applied to the individual points of this surface BG ; then the force, by which the surface of the cylinder itself is impressed by all these forces applied, whether it arises from the pressure against the plane mM , Aa , or from the traction from the same plane, will be set out by the parallelepiped of base AZ and with the height Mm ; and thus is equal to the force, by which the plane $mMAa$ is affected, if equally its individual points undergo equal impressions of the forces. For on the surface of the cylinder just as many curves are understood to be similar and parallel to AOM itself, as there are points in the height of the cylinder or of the prism Mm , any of which is impressed by a force equal to the rectangle AZ . And thus the force, by which all curves, that is cylindrical surfaces, may be acted on, may be expressed by all the rectangles equal to AZ itself, which may be present in the parallelepiped, that is by the parallelepiped itself, of the base AZ , and with the height Mm . And just as great a force also may be acting on the right line $MmaA$ by the equal forces applied to its individual points.

DEFINITIONS.

IV.

64. Fig. 19 & 20; For any solid ABCD is said to be an *unyielding solid* [which henceforth we will call *rigid*], of which the individual points of a curve CBAD on the surface are pressed on by any forces perpendicular to the surface, BM , bu &c., but still thus, so that the individual points of the curve cbd , parallel to the solid section of the base, may be affected by the same force bu or by forces equal to this. Truly the action of the forces on rigid solids hence will be designated by the name *Pressure* [*Pressionis* : a pressing down; we are tempted to call these stresses, a term introduced a century later by Thomas Young].

V.

65. The *pressure is internal* when the forces BM , bu , &c. applied at the points B , b &c. of the internal surface near to BM , bu , press that outwards.

VI.

66. Truly the *pressure is external*, when the points B , b &c. near MB , nb , &c. of the convex surface are pressed inwards by the applied forces themselves, Therefore from the order of the letters, from which the representative lines of the applied forces are signified, it can be distinguished at once whether the applied forces may press inwards or indeed press outwards. For if the letter or sign, by which a point of the surface is specified, to which the force is applied, may hold the first place, the pressure is internal but truly external, if it is in the following place. Thus BM , bu &c. denote internal

pressures, because the points of application B, b & c . hold the first place, & MB, ub , & c . signify external pressures. [i.e. the pressures or forces are going from or towards a point on the surface such as B or b .]

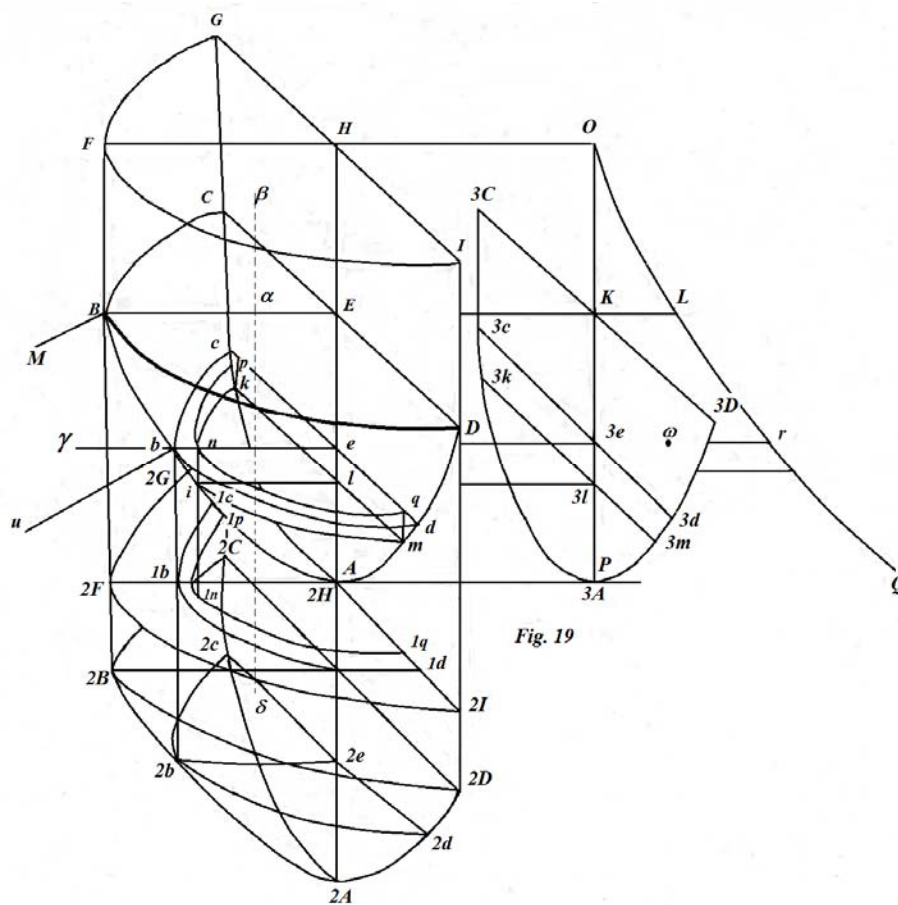
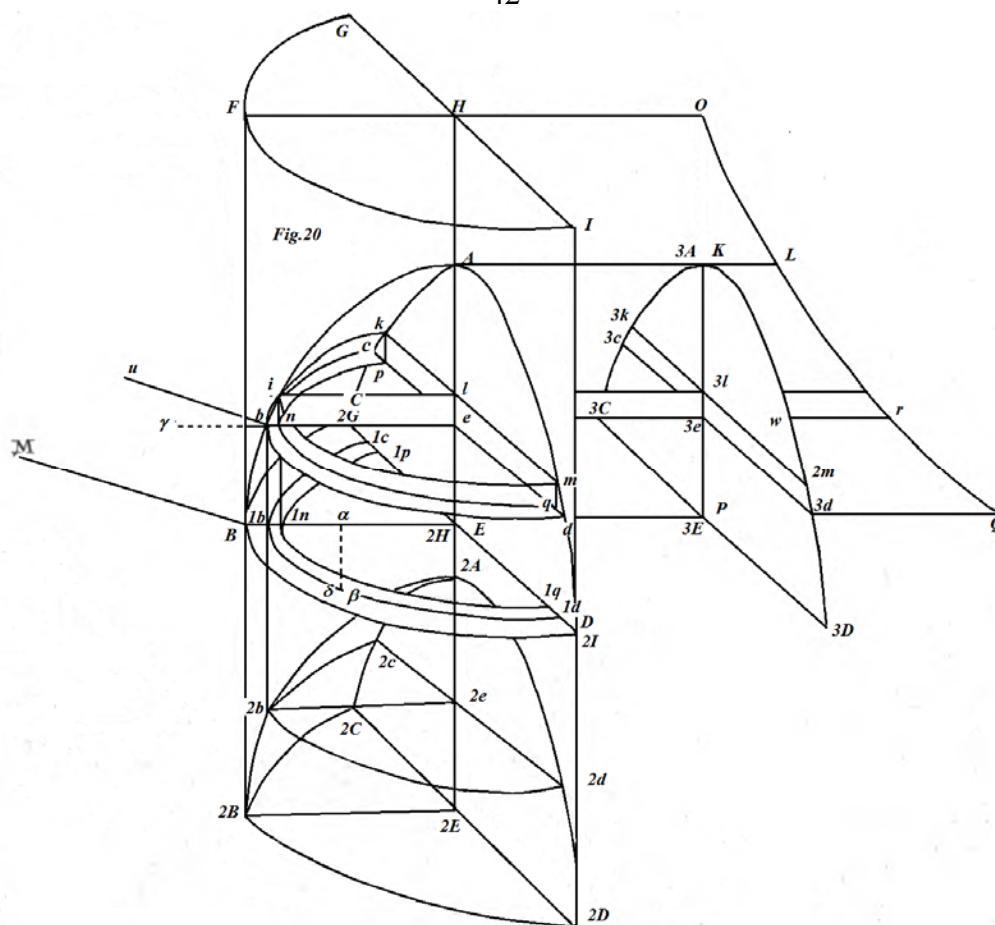


Fig. 19



VII.

67. By the *base of a rigid solid*, I understand some plane or plane section of that CBD equidistant from the horizontal.

VIII.

68. Truly a right section of a rigid solid for us is a plane figure CAD of our solid, the right section of its base, of which right section the *vertex A* is the point of the curve CAD at a maximum distance from its base CD, and the line AE drawn perpendicularly from the vertex A to CD may be called the *axis of the section*, and perhaps also of the rigid solid itself.

IX.

69. The figure of curved and straight lines KLrQP is a *scale diagram of the forces* acting on the rigid solid, in a plane at right angles to a right section of the rigid solid, and passing through the axis of the right section AE, any ordinate ${}_3er$ of which figure of its axis KP, which is put parallel to the perpendicular AE, displays the force, by which each point of the curve *cbd* parallel to the section *cbdc* of the base CBDC is affected. The *origin* or vertex of this curve O is the crossing point of the curve QrL with its axis KP.

43

X.

70. *The highest plane* is GFI, which is parallel to the base of the rigid solid, passes through the origin O of the scaled diagram of the forces.

XI.

71. *The lowest plane*, which is parallel to the highest, passes through the maximum ordinate PQ of the scale of the forces.

XII.

72. The *excess* [or extended] *prism* is the solid GFBADIG, which is put together, Fig.19, from the rigid solid CBAD with its vertex pointing down, and from the right prism GFBDI above the base CBD and ending on the upper plane GFI.

XIII.

73. The *deficient prism* is the solid GFBADIG, Fig. 20, clearly which remains with the rigid solid CBAD removed, with its vertex A now turned upwards from the right prism GFBDI on top of the CBDE, and likewise with the end on the upper plane GFI.

XIV.

74. Fig.19; The solid ${}_2C_2B_2A_2D$ is analogous [i.e. in proportion] to the rigid solid next to the scale of the forces, of which any section ${}_2c_2b_2d$ is similar to the homologous section cbd in the rigid body, equally distant and parallel to the interval ${}_2H_2e$ of the respective equal ordinate ${}_3er$ of the scale of the forces from the lowest plane ${}_2G_2F_2I$, while the homologous section cbd of the rigid body is distant by the interval He equal to the abscissa O_3e , pertaining to the ordinate ${}_3er$.

XV.

75. The analogous solid of the superfluous prismatic body is the body in Fig.19, ${}_2G_2F_2B_2A_2D_2I$, which is composed from the analogous ${}_2C_2B_2A_2D$ and from the right prism ${}_2G_2F_2B_2D_2I$, on top of the base ${}_2G_2B_2D$, as far as to the end on the lowest plane ${}_2G_2F_2I$.

XVI.

76. Fig. 20. *The solid analogous to the deficient prismatic solid* is the solid ${}_2G_2F_2B_2A_2D_2I$, which is left on taking the analogous rigid solid ${}_2C_2B_2A_2D$ from the right prism ${}_2G_2F_2B_2D_2I$, also terminated at the lower plane ${}_2G_2F_2I$.

XVII.

77. Fig.19, 20. *The pseudo-wedge* is a solid formed from the right cross-sections of the rigid solid, the horizontal cross-sections of which wedges are rectangles, parallel to the lowest or highest plane, the bases of which are the ordinates of a certain figure, of the similar and equal section of the rigid solid; truly just as the heights are the corresponding ordinates of the scale of the forces for the named ordinates.

[Thus, the diameter of the solid section at some height is one side of the base of the wedge section, at some ordinate of the graph, while the other side of this rectangular section is the force corresponding to ordinate. The complete wedge is the sum of all these sections in Fig. 20. Calculus can be used to find the position of the centre of gravity/mass ω of all these incremental wedges under the action of the horizontal forces applied.]

Around the axis KP, the figure ${}_3C_3A_3D$ shall be similar and equal to the right section of the rigid solid CAD, of which any ordinate ${}_3c_3d$ hence will be equal to the ordinate cd passing through the point e , which corresponds to the point ${}_3e$, through which the ordinate ${}_3c_3d$ has been drawn, which ordinate has a correspondence on the scale of the forces ${}_3er$. The pseudo-wedge of the right section CAD is a solid, of which the individual sections are rectangular parallelograms parallel to the plane GFI, of which the bases are the ordinates ${}_3c_3d$ and the heights of the ordinates ${}_3er$ of the scale of the forces, placed in the same plane section, that is, the rectangle ${}_3c_3d.{}_3er$. &c.

These pseudo-wedges also will be understood to be generated according to this account : with the above figures ${}_3C_3A_3D$ and PLQ, taken as the basis, right prisms are understood to be erected, which, on account of their bases crossing each other in turn at right angles, will pass through each other. That solid, which is intercepted by the prisms at right angles passing through each other in turn, is the pseudo-wedge, from which it is generated.

If the scale of the forces is a triangle or a trapezium [i.e. the force vs distance curve OQ is a straight line or a constant or proportional force; the next Prop. IX examines the case where the surface is a conic, and the force is constant], then the pseudo-wedge shall become a true wedge.

XVIII.

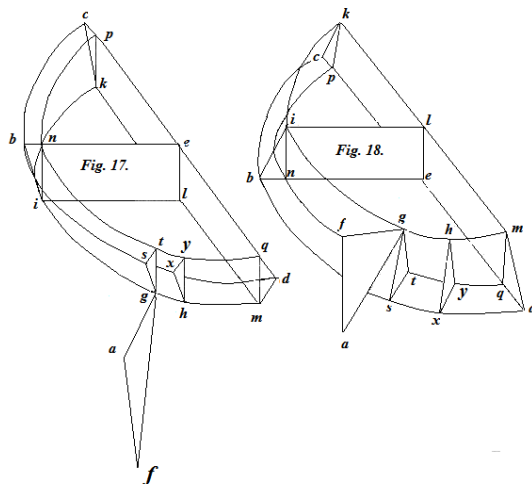
78. The *projection* of each figure in a certain plane, which thus also is called the *plane of projection*, is the figure, which is formed in the plane of projection, by sending perpendiculars from the individual points of the figure of projection to the plane of projection. Thus the projection of the curve *kim* onto the plane of projection ${}_2G_2F_2I$ is the curve ${}_1p_1n_1q$, which are formed from the perpendiculars sent from the individual points of the curve *kim* to the plane ${}_2G_2F_2I$.

[Thus, a vertical summation of the incremental cross-sections can be performed by projecting each onto a lower section in Fig. 19, and vice versa for the other figure; this can be used in a summation over the vertical forces to find the vertical position of the centre of gravity δ .]

PROPOSITION IX. THEOREM.

79. Fig. 17, 18. If, at the individual points of the conic surface *kcbimd*, with the two planes *cbd* and *kim*, with an indefinitely small distance el , ni , or pk placed in turn between themselves, and terminated by the incremental lines *ck* and *dm*, the force *fg* applied at a right angle, shall be the pressure that the whole conical surface endures, which will be equivalent to the pressure, which the ringlet *pbq*, intercepted by the two curves *cbd* and *pnq* (which is the projection of the other curve *kim* in the plane *cbd*), and

terminated by the lines cp and dq ; and the cylindrical surface knm for the base $kigm$ and erected with the height el or ni and terminated by the incremental lines pk and qm , and which will be apparent to be the same force gf , applied to the individual points of the



ringlet and to the points of the surface perpendicularly.

Some infinitesimal arc sx may be taken on the exterior of the curve cbd , and through the points s, x of the curve perpendiculars are understood to be drawn st, xy crossing to the interior pnq at the points t, y . Again the solid section $cbmd$ may be considered the section from the two planes drawn through the incremental lines st, xy and with the right plane cbd , and the solid will be $cbmd$, and the common sections of the planes will be the prism txg placed between the triangles stg, xyh and the rectangles. From which it is understood, the pressures, which the shapes $tghy$ and $styx$ endure (§. 63.) are equivalent to the pressure (which are to be considered, just as the continuous surface $stghyxs$ can be resolved into components of curvature ty having the base $sghx$), where the base $sghx$ is affected by same force gf . And since this may happen, with respect of any other $sghx$ on the surface of the wedge kbm , and of the homologous incremental areas $styx$ in the ringlet, & $tghy$ on the surface of the cylinder; it is evident, the pressures which all the elemental areas $sghx$ endure on the surface of the cone, to be equivalent to the pressures, which all the elemental areas $tsxy$ sustain, which are on the ringlet, and which all the $tghy$ support, which are encountered on the surface of the cylinder knm . And therefore the pressure, which the conical zone endures kbm from all these equal forces gf applied at right angles, is equivalent to the pressures, which are apparent from all the equal forces gf themselves applied at right angles to the ring pbq and the cylindrical surface knm . Q.E.D.

[A proof of this kind is still offered to show that fluid pressure is the same in all directions at a point in the fluid at rest; here the argument is used to say that the pressure or stress is the same in all directions in an inflexible solid at rest: this presumably must be true, as no work is done on the particles of the solid due to its infinite rigidity. Whether Hermann considers any rigid body as totally inflexible I do not know at this point.]

PROPOSITION X. THEOREM.

80. Fig. 19, 20. *If indeed unequal forces MB, ub &c. or BM, bu &c., shall be applied to the individual points of the convex or concave surfaces CBAD respect., from which it is apparent the solid CBADC may be terminated by the part CBAD, and perpendicular to the surfaces, but all of these forces in any case, which are applied to the individual points of any sections bcd parallel to the base CBD, shall be equal to these and shown by the homologous ordinate ${}_3er$ of the scale of the forces ; The pressure, which the surface of the curve of the rigid solid endures, will be equivalent to the pressure, which the base of the rigid solid CBD may be acted on along the direction $\alpha\beta$ of the perpendicular to the base drawn through the centre of gravity δ of the solid ${}_2G_2F_2B_2A_2D_2I$, which is analogous to the excess (or deficient) prism GFBADI, as this analogous solid shall show the same pressure ; and likewise also for the pressure which the right section of the pseudo-wedge shows in the rigid solid, or the solid ${}_3C_3ALQ_3D$ being extended in the right section of the rigid solid CAD along the direction er perpendicular to this section, and drawn through the centre of gravity ω of the pseudo-wedge.*

[Thus, a lengthy verbal description is given of what amounts to integrations in 3 dimensions, where the pressure exerted by symmetric external/internal forces for the outer /inner pressures exerted horizontally on the surface of any section is equal to the vertical pressure exerted from above, along a line passing vertically through the c. of g. of the solid; the magnitude of the vertical pressure is set equal to that exerted laterally for any section, and the size of the force horizontally is put, for each horizontal increment, equal to the area of a rectangle with sides equal to the diameter of the section and the applied force expressed as a length : these rectangles of incremental height can then be stacked together as a wedge with curved sides, from which the resultant horizontal force on the whole solid can be evaluated passing through the centre of gravity or the wedge, to be balanced by the force through the centre of gravity of the solid, as is the case for the resultant vertical force. The moments of these forces are considered in the corollaries.]

That is, if β may designate that solid analogous to the excess/ deficient prism, and ω the pseudo-wedge mentioned in the proposition ; The forces β and ω are to be applied to the rigid solid in the directions $\alpha\beta, e\omega$, or in the directions $\beta\alpha$ and ωe , understood to be in equilibrium with all the forces BM, *bu* &c. of the inner surface CBAD, or with all the forces MB, *ub* applied to the outer surface.

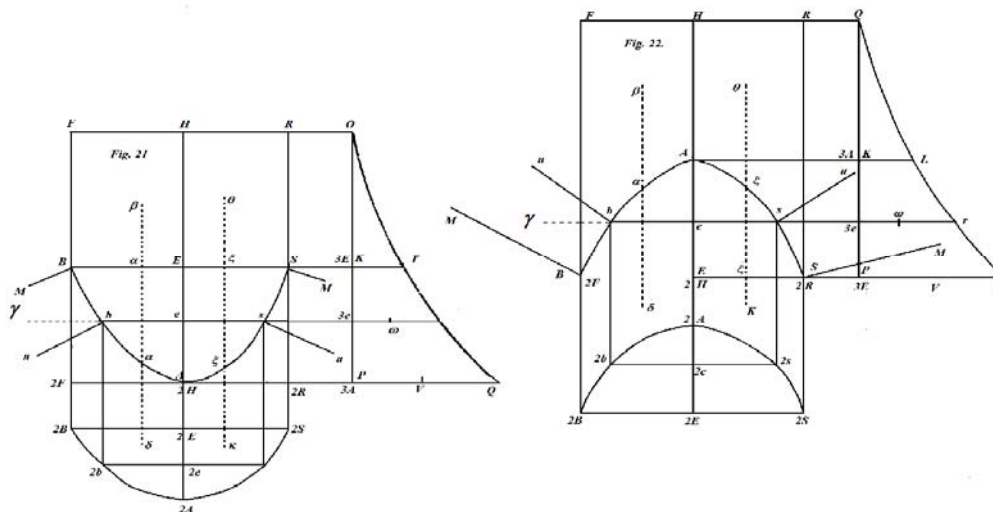
A section of the rigid solid is understood to be formed from the two planes *cbd*, *kim* parallel to the base CBD and indefinitely close together. The part of the surface of the rigid solid cut off by the two intersecting planes, that in the following we will designate by *kbm*, is an instance of the two curves *cbd* and *kim* of the conical surface and bounded by the incremental lines *ck*, *dm*. The projection of the curve *cbd* on the lower plane ${}_2G_2F_2I$ shall be ${}_1c_1b_1d$, and the projection of the curve *kim* on the same lower plane shall be ${}_1p_1n_1q$, truly *cbd* in the plane, is projected onto the curve *pnq*.

Now the pressure which the conical surface kbm endures from the force bu (following the hypothesis) $= {}_3er$, applied perpendicularly at its individual points (§.79.), will be equivalent to the pressures, which are permitted by the ring pbq (such as were considered in the preceding proposition) and the cylindrical surface knm , if also the individual points of the ring and of the cylindrical surface may be affected perpendicularly by the same force $bu = {}_3er$; and the force by which the ring pbq is affected will be, as the volume of this ring multiplied by the force bu or ${}_3er$, by which its individual points may be acted on, that is, it can be explained in terms of the solid $pbq{}_3er$, or because the ringlet $phq =$ its projection ${}_1p{}_1b{}_1q$, and (§.74) ${}_3er = {}_2H{}_2e = {}_1b{}_2b$, by the solid ${}_1p{}_1b{}_1q$ at ${}_1b{}_2b$, that is by the half pipe ${}_1p{}_1b{}_2b{}_1q{}_1d$, described by the solid ${}_2G{}_2B{}_2A{}_2I$. Truly the pressure which the cylindrical surface knm may be permitted from the applied forces, is equal to the pressure (§.63.), which the rectangle $kpqm$ endures because the cylindrical surface which the cylindrical surface subtends, may be established by the solid or parallelepiped $km.el.{}_3er$, or by the parallelepiped ${}_3k{}_3m{}_3el.{}_3er$, because it is equal to the former and to the inscribed pseudo-wedge. From which, because the pressure, which any other incremental conical region kbm of the rigid solid may allow from the applied forces, similarly is equivalent to the pressures, by which the homologous rings pbq , and the homologous cylindrical surfaces knm may be acted on by the same forces applied perpendicularly to the points of the conical zones: it follows further to be the case that the pressures, which all the incremental zones of the wedge endure, may be contained in the whole surface of the rigid solid, is that pressure, which the curved surface itself of the rigid solid will undergo, and they may be equivalent to the pressures, which all the ringlets, and all the homologous cylindrical surfaces from the conic zonules will allow from the applied forces; and since it was shown a little before, again the pressure from any ringlet pbq is to be established from the homologous cylindrical half-pipe, of which the projection is ${}_1p{}_1b{}_2b{}_1q$, the base of which is formed from the two curves ${}_1p{}_1b{}_1q$ and ${}_1p{}_1n{}_1q$ and by the incremental lines ${}_1c{}_1p$ & ${}_1q{}_1d$ of the ringlet pbq intercepted equally, truly the height ${}_1b{}_2b$ (§.74.) is equal to the ordinate ${}_3er$ of the chart [graph] of the forces, and the pressure of each surface of the cylinder knm to be expressed by the parallelepiped ${}_3k{}_3m.{}_3el.{}_3el$. For the inscribed pseudo-wedge, the pressure of all the ringlets may be expressed by all the cylindrical semi-tubes ${}_1p{}_1b{}_2b{}_1q$, which are to be contained in the solid ${}_2G{}_2F{}_2B{}_2A{}_2D{}_2I$ of the extra analogous cylinder (in which none of the internal structure has been given), that is by this analogous solid itself, and the force arising acting on the rigid solid by all these cylindrical surfaces knm , will be expressed by all the parallelepipeds from the inscribed pseudo-wedges; or because with the parallelepipeds of the figure of this kind taken together, to which they are inscribed, and in which finally they vanish, they will be equal through the pseudo-wedge itself (as hence all the semi tubes inscribed in the analogous solid, to be redundant or lacking this solid prism). Therefore all the forces BM , bu &c. applied to the whole surface at the individual points of the curve $CBAD$ or MB , ub &c. for the two equivalent forces, of which the one, being expressed by the solid ${}_2G{}_2F{}_2B{}_2A{}_2D{}_2I$, may act perpendicularly along the direction $\alpha\delta$ (§. 54.) of the rigid solid base CBD and passing through the centre of gravity δ of the solid in the manner named, as for brevity the centre of gravity is designated by the single letter δ , henceforth we will indicate, or what amounts to the same thing, of all the forces acting together and in

the plane of the perpendicular CBD, which is expressed by the semi tube $1p_1b_1q$ inscribed for that δ ; truly the other by the pseudo-wedge $Q_3C_3A_3D$ (likewise by which its centre of gravity hereafter being indicated by the same single letter ω) along the direction re or $b\gamma$ through the centre of gravity ω drawn through the pseudo-wedge, and perpendicular to the right section CAD of the rigid solid, since the parallelepipeds inscribed in the pseudo-wedge represent the forces acting together and applied perpendicularly to the plane CAD, of which the resultant direction (§. 54.) will pass through the centre of gravity of these ω . Therefore the force δ applied in the direction $\alpha\delta$ perpendicularly to the plane CBD, and the force ω applied normally to the plane CAD in the direction $e\gamma$, acting likewise produce the same effect as the forces BM, bu &c. and thus the same forces δ & ω acting in the opposite direction, along the directions $\alpha\beta$ & $e\omega$, which generally are in the opposite directions $\alpha\delta$ & b , which are agreed to be in equilibrium with the forces BM, bu &c. at the individual points B, b applied everywhere to the surface of the curve CBAD. But the same forces δ & ω acting along the directions $\alpha\delta$ & $e\gamma$ also will remain in equilibrium with all the forces MB, ub &c. Which were to be demonstrated.

COROLLARY I.

81. Fig. 21, 22. If now in place of the solid BAE the curved surface BbA be terminated in turn by the two right planes AE & BE, it may be understood the curved surface BbAsS of the solid BAS be terminated by the single plane BS, which shall be the whole of the base of the solid (§.67.), and the plane EA (§. 68.) shall be a right section of this, and for the other part of the solid EAsS there shall be the same scale of the forces, as for the parts



of the rigid solid EAbB, thus so that the forces bu , su for the individual points of the section bs applied parallel to the plane BS shall be equal to each other, and $3er$ the ordinate of this scale of the forces; all the lines may be designated, in the analogous solid with the same letters, as in the proposition, in which the homologous lines in the rigid solid, but by a small 2 as a distinguishing mark, and thus the solids will be $2F_2B_2b_2A_2H$,

& ${}_2R_2S_2S_2A_2H$ and with the unused prisms (*i.e.* lacking in details) $FBbAH$, $RSsAH$. Those analogous solids will be indicated by the letters δ & κ , by which the centre of gravity of these may be designated also, and as in the preceding article, ω will designate both the right section of the pseudo-wedge AE in the rigid solid BAS , as well as the centre of gravity of the same pseudo-wedge, with which in place all the forces SM , su applied to the solid EAS will not be carried off by any other force, as the force κ pulling the curve AS along the direction $\xi\kappa$ of the right section AE parallel to and passing through the centre of gravity κ , together with the force ω pulling the solid along $e\omega$ parallel to the base BS , and thus all the forces BM , su , bu , SM &c. applied to the whole solid BAS are equivalent to the forces δ , & κ applied to the solid in the directions $\alpha\delta$ & $\xi\kappa$, with its vertex A shown downwards as in fig. 22, or in the directions $\alpha\beta$ & $\xi\theta$ with the vertex of the solid turned upwards as shown in fig. 22; and in addition to the force ω with the others AbB , AsS hence applied to the solid thence in the directions $b\gamma$, $s\omega$ in each case. But if the forces applied to the rigid body are pressing on the exterior, these can be considered as acting along MB , ub , MS , us &c. and in that case these forces will be equivalent to the same as the former, δ , κ , ω , but the solid BAS in the case of Fig. 21, with the pressures in the directions $\delta\alpha$, $\kappa\xi$, γb & ωs , and along the directions $\beta\alpha$, $\theta\xi$ in Fig. 22.

COROLLARY II.

82. Indeed, if the parts BbA and SsA of the solid BAS be interchanged about the vertex A and in place some others to be deduced in turn and that may be able to be bound together again mutually, the moment from all the forces γ or ω , & δ will be $\gamma.Ae + \delta.\alpha E$, which will be equivalent to the individual forces BM , bu , &c. or MB , ub &c. applied perpendicularly to the curved surface AbB . And $\omega.Ae + \kappa.\xi E$ will be the total moment of the forces of the part of the solid AsS applied ω & κ and being equivalent to the forces SM , su &c. or MS , us &c. applied to the individual points of the curved surface AsS .

COROLLARY III.

83. Fig. 21 & 25 ; If now the rigid solid $aBAS$ shall be terminated by a single curved surface, the individual points of which are acted on by perpendicular forces, and which forces will be determined by the scale of the forces $KLQP$, thus so that the highest point a may be acted on by the force $ba = KL$, the lowest point A by the force ${}_2A_2H = PQ$, and the individual points of the circumference of the section BS by the force $MB = MS = {}_3er$, and thus with the rest respectively: such a solid $aBAS$ high will be impressed along the direction of the plane FO , or the perpendicular ${}_2FQ$, expressed by a force which is analogous to the rigid solid ${}_2a_2B_2A_2S$ (§.74.). For just as above (§.81.), the equivalent force or strength applied perpendicularly to the whole of the curved BAS (Figs. 21. & 22.), is agreed from the equal and opposite forces γ & ω constantly acting on the same side, as if not to be present ; now these forces, which indicate the effectiveness of these forces, δ & κ taken at the same time indicate the whole solid

${}_2F_2B_2A_2S_2R$ which is redundant to the prism (fig. 21) or is deficient to the analogous FBAS (fig. 22). Thus also in our solid $aBAS$ the forces directly opposing put in place through the pseudo-wedge ω mutually destroy each other, truly these themselves, which act along the directions to the planes BS or perpendicular to FR , are considered to arise here ; for the upper part BaS of our solid ${}_2F_2B_2A_2S_2R$ will be forced downwards by its force or strength , and the lower force BAS , as the solid ${}_2F_2B_2A_2S_2R$ establishes, which is analogous to the redundant prism $FBASR$, is forced up in height along the direction at right angles to the plane BS . And thus (§. 38.) with the smaller force taken away, which is set out by the solid ${}_2F_2B_2a_2S_2R$ analogous to the deficiency of the prism $FBaSR$, from the greater force, which the analogous solid $FBASR$ sets out for the redundant prism, since the smaller force acts directly downwards on the rigid body, truly the greater the body is urged to rise ; and the body force will remain at a height acting together on the solid ${}_2a_2B_2A_2S$, which is analogous to the rigid body $aBAS$.

CAPUT I.

*De proportione inter Solicitationes gravitatis, seu pondera corporum, cum proportione
Massarum eorumdem corporum collata.*

Apud Philosophos Geometras elegans corporum proprietas celebratur, in eo consistens quod corporum pondera in eadem prorsus ratione crescant in qua ipsorum Massae seu Quantitates Materiae ; aut Geometrica phrasi eandem proferendo ; *quod corporum gravitates seu pondera Massis ipsorum proportionalia sint.* Illustr. Newtonus, sumtis accuratissimis pendulorum experimentis hanc, gravium proprietatem se semper comperisse testatur pag. 305. Princ.Nat.Math. primae edit. quam etiam ex Coroll.I. Prop. XXIV. Lib.II. eleganter elicit. Nobiliss. Hugenius hanc proprietatem non solum admittit, sed etiam pag. 140. Diatribae De Causa Gravitatis ex regulis motus deducit. Et quia haec corporum affectio per universam Philosophiam Naturalem magni momenti est, aliam adhuc ejus demonstrationem adducere tentabo, admissa unica quae sequitur hypothesi.

HYPOTHESIS.

27. Eadem manente Materiae quantitate, & directionibus gravium existentibus parallelis, seu in Centro indefinite distante convergentibus, pondera corporum non mutantur, variatis eorum positione respectu horizontis & figurae.

Hoc est, corpus quodvis idem pondus retinet, quocunque modo id positum sit respectu horizontis, nec ejus pondus mutari censendum, si ejus figura in aliam, quamcunque scilicet, in globum, cylindrum, conum &c. mutata sit, modo eadem manserit materiae quantitas, seu massa.

PROPOSITIO I. LEMMA.

28. Causa gravitatis, quaecunque es sit, non tantum agit in partes exteriores corporum, sed etiam in interiores omnes.

Nam si gravitatis causa in exteriores tantum corporum partes ageret, in eas solum ageret, quae horizonti aversae sunt corporum superficies, atque adeo corporum pondera hisce superficiebus proportionalia essent atqui, pro varia corporis positione & situ, superficies horizonti aversa modo major modo minor est; ergo, si gravitatis impressiones superficiebus hisce proportionales essent, sequeretur, mutari corporis pondus, pro alio atque alio ejus situ contra hypothesin.

Deinde, si gravitas tantum ageret in exteriores corporum partes, non vero in interiores sequeretur, quod, mutata figura corporis, sed eadem manente materiae quantitate, aliud atque aliud inde resultet corporis pondus ; quandoquidem variatis figuris variantur simul particularum & superficierum horizonti aversarum positiones ac quantitates, atque adeo variarent corporum pondera eadem manente eorum materiae quantitate. Quae omnia sunt contra hypothesin articulo 27 assumtam.

COROLLARIUM.

29. Idcirco nullius corporis pondus in omni positione & figura idem manere potest, nisi singula ejus elementa aequalia aequales gravitatis impulsus excipiant. Quo posito facilis erit probatu sequens

PROPOSITIO II. THEOREMA.

30. *Pondera corporum quantitatibus materiae seu massis eorum proportionalia sunt.*

Sint primum duo corpora commensurabilia C, c, unum eorum elementum e, numerus elementorum in Corpore C, N & numerus in altero c, dicatur n, & denique massae corporum seu materiae quantitates dicantur respective M, m. Quibus positis, quia (§.29.) singula utriusque corporis elementa aequalem a gravitate impressionem subeunt, si una ejusmodi impressio nominetur i, pondus corporis C seu sollicitatio totalis a gravitate, qua corpus ad descensum urgetur (§.23.) est aggregatum sollicitationum, quibus singula elementa urgentur, & quoniam harum sollicitationum directiones ex hypothesi parallelae sunt, ipsae sollicitationes seu gravitatis impressiones conspirantes erunt; atque adeo sollicitatio totalis gravitatis in corpore C erit una sollicitatio seu impressio i ducta in numerum N elementorum, quae in hoc corpore continentur, hoc est pondus ipsius C quod indicabo per pC fiet $= N.i$ ac $pc = n.i$. Est vera $N.i : n.i = N.e : n.e$; ergo quia N.e est aggregatum particularum e in corpore C, seu hujus corporis massa M, & n.e massa alterius c seu m, erit omnino $pC : pc = M : m$ hoc est pondera corporum C, sunt massis eorum proportionalia. Q.E.D.

COROLLARIUM I.

32. Si corporis massa, quae subinde eadem litera designabitur, qua ipsum corpus signatur, ducetur in sollicitationem seu gravitatis impressionem (haec enim duo hoc loco & ubique unum idemque significant) qua unum corporis elementum urgetur, factum exponet semper per pondus absolutum corporis, ut si corpus fuerit C, & gravitatis sollicitatio, qua singula ejus elementa afficiuntur, nominetur G, quantitas seu factum C.G exponit pondus absolutum corporis C, intelligendo per hanc literam C massam hujus corporis.

COROLLARIUM II.

32. Hinc si, ut supra, Corporis hujus volumen & densitas dicantur V & D, & pondus pC , erit pC ut G.D.V. Id est, pondus cujusque Corporis est ut factum ex sollicitatione gravitatis G, qua singula ejus elementa e urgentur, & densitate D in Volumen V. Nam (§.32) pC est ut CG, & (§. 28) C, quae hoc loco idem significat ac illic M, scilicet massam corporis, est ut D.V, ergo pC seu CG est sicut G.D.V.

COROLLARIUM III.

33. Et quia gravitas specifica corporum consistit in proportionem ponderum absolutorum corporum, sub voluminibus aequalibus, ideo gravitates specificae densitatibus

proportionales erunt, cum (§.26.) massae corporum volumine aequalium densitatibus proportionales sint & per praesentem propositionem pondera corporum, eorum massae. Idcirco si corporis nostri C gravitas specifica dicatur S, erit ejus pondus absolutum seu pC ut S.V., quandoquidem S est ut D.G.

SCHOLION.

34. Haec ipsa propositio ex theoria motus pendulorum aliter adhuc demonstrari potest, ut a Cel. Newtono, in loco supra in proemio hujus capitis jam indicato, praestitum. Similem propositionem ex principiis, quibus ad derivandi symptomata motuum acceleratorum gravium utemur, facillimo etiam negotio eliciemus.

CAPUT II

De Solicitationibus quibus Corpora rigida, id est inflexibilla, urgentur, earumque mediis directionibus.

DEFINITIONES.

I.

35. Cum quaelibet vires, ac per consequens etiam sollicitationes

sint ex genere quantitatis per lineas rectas hisce sollicitationibus proportionales recte possunt exponi. Idcirco si puncta A, C, E a rectae AE sollicitationibus secundum directiones AB, CD, EF agentibus urgentur, hae sollicitationes seu potentiae exprimentur rectis AB, CD, EF in ipsis earundem directionibus homologue sumtis, adeo ut hae lineae sollicitationes vel potentias earumque

directiones simul designent, & hoc in sequentibus tantum non semper observabitur.

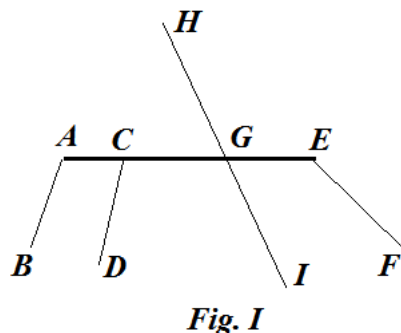


Fig. I

II.

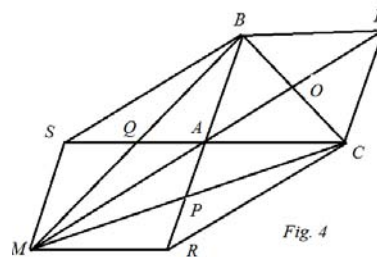
36. Si plurium sollicitationum vel potentiarum, in aequilibrio consistentium, nonnullae positae sint ad unam partem corporis ad quod pertinent, reliquae vero vel reliqua ad partem oppositam, Sollicitationes, quae sunt ad unam partem, reliquis in parte opposita *aequipollere* dicentur. Ut si rectae AE, a sollicitationibus aut cuilibet alii corpori potentiae AB, CD, EF applicatae sint ab una parte, in parte vero opposita potentia GH agens ex G in H, & haec cum reliquis ipsi oppositis in aequilibrio consistat. Potentia unica GH, reliquis omnibus AB, CD, EF *aequipollere* dicitur. Caeterum monendus est tyro, quod potentia vel sollicitatio, quae reliquis *aequipollere* dicitur, ipsis ideo aequalis censenda non sit; etenim *aequipollere* & *aequale esse* in Mechanicis non sunt phrases synonymae. Unicus enim est casus, quo potentia GH oppositis AB, CD, EF *aequipollens* ipsis simul sumtis aequalis est, tunc scilicet, cum hae conspirantes sunt (§.22.) ipsisque altera GH (§.23) directe contraria.

Demonstr. Cogitetur mobile A rectae mobili AB inhaerere; sed ita tamen ut in ea libere moveri queat ex A versus B, lineam vero AB duntaxat motu parallelo sibi & rectae CD super AC ferri posse, deinde rectam mobilem AB corpus A secum deferentem urgeri versus CD sollicitatione AC; mobile vero A in linea AB urgeri sollicitatione AB; & manifestum erit conatum A_2A , quo linea mobilis AB ad rectam ipsi parallelam CD accedere conatur, urgente sollicitatione AC, se habere ad conatum ${}_2A_2D$ urgente sollicitatione AB corpus A in hac recta mobili, ut AC se habet ad CD; jam quia hypothesi ${}_2A_2B$ ipsi CD parallela est, & ut modo vidimus, $A_2A : {}_2A_2D = AC : CD$, erit punctum ${}_2D$ in diametro AD parallelogrammi BC. Porro, quia mobile A participat motum A_2A , vel potius conatum ad motum lineae mobilis AB, in qua moveri posse intelligitur, & motum proprium, seu potius conatum, ad hunc motum

${}_2A_2D$, ac quia, si linea mobilis & corpus spatiola utlibet parva A_2A & ${}_2A_2D$ simul describere potuissent, idem id suisset quam si corpus A spatium A_2D transmisisset ; liquet utique, ex conatibus lineae mobilis & corporis, A_2A & ${}_2A_2D$ nasci conatum A_2D juxta directionem AD, qui se habeat ad A_2A vel ad ${}_2A_2D$ sicut AD ad AC, vel ad AB ; ergo ex sollicitationibus AC & AB resultat sollicitatio AD in directione A_2D . Quod erat demonstrandum.

SCHOLION.

40. Sollicitationes, quae exponuntur lateribus AC, AB parallelogrammi BC, ideo dici possunt sollicitationes *laterales*. Deinde in communi Mechanicae scriptorum phraseologia motus vel sollicitatio juxta diagonalem AD ex motibus seu sollicitationibus lateralibus AC, AB componi dicitur. Sed quia video non deesse Autores, qui nescio quae difficultatum spectra in hac re sibi somnient, perperam existimantes per compositionem & resolutionem, additionem vel subductionem motuum vel potentiarum componentium intelligendas esse, ne ejusmodi aequivocatio tyronibus negotium facesseret, motus vel potentias ex aliis componi dici solitas, ex iis nascentes vel resultantes appellare, consultius duxi.



COROLLARIUM I.

41. Producta parallelogrammi diametro in M, ac facta $AM = AD$, sollicitatio AM in hac directione AM reliquis AC & AB aequipollebit. Nam sollicitatio AM alteri aequali & directe contrariae AD aequipollet, atqui AD nascitur ex lateralibus AC & AB; neque plus aut minus praestare potest quam sollicitationes hae laterales simul agentes, proptereaque perinde est sive mobile A unica sollicitatione AD, sive duabus collateralibus AC, AD urgeatur, ergo AM ipsi AD contraria & aequalis lateralibus AC & AB omnino aequipollet.

COROLLARIUM II.

Hinc si tres sollicitationes eidem mobili impressae AM, AB, AC in aequilibrio consistent, erit quaelibet earum ad alterutram ex reliquis, id est AM ad AB, ut sinus anguli BAC a reliquis comprehensi, ad sinum anguli MAC a sollicitatione AM & altera AC ex reliquis formati. Nam (§.42.) $AM = AD$, atque adeo $AM:AB$ sicut AD ad AB, verum ex trigonometria constat, esse AD ad AB, ut sinus anguli ABD ad sinum anguli BCA, ergo etiam AM est ad AB ut sinus anguli ABD, vel ejus supplementi ad duos rectos BAC ad sinum anguli BDA, vel ejus alterni aequalis DAC, aut etiam hujus ad duos rectos supplementi MAC. Et sic externis sollicitationibus, ad aequilibrium compositis, duae quaelibet sunt inter se ut sinus angulorum ipsis oppositorum, hoc est $AB : AC = \sin. MAC : \sin. MAB$, & $AM : AC = \sin. BAC : \sin. MAB$.

Et sic ubique.

Elegans hoc theorema a Cel. Petro Varignon primum, quod sciam, demonstratum est, cui ferme totam suam novam Mechanicam, anno 1687. Lutetiae editam, superstruxit.

COROLLARIUM III.

43. Ex hisce etiam patet ratio theorematis non inelegantis, quod Robervallius Torricellio quondam proposuit, sed sine demonstratione, ut videre est in Opere Academiae Regiae Scientiarum Parisientis, quod inscribitur *Divers Ouvrages de Mathematique & de Physique de Messieurs de l'Academie Royale de sciences*, fol. 302. Theorema vero nostris loquendi phrasibus expressum, est hujusmodi Si per terminos rectarum AM, AB, AC trium solitationum eidem puncto A applicatarum, repraesentatricium, ducantur rectae MB, BC & MC, punctum A, cui solitationes in aequilibrio consistentes impressae sunt, existet in centro gravitatis trianguli MBC. Circa AM, AB & AM, AC descripta sint parallelogramma AMSB, AMRC, quibus descriptis, & quia tres solitationes AM, AB, AC in aequilibrio sunt, secundum hypothesin AC aequipollebit (§.36.) ipsis AM & AB atque adeo (§.42.) $AC = AS$, & hae duae in directum positae erunt, unde CA producta latus trianguli MB secabit in medio Q, similiter AB producta secabit latus trianguli MC in ejus medio P; jam ex staticis constat trianguli MBC centrum gravitatis esse in singulis lineis CQ, BP ex quibusvis trianguli angulis C & B ad puncta Q & P in medio laterum BM ac CM angulis illis oppositorum ductis, propterea centrum gravitatis trianguli BMC, est in communi intersectione A harum linearum CQ & BP.

PROPOSITIO IV. LEMMA.

44. Si singula corpora positione data A, B, C, D &c. in suam quodque distantiam à plano positione dato a Pd ducantur, erunt omnium facto ex aggregato omnium corporum in distantiam Ee centri eorum gravitatis E a dato plano, in casu quo omnia corpora ad eandem planii a Pd partem sita sunt.

Sin vero nonnulla corpora A, B &c. posita fuerint ad unam plani PQ partem, & reliqua C, D ad alteram partem; Excessus, quo facta ex corporibus ad partes centri omnium gravitatis E sitis, in suas homologas a plano PQ distantias, superant facta ex corporibus reliquis in suas ab eodem plano distantias, aequabitur facto ex omnibus corporibus, simul sumtis in distantiam eP, communis omnium centri gravitatis E a plano PQ.

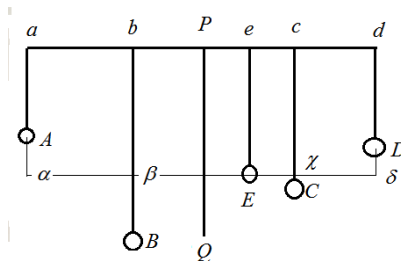


Fig. 5.

Casu primo ostendi debet, quod, positis $A + B + C + D + \&c. = M$, sit $A.Aa + B.Bb + C.Cc + D.Dd = M.Ee$. Et secundo casu, esse $C.cP + D.dP - A.aP - B.bP = M.eP$, designantibus cP, dP, aP, bP distantias corporum C, D, A, B &c. a plano PQ

*Demonstr.*I. Per centrum gravitatis E omnium corporum transeant $\alpha\delta$ parallela plano ae, & eE parallela plano PQ alteri ad recto, & $\alpha\delta$ considerari potest tanquam axis

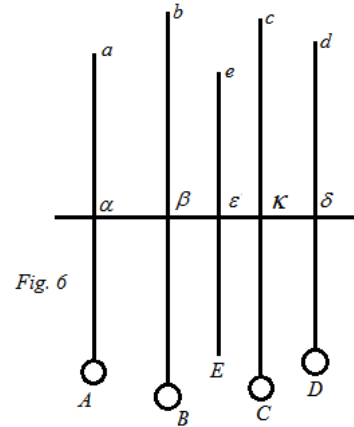
aequilibrii ponderum A, B, C, D, atque adeo erit ob casum aequilibrii
 $A.A\alpha + D.D\delta = B.B\beta + C.C\chi$, vel quod eodem recidit,
 $B.B\beta + C.C\chi - A.A\alpha - D.D\delta = 0$. Item $A.ae + B.be = C.ce + D.de$, vel
 $C.ce + D.de - A.ae - B.be = 0$ respectu axis aequilibrii eE.

II. Quia $Aa = \alpha a - \alpha A = Ee - Aa$, $Bb = Ec + B\beta$, $Cc = Ee + C\chi$, & $Dd = Ee - D\delta$,
 $A.Aa + B.Bb + C.Cc + D.Dd = A.Ee + B.Ee$
 erit
 $+C.Ee + D.Ee + B.B\beta + C.C\chi - A.A\alpha - D.D\delta = M.Ee$,
 quia
 $A + B + C + D = M$, & (num.1 hujus.) $B.B\beta + C.C\chi - A.A\alpha - D.D\delta = 0$.
 Quod est primum.

III. Quia $aP = ae - eP$, $bP = be - eP$, $cP = ce + eP$; & $dP = de + eP$, erit
 $C.cP + D.dP - A.aP - B.bP = C.eP + D.eP + A.eP + B.eP + C.ce + D.de - A.ae - B.be = M.eP$,
 quia (num.2 hujus.) est $C.ce + D.de - A.ae - B.be = 0$, & $C + D + A + B = M$ ut antea.
 Quae omnia erunt demonstranda.

COROLLARIUM I.

45. Si corpora recta ascendant in lineis Aa, Bb, Cc, Dd
 &c. horizontali perpendicularibus, eorumque centrum
 gravitatis E in recta Ee illis parallela, erit
 $A.Aa + B.Bb + C.Cc + D.Dd = M.Ee$.
 Nam ducta ut libet recta $\alpha\delta$ ipsis Aa, Bb
 &c.perpendiculari, eritque (§.44.)
 $A.A\alpha + B.B\beta + C.C\gamma + D.D\delta = M.E\varepsilon$,
 and
 $A.\alpha a + B.\beta b + C.\gamma c + D.\delta d = M.\varepsilon e$.



COROLLARIUM II.

46. Si omnia corpora A, B, C, D &c. sint aequalia, omnes eorum Fig. 5 distantiae a
 plano aPd, erunt multiplum distantiae communis eorum
 centri gravitatis E ab eodem plano secundum ponderum numerum. Excessusque quo
 distantiae illorum a plano PQ secundi casus, quae ad partes centri omnium gravitatis
 posita sunt, excedunt distantias illorum, quae sunt in parte opposita, est distantiae centri
 omnium gravitatis E ab eodem plano PQ multiplum, secundum corporum A, B, C, D
 &c. numerum.

SCHOLION I.

47. Ex hisce nunc tam facile deduci potest regula Guldini jam a Pappo diserte indicata, ut haud abs re futurum existimem, si ean hoc loco demonstratam darem, quanquam ea a pluribus jam demonstrata sit. Guldini Regula ita habet *Figuram ex conversione cujuslibet magnitudinis circa aliquam rectam positione datam oriundam, aequari facta ex Magnitudine genitrice in viam centri ejus gravitatis.* Figura genitrix dicatur F, distantia centri cuius gravitatis ab axe totationis D, ordinata figurae y ad axem totationis, dx elementum axis, solidum ex conversione figurae F circa axem x dicatur S, ejus elementum dS; quibus positis per praesentem propositionem est

$$D.F = \text{summae momentorum elementorum magnitudinis } F = \int \frac{1}{2} y y dx,$$

nam elementum ipsius F est ydx, & hujus momentum $\frac{1}{2} y y dx$. sit p circumferentia circuli cujus radius est I, & ducatur antecedens aequatio in p, ut fiat $pD.F = \int \frac{1}{2} p y y dx$. Jam pD est circumferentia radii D, & py circumferentia radii y, atque adeo $\frac{1}{2} p y y dx$ area circuli ejusdem radii y, & per consequens $\frac{1}{2} p y y dx$ cylindrulus solido S inscriptus, seu ejus elementum dS; ergo $pD.F = \int dS = S$. Quod erat ostendendum pro solidis rotundis; eadem est demonstratio in aliis.

SCHOLION II.

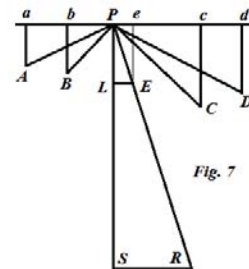
48. Quanquam in hac propositione gravitas in singulis corporibus A, B, C, D eadem seu uniformis sumta sit, ita ut pondera eorum absoluta massis proportionalia sint, quoniam tamen gravitas diversa esse posset, & hoc lemma etiam generaliter verum est, scilicet tunc etiam, cum revera gravitatis sollicitationes ipsis corporum massis A, B, C, D proportionalia non existunt, id saltem indicandum fuit hoc loco. Nam si per A, B, C &c. non amplius, ut in propositione, massae horum corporum, sed pondera eorum absoluta intelligantur, canon propositionis adhuc obtinebit. Verum satius foret, per A, B &c, ut in propositione, intelligere massas horum corporum, & juxta articulum 32, eorum pondera designare per facta ex massis in gravitatis sollicitationes P, Q, R, S quibus singula A, B, C, D respective urgentur, id est per AP, BQ, CR, & DS &c. Quod alibi in applicatione observabimus.

PROPOSITIO V. THEOREMA.

49. *Sollicitationum quarumlibet PA, PB, PC, PD &c. eidem mobili P impressarum media directiio PE est recta jungens centrum gravitatis mobilis P & centrum gravitatis E omnium punctorum A, B, C, D &c. quibus recte sollicitationum repraesentatrices terminantur; & sollicitatio ex omnibus corpori impressis resultans exponi debet multiplo*

rectae PE, secundum punctorum seu sollicitationum impressarum numerum.

Per mobilis P centrum gravitatis ducta intelligantur plana PS & ad sibi invicem recta, & per terminos rectarum PA, PB &c. perpendiculares ad planum *ad* demittantur Aa, Bb, Cc, Dd, & ex centro gravitatis E punctorum A, B, C &c. normalis Ee. Sollicitationum numerus dicatur N, & in producta PE sumatur PR = N, PE. Dico PR exponere sollicitationem ex omnibus mobili P impressis PA, PB, PC, PD &c. nascentem.



I. Per E & R agantur EL & RS plano *aPd* aequidistantes, ob triangulorum PLE & PSR similitudinem, & propter PR = N, PE, erunt PS = N, PL & SR = N, LE.

II. Quia E est centrum gravitatis punctorum A, B, C, D &c. quorum numerus secundum hypothesin est N, erit (§. 46.)

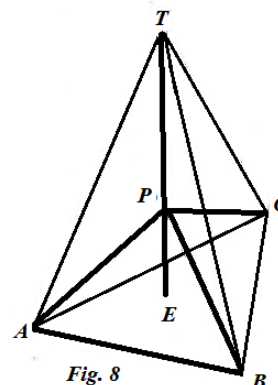
$$aA + bB + cC + dD + \&c. = N.eE, \& Pc + Pd + \&c. - Pa - Pb - \&c. = N.Pe.$$

III. Quia singulis sollicitationibus obliquis (§. 39.) aequipollent sollicitationes earum laterales plano *aPd* perpendiculares & parallelae, scilicet ipsi PA aequipollent laterales *aA*, *Pa*, ipsi PB vero laterales *bB* & *Pb* & sic in caeteris, omnibus obliquis aequipollebunt omnes earum laterales sollicitationes, id est conspirantes *aA*, *bB*, *cC*, *dD*, & oppositae *Pa*, *Pb* ex una & *Pc* ac *Pd* ex altera plani PS parte; atque ex conspirantibus resultat sollicitatio ipsis omnibus (§.38.) aequalis id est sollicitatio $aA + bB + cC + dD + \&c. (n.11.) = N.eE (n.1.) = PS$, & ex omnibus contrariis (§. 38.) nascitur sollicitatio aequalis excessui quo fortiores excedunt debiliores, id est ipsi $Pc + Pd + \&c. - Pa - Pb - \&c. (num.11.) = N.Pe (num.1.) = SR$, ergo omnibus obliquis PA, PB &c. aequipollent duae laterales PS & SR., & (§. 39.) hisce unica PR haec ergo denonat medium sollicitationum PA, PB, PC, PD &c. & sollicitationem ex ipsis nascentem, vel quod idem est, ipsis aequipollentem. Quod erat demonstrandum.

Elegans hoc theorema Illustri Liebnitio acceptum est ferendum, utpote quod in aliqua ejus epistola, jam pridem ad Wallisium data. Sine demonstratione exhibetur. Vid. Tom. III. Oper. Wallisii fol. 687.

COROLLARIUM I.

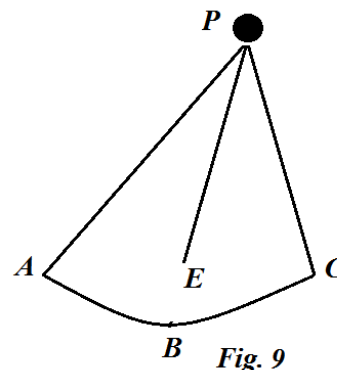
50. Hinc punctum P, cui quatuor sollicitationes PA, PB, PC, PT ad aequilibrium quidem compositae; sed non in eodem plano existens, impressae sunt, erit in centro gravitatis pyramidis TAC, in cujus angulis sollicitationum repraesentatrices lineae terminantur. Sit E centrum gravitatis punctorum A, B, C & jungitur PE, quae media directio erit sollicitationum PA, PB & PC, ac hisce aequipollebit sollicitatio 3.PE; hinc quia omnes in



aequilibrio sunt, erit $PT = 3.PE$; atque adeo est
 $PT : PE = 3 : 1 = A + B + C : T$ considerando
 haec puncta tanquam ponduscula aequalia, ac proinde P
 est in centro gravitatis punctorum quatuor angularium pyramidis A, B, C, T. Sed idem
 est centrum gravitatis pyramidis, quod punctorum ejus angularium. Ergo liquet assertio.
 Hoc est alterum theorema, quod Robervallius Torricellio olim proposuerat in loco
 supra (§.43.) jam indicato.

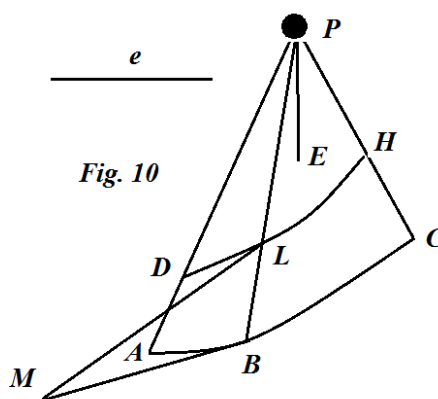
COROLLARIUM II.

52. Hinc etiam, si mobile P ad singula superficie
 curvae ABC puncta urgeatur sollicitationibus expressis
 punctorum superficiei distantis a mobilis centro ; media
 omnium sollicitationum directio erit recta PE, jungens
 centra gravitatis mobilis P & superficiei curvae ABC, ac
 sollicitatio ex omnibus nascens ipsis aequipollens
 est, ut factum ex superficie ABC in distantiam EP centri
 ejus gravitatis a centro corporis P. Nam numerus
 punctorum seu sollicitationum est ipsa superficies ABC,
 ejusque centrum gravitatis est centrum omnium
 punctorum, quibus rectae sollicitationum repraesentatrices terminantur.



COROLLARIUM III.

52. Quinimo si idem mobile P, ad singula
 curvae ABC puncta urgeatur, sed sollicitationibus
 non ipsis PB distantis punctorum a mobili, sed
 ipsis LP in singulis BP sumtis, & in curva
 quacunque DLH terminatis, proportionalis. Erit
 etiam nunc omnium sollicitationum PL media
 directio PE recta jungens centra mobilis P &
 curvae DLH, ac sollicitatio ex illis omnibus
 resultans, factum ex gravitate curvae DLH in
 rectam EP; sed hujus curvae partes non
 uniformiter graves intelligendae, verum
 difformiter ; ita ut ductis per homologa utriusque
 curvae AB & DLH puncta B, L tangentibus BM &
 LM in puncto M concurrentibus, gravitas puncti B curvae ABC uniformiter gravis, sit
 ubique ad homologi puncti L curvae DLH, gravitatem ut rectangulum MBP ad
 rectangulum MLP.



PROPOSITIO VI. PROBLEMA.

53. *Invenire mediam directionem sollicitationum quarumvis AG, BG, CG, DG quibus puncta A, B, C, D lineae rectae inflexilis AD urgentur.*

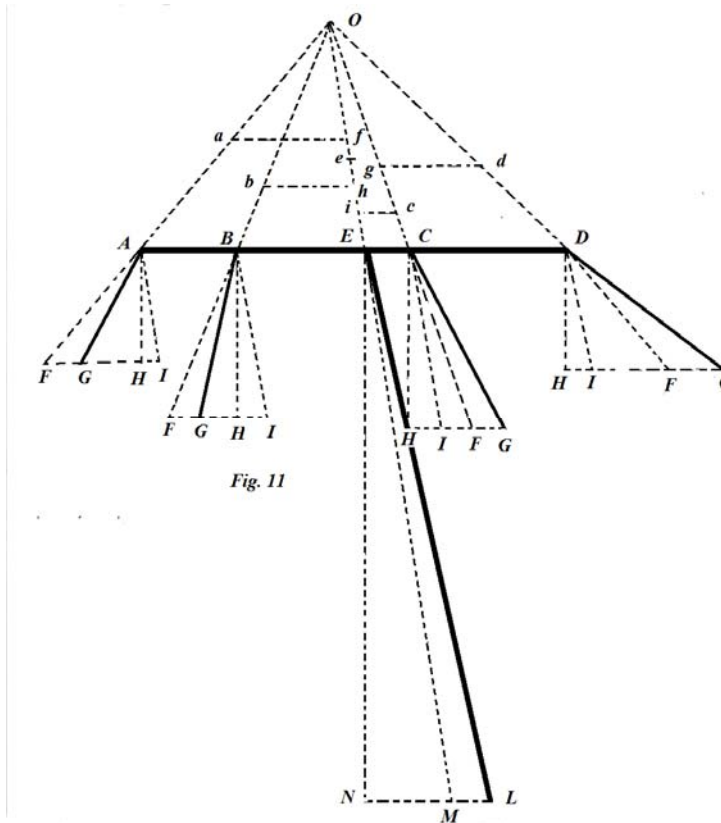


Fig. 11

Ex quolibet extra lineam AD puncto O agantur per singula linea; puncta A, B, C, D in quibus linea juxta directiones AG, BG &c. urgetur, rectae indefinitae OAF, OBF, OCF, ODF, ductisque per singula puncta G in quibus lineae sollicitationum aut potentiarum applicatarum repraesentatrices terminatur rectis indefinitis FG lineae AD parallelis & lineis OA, OB, OC, OD &c. quantum opus est productis in punctis F occurrentibus. Dehinc singulis AF, BF, CF, DF &c. in suis directionibus transferantur, a puncto concursus O, respective aequales Oa, Ob, Oc, Od &c punctorumque a, b, c, d centrum gravitatis e inveniatur atque in linea OM per hoc centrum e, & punctum O ducta lineaeque AD occurrente in E, sumatur EM, quae sit ad Oe ut numerus punctorum a, b, c, d ad unitatem; ductaque per punctum M recta NL aequidistante ipsi AD, assumatur in ea ML omnibus FG inter AF, BG, & BF, BG &c. interceptis, aequalis, ∞ casu, quo omnes GF conspirantes sunt, vel juxta ordinem literatum ex F, versus G sumtae omnes ad eandem plagam vergunt, tunc enim ML in eandem plagam & omnibus FG aequalis sumenda est; sin vero nonnullae FG vergerent ad unam plagam, reliquae vero FG in plagam oppositam, eo casu ML aequalis facienda esset excessui quo fortiores

FG simul sumtae, excedunt reliquas FG in plagam priori oppositam agentes eamque in partem ducenda esset in quam agunt fortiores FG Linea EL jungens puncta E, L dabit mediam directionem omnium sollicitationum AG, BG, CG, DG &c. ejusque longitudo EL exponet sollicitationem omnibus applicatis aequipollentium in hac media directione EL, seu etiam onus quod sustinaculum V puncto E subjectum ab iisdem applicatis potentiis AG, BG, CG &c. sustineret. Quod erat inveniendum.

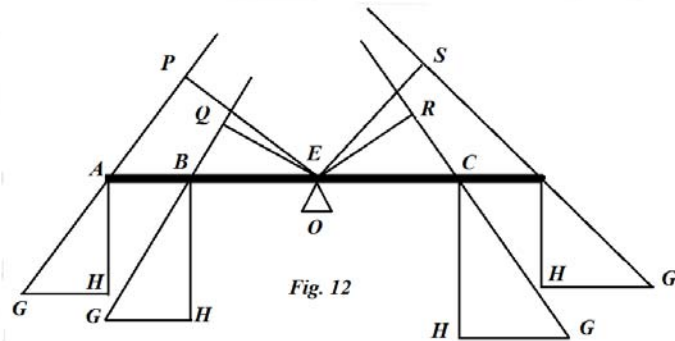
Demonstratio. Quaelibet sollicitatio vel potentia applicata AG aequipollet (§. 39) lateralibus AF & FG; atque adeo omnes AG, BG, CG &c. aequipollebunt omnibus AF, BF, CF, DF secundum hypothesin in puncto O convergentibus, omnibusque harum collateralibus FG. Cogitetur jam virga seu linea AD, cui potentiae applicatae vel sollicitationes AG, BG &c. impressae sunt, plano gravitatis experti, perinde ac ipsa linea, infixae esse, ita ut linea moveri nequeat, quin planum simul moveatur hoc enim modo fiet ut potentia AF eandem vim habeat movendi planum, in quocunque directionis OA puncto ipsi plano applicata fit, sive in puncto A, aut in quovis alio *a*, O &c. hoc per se satis evidens est, nam si potentia AF rectae AD in A applicata in directione OA majorem vim haberet ad movendum planum AODF quam haec eadem potentia AF ac in eadem directione OA sed plani AOD puncto O applicata, sequeretur potentiam AF cum alia ipsi aequali, puncto O applicata in eadem directione, sed in partem oppositam ex *a* versus O, vel potius in linea AO extrorsum trahent punctum plani O, non esse in aequilibrio, sed ipsi praevalere, aut hanc illa fortiolem existere; atque adeo duae potentiae aequales AF & aO juxta eandem lineam AO in oppositas partes agentes se mutuo in aequilibrio non tenerent, quod est absurdum. Ergo potentia AF rectae AD applicata eundem directionem essectum praestat ac potentia Oa ipsi AF aequalis & in eadem directione OA puncto plani O applicata, quod de reliquis BF, CF &c. etiam intelligendum ac proinde potentiarum AF, BF, CF, DF eadem media directio, quae potentiarum hisce aequalium Oa, Ob, Oc, & Od &c. atqui harum media directio OeM (§. 49.) est linea jungens punctum O & centrum gravitatis *e* punctorum *a*, *b*, *c*, *d* &c. & potentia omnibus applicitis puncto O, aut rectae in punctis A, B, C, D &c. aequipollens est multipla ipsius Oe secundum punctorum *a*, *b*, *c*, *d* &c. numerum, atque adeo exponitur recta EM, quae (constr.) est ad Oe ut numerus punctorum *a*, *b*, *c* &c. ad unitatem. Aequipollet ergo potentia EM omnibus AF, BF, LF & DF, ex omnibus vero FG lineae AD parallelis nascitur sollicitatio ipsis omnibus aequalis, si conspirantes fuerint, vel aequalis excessui, quo fortiores FG superant debiliores & oppositas FG, ac proinde utroque casu exponitur recta ML quae pro varietate circumstantiarum (constr.) aequalis est summae omnium FG eandem plagam respicientium, vel differentiae earum quae in diversas plagas tendunt, unde cum omnibus AF, BF, CF, DF & omnibus FG aequipolleant duae laterales EM & ML & hisce duabus (§. 39) unica EL, omnino patet hanc EL etiam aequipollere omnibus AG, BG, CG, & DG, quae lineae AD applicatae sunt. Propterea EL significat mediam directionem potentiarum applicatarum & sollicitationem in hac media directione ipsis omnibus aequipollentem. Quod erat demonstrandum.

COROLLARIUM I.

54. Si lineae AD perpendiculares AH, BH, CH, DH agantur ipsis FG occurrentes in punctis H, hae perpendiculares totidem potentias lineae AD applicatas repraesentabunt, momentaque omnium earum, quae sunt ex una parte puncti E, aequabuntur momentis earum, quae sunt ex altera parte, id est, $AH.AE + BH.BE = CH.CE + DH.DE$. Ductis enim af, bh, dg, ci rectae AD & AI, BI, CI, DI &c. rectae OM parallelis, eritque propter centrum gravitatis e , $af + bh = dg + ci$, vel adscita communi altitudine OE erunt facta seu rectangula $OE.af + OE.bh = OE.dg + OE.ci$. Vel
 $Of.AE + Oh.BE = Og.DE + Oi.CE$, quia $OE.af = Of.AE$ propter triangulorum OAE & Oaf, item $OE.bh = Oh.BE$, ob triangulorum OBE & OBh similitudinem, & quia ob similes rationes rectangula reliqua $OE.dg, OE.ci$ rectangulis; $Og.DE, Oi.CE$ respective aequantur. Porro ob AI, BI, CI &c. parallae (constr.) rectae OM, & singulas Oa, Ob, Oc, Od aequales singulis AF, BF, CF, DF, aequabuntur etiam Of, Oh, Oi, Og ipsis AI, BI, CI, DI, quia quodvis triangulum Oaf respectivo AFI simile & aequale est, propterea substitutis in rectangulis aequalibus $Of.AE + Oh.BE = Og.DE + Oi.CE$, loco rectarum Of, Ob, Og & Oi , aequalibus AE, BI, DE ac CI eodem ordine sumtis, fiet
 $AI.AE + BI.BE = DI.DE + CI.CE$. Vel substitutis adhuc loco ipsarum AI, BI, CI, DI, aliis rectis AH, BH, CH, & DH, quae ipsis propter triangula similia AHI, BHI, CHI, DHI analogae sunt, hoc est, proportionalia, erunt tandem
 $AH.AE + BH.BE = DH.DE + CH.CE$. Porro sicut
 $EM = Of + Oh + Og + Oi = AI + BI + DI + CI$, ita etiam EN rectae ML perpendicularis, erit $= AH + BH + DH + CH$.

COROLLARIUM II.

55. Ex centro aequilibrum E demissis ad directiones potentiarum AG, BG, CG, DG productas perpendicularibus EP, EQ, ER & ES, momenta potentiarum obliquarum ex una parte, aequalia erunt momentis potentiarum in parte altera. Hoc est,



$AG.EP + BG.EQ = DG.ES + CG.ER$. Nam quia quaelibet triangulorum PAE, & HAG paria similia sunt, fient rectangula AH.AE, BH.BE, DH.DE ac CH.CE. aequalia homologue rectangulis AG.EP, BG.EQ, DG.ES & CG.ER., unde quia (§.54) $AH.AE + BH.BE = DH.DE + CH.CE$, erit etiam $AG.EP + BG.EQ = DG.ES + CG.ER$.

64
SCHOLION.

56. Casus Corollarii hujus secundi obtinet non solum tunc, cum linea AD est recta, cui potentiae obliquae AG, BG &c. applicantur ; sed etiam in casu, quo ipsa linea applicatas potentias habens est curva, imo etiam in rotis aliisque ejusmodi organis. Uno verbo, si circa aliquod punctum potentiae aut sollicitationes quaecunque in equilibrio sunt, momenta potentiarum, quae agunt in unam partem, aequalia sunt momentis potentiarum, quae agunt in partem oppositam; atque sic inopinato incidimus in demonstrationem directam & immediatam principi Archimedei de aequalitate momentorum, in casu aequilibrii potentiarum inter se commensurarum, quod varii varie demonstrare conati sunt. Caeterum ope Corollarii primi hujus, ipsum problema hujus propositionis multo simplicius jam solvetur, imo plura alia praestabuntur quam antea, ut ex sequentibus facile patescet.

PROPOSITIO VII. THEOREMA.

57. Si lineae inflexili & gravitatis experti (qualis etiam in praecedenti Propositione est ponenda) AD quotcunque sollicitationes seu potentiae obliquae A2A, B2B, C2C, D2D applicatae, atque hae potentiae omnes in suas laterales potentias lineae AD perpendiculares & parallelas resolutae fuerint, erit, ductis per centrum aequilibrii H potentiarum lineae AD perpendicularium A2A, B2B, C2C, D2D, recta HI ipsis omnibus simul sumtis aequalis & parallela, & per punctum I recta IK lineae AD parallela ac aequali excessui, quo potentiae lineae aequidistantes, quae sunt ad unam lineae HI partem ut D2D, superant eas, quae ipsi in parte contraria oppositae sunt 2A2A, 2B2B, 2C2C, recta jungens puncta H, K erit media directio potentiarum obliquarum virgae seu lineae AD applicatarum, ejusque magnitudo exponet onus, quo hypomochlion, seu sustinaculum puncto virgae H subjectum, juxta mediam directionem HK, ab omnibus potentiis urgetur.

Cum quaelibet sollicitatio seu potentia obliqua A2A (§.39.) spectari possit, tanquam nascens ex suis lateralibus sollicitationibus A2A & 2A2A virgae AD perpendiculari & parallela, omnes potentiae obliquae considerari possunt tanquam nascentes ex omnibus perpendicularibus A2A, B2B, C2C, D2D, & ex omnibus parallelis D2D, 2C2C, 2B2B, & 2A2A. Atqui (§. 54) ex omnibus perpendicularibus resultat sollicitatio vel potentia ipsis omnibus simul aequalis & parallela, transiens per earum centrum aequilibrii H, id est (const.) sollicitatio $HI = A2A + B2B + C2C + D2D$, ex parallelis vero nascitur sollicitatio aequalis (§. 38.) excessui, quo fortior vel fortiores excedunt debiliores ipsis

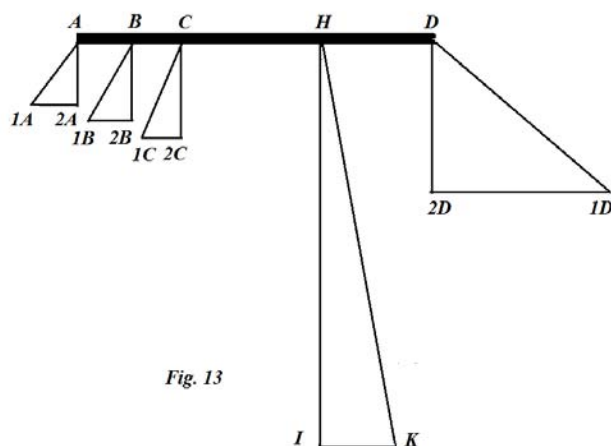


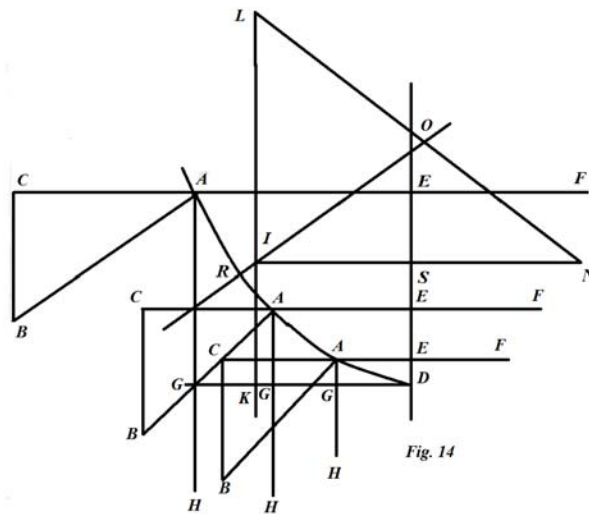
Fig. 13

contrarias, unde cum (constr.) sit $IK = 2D1D - 2A1A - 2B1B - 2C1C$, haec IK exponet sollicitationem ex sollicitationibus virgae parallelis nascentem; adeoque cum ex omnibus potentiis lateralibus, virgae perpendicularibus & parallelis nasci intelligantur, tum omnes obliquae A2A, B2B&c. tum etiam duae laterales H1 & IK & ex hisce (§.39) unica HK, patet hanc sollicitationem HK omnibus obliquis aequipollere ejusque directionem harum esse Mediam directionem, atque ideo punctum, virgae H in hac media directione urgeri sollicitatione seu onere, quod exponitur magnitudine rectae HK. Quod erat demonstrandum.

SCHOLION.

58. Eadem methodo, mutatas mutandis, obtinetur media directio potentiarum lineae curvae DAA utlibet applicatarum. Nam si potentiae vel sollicitationes applicatae repraesententur per lineas AB, quarum unaquaeque resolvatur in suas laterales potentias AC, axi curvae DE perpendiculares, & CB eidem aequidistantes, productisque CA ultra axem DE, ut singulae EF singulis AC aequales fiant, potentia IN singulis EF simul sumtis aequalis & per earum centrum aequilibrii ducta axique perpendicularis (§.54) erit potentia seu sollicitatio omnibus EF aequipollens, unde cum singulae hae EF singulis AC aequales atque directae oppositae sint, omnes EF cum omnibus AC in

Fig. 14



aequilibrio manebunt, ergo etiam unica potentia IN universis EF aequipollens cum iisdem AC in aequilibrio consistet; pariter ductis DG axi DE normali, & rectis indefinitis AH eidem axi parallelis, & in iis factis GH singulis CB respective aequalibus, erit potentia IL universis GH aequalis & parallela transiens per centrum aequilibrum K harum GH, cum iisdem GH (§. 54.) in aequilibrio, ergo duae potentiae laterales IN & IL cum universis AC & univesis CB in directionibus AG agentibus, id est, universis GH in aequilibrio consistent, atque adeo cum omnibus obliquis, at vero ex potentiis IN & IL (§.49.) nascitur potentia, cujus directio IO transit per punctum I & centrum gravitatis O punctorum L & N, & quae exponitur per multipulum rectae IO secundum punctorum L & N numerum, hoc est per $2 \cdot IO$; ergo talis potentia sola in aequilibrio erit cum universis AB curvae DAA applicans, idcirco (§37.) recta OI producta in M, dat mediam directionem IM omnium potentialium AB, & hypomochlium R curvae DAA applicatum sustinebit onus aequale $2 \cdot OI$ ab omnibus applicatis potentiis AB. Quod erat inveniendum. Caeterum etsi punctum I, quod est communis intersectio rectarum KL & SN in curva DA esse videtur, in eam tamen incidere non debet, ac propterea puncta I & R sunt semper diversa.

PROPOSITIO VII. THEOREMA.

Sic Bb elementum curvae AB, per cujus terminum b ducantur bd rectae BD & bf lineae BF parallelae, tum etiam per terminum G potentiae BG recta GH axi parallela, quae ordinatae CB productae occurrat in H, & denique demittatur ba perpendicularis ad GT, quibus positis,



sumti etiam recto aequuntur) aequales, adeo ut ablato communi VBS remaneant aequales bBa & VSB , & propter angulos ad a & V rectos, similia sunt; habebimus insuper BV $BS = ba$ Bb , triacula vero similia $bB\beta$ & BCS praebent $BS : BC = Bb : B\beta$, igitur denuo ex aequo $BV : BC = ba : B\beta$. Sed initio hujus inveniebatur etiam $BV : BC = CD : BH$, ergo $CD : BH = ba : B\beta$, adeoque $BH.ba = CD.B\beta = \text{rectangulo } C\delta \text{ areae } AXDC$ inscripto.

II. Quonium (constr.)

$EF : BG = BV.CT : SC.BT$, & $BG : GH = BT : CT = SC.BT : SC.CT$, erit ex aequo $EF : GH = BV.CT : SC.CT = BV : SC$. Praeterea, ut num. 2. ostensum, est etiam $BV : BS = ba : Bb$, & $BS : SC = Bb : b\beta$, ergo iterum ex aequo $BV : SC = ba : b\beta$, at modo habuimus etiam $BV : SC = EF : GH$, ergo $EF : HG = ba : b\beta$, atque adeo $HG.ba = EF.b\beta = \text{rectangulo } E\phi \text{ areae } AXFE$ inscripto.

III. jam quia potentia BG curvae puncto B applicata (§. 39.) resultat ex lateralibus BH axi AT perpendiculari, & HG eidem parallela, & singula puncta elementi Bb ab eadem potentia obliqua BG urgentur, potentia, qua totum elementum afficitur, est ut rectangulum $BG.ba$ cujus altitudo ba multitudinem potentiarum obliquarum omnibus elementi Bb punctis applicatarum exponit, rectangulum $BH.ba$, exponet potentiam axi perpendiculararem, & $HG.ba$ potentiam eidem axi parallelam, quae ex obliqua $BG.ba$ ad elementum curvae Bb pertinente derivantur, & ex quibus haec. (§.40.) resultat. Unde, quia (num.I. & II.) ostensum $BH.ba = \text{rec} - \text{lo } C\delta$ & $HG.ba = \text{rec} - \text{lo } E\phi$, omnes potentiae $BG.ba$ resultabunt ex omnibus $C\delta$ areae $AXDC$ inscriptis, & ex omnibus $E\phi$ areae $AXFE$ inscriptis, cum singulae $BG.ba$ ex singulis $C\delta$; & $E\phi$ nascentur. Atqui ex omnibus potentiis $C\delta$ (§. 54.) resultat potentia $A.MR$ cum hoc rec-lum (constr.) aequale sit omnibus $C\delta$, seu areae $AXDC$, cui inscripta sunt, & potentia $A.MR$. directio transeat per centrum gravitatis areae $AXDC$; & ex omnibus potentiis $E\phi$ resultat potentia $A.OM$, quoniam (constr.) hoc rec-lum $A.MO = AXFE$ ejusque directio OQ transit per centrum gravitatis Q ejusdem areae $AXFE$. Propterea potentiae $A.MO$ & $A.MR$. in directionibus suis MO , MR in aequilibrio consistent cum omnibus potentiis obliquis BG , BG , &c curvae AB in singulis punctis applicatis. Sed ex potentiis $A.MO$ & $A.MR$. resultat (§.49.) potentia $2.MI.A$ cujus directio MI transit per centrum gravitatis I punctorum O & R , quorum gravitas fingitur esse A . Ergo etiam haec potentia $2.MI.A$ in aequilibrio erit cum omnibus BG , &c., atque adeo ejus directio MI in contrarium partem P producta MP (§.37.) erit media directio omnium potentiarum BG , BG &c. & $2.MI.A$ exponet onus quod hypomochlium Z in media directione MP a potentiis rectis sustinebit. Quod erat demonstrandum. In figura male confunduntur puncta Z & M , quorum hoc non in curva AB , sed intra figuram ABC situm esse debet.

COROLLARIUM I.

60. Si omnes potentiae BG curvae AB perpendiculares sint, evanescent anguli TBS , eritque ubique $CD = EF = BG$. Nam coincidentibus BF & BS , rectae BV & BT aequales fient ipsi BS , & $CT = CS$; unde cum (constr.) sit $CD : BG = BV : BT$ fiet omnino $CD = BG$. Sic etiam quia $EF : BG = BV.CT : SC.BT$, & hoc

casu $BV = BS$, $CT = CS$, & $BT = BS$, ratio $BV.CT$ ad $SC.BT$ erit ratio aequalitatis, atque adeo $EF = BG$.

COROLLARIUM II.

61. Et si praeterea omnes BG inter se aequales fuerint, figurae mixtilineae $AXCD$ & $AXFE$ mutabuntur in parallelogramma rectangulia CAX & EAX .

COROLLARIUM III.

62. Unde si singulis curvae AOM punctis potentiae perpendiculares curvae, & omnes inter se aequales BG applicatae sint; vis totalis qua curva AOM urgetur, eadem erit ac vis totalis, qua urgeretur curvae basis AM , si etiam singulis ejus punctis potentiae aequales ipsis BG , & basi AM etiam perpendiculares applicatae essent, atque adeo exponetur rectangulo $MAXZ$. Nam descriptis circa curvam rectangulo $ME.EA$, cujus latus EE basi aequalis tangat curvam in O , & ad latera ME , AE rectangulis $MXFE$, $AXFE$, consideretur basis MA instar axis curvae AOM , & quoniam secundum hypothesin omnes potentiae BG curvae normales sunt atque inter se aequales, erunt (§.61.) $ef = CD = BG$ & hisce aequales existent homologae lineae respectu cujusvis alius curvae puncti B ; adeoque omnes potentiae curvae parti ABO normales, aequipollebunt duabus potentiis, quarum una axi perpendicularis exponitur recto $AXQP$, alteraque axi parallela exponatur rectangulo $AXFE$. Sic etiam omnes curvae OBM potentiae aequipollent duabus, quarum una axi perpendicularis alteraque eidem parallela, & quarum haec rec-to $MXFE$, illa vero rec-to $PQZM$. Atqui ob aequalia. rec-la $MEFX$ & $AEFX$, potentiae axi AM parallelae & directe oppositae pertinentes ad curvae partes ABO , MBO , utpote inter se aequales, se mutua elidunt, solaeque potentiae axi perpendiculares considerandae veniunt; illae scilicet, quae exprimuntur rec-lis $XAPQ$, & $ZMPQ$; ergo vis, qua tota curva AOM ab omnibus ipsi applicatis potentiis BG urgetur, exponitur rectangulo $AXZM$. Sed eandem vim basis AM quoque subibit, sicubi in singulis suis punctis ejusmodi aequalibus potentiis perpendiculariter premitur vel trahitur. Potentia enim, qua tota basis afficitur, est, ut haec basis ducta in potentiam, qua singula ejus puncta urgentur, id est ut rectangulum $AMXZ$.

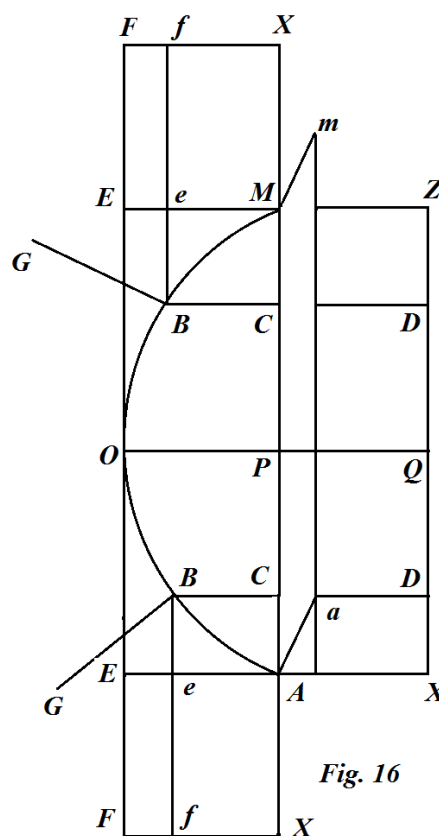


Fig. 16

COROLLARIUM IV.

63. Pariter, si loco curvae AOM intelligatur superficies cylindri recti, cuius altitudo fit Mm ; ac superficiei isti in singulis punctis potentiae perpendiculares & aequales BG applicatae cogitentur; tunc potentia, qua cylindrica superficies ab omnibus ipsi applicatis potentiis urgetur, sive id fiat pressione ejus contra planum mM , Aa , sive tractione ab eodem plano, exponetur per parallelepipedum baseos AZ altitudinisque Mm ; adeoque aequatur potentiae, qua afficeretur planum $mMAa$, si pariter singula ejus puncta aequalium potentiarum impressiones subirent. Nam insuperficie cylindrica tot curvae intelligi possunt ipsi AOM parallelae similes & aequales, quot puncta sunt in altitudine cylindri vel prismatis Mm , quarum quaelibet urgetur a potentia exprimenda rec-lo aequali AZ . Adeoque vis, qua omnes curvae, id est cylindrica superficies, urgentur, exponetur per omnia rec-la ipsi AZ aequalia, quae in parallelepipedo continentur, id est per parallelepipedum ipsum, sub basi AZ , & altitudine Mm . Et tanta potentia etiam urgetur rec-lum $MmaA$ a potentiis aequalibus, singulis ejus punctis applicatis.

DEFINITIONES.

IV.

64. *Solidum patiens* dicitur omne solidum $ABCD$, cujus singula superficiei curvae $CBAD$ puncta a potentis quibuslibet superficiei perpendicularibus, BM , bn &c. urgentur, sed ita tamen, ut singula puncta curvae cbd , sectionis solidis basi parallelae, eadem potentia bn vel huic aequalibus; afficiatur. Potentiarum vero *actio* in solidum patiens generali nomine *Pressionis* deinceps insignietur.

V.

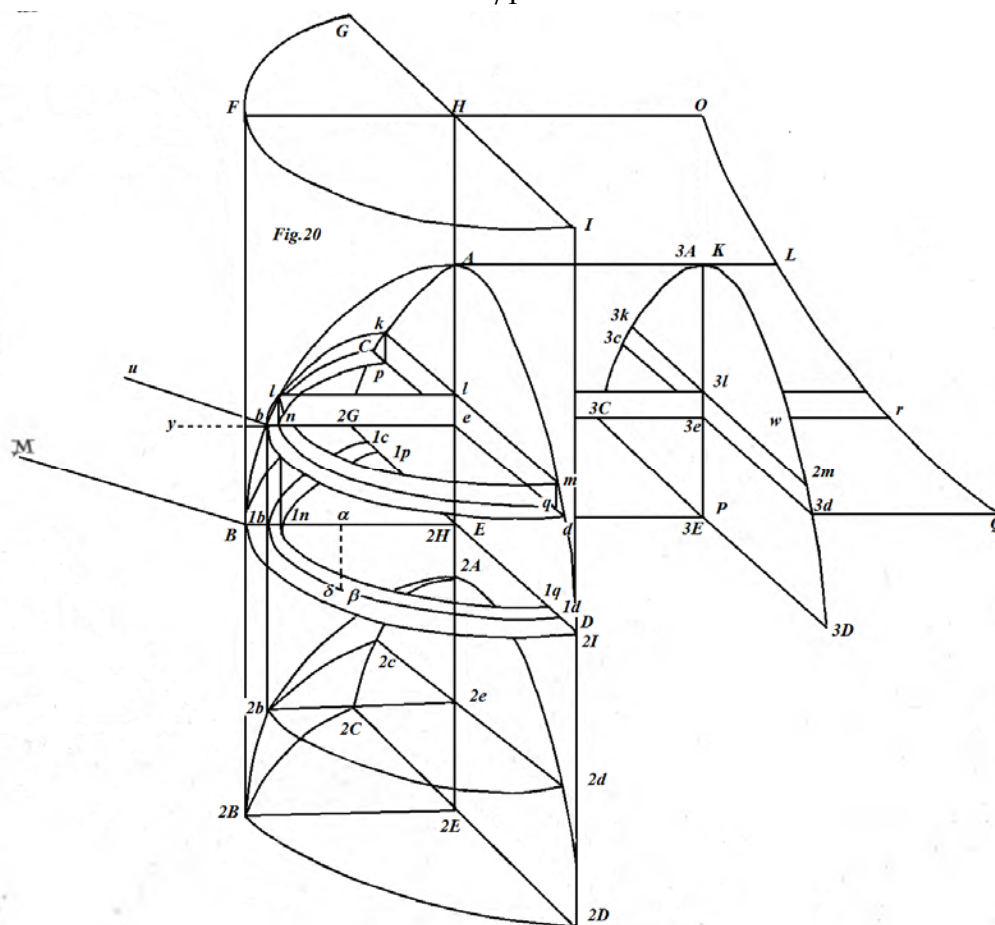
65. *Pressio interna* est, cum potentiae BM , bu , &c. cavae superficiei punctis B , b &c. applicatae, eam extrorsum juxta BM , bu premunt.

VI.

66 *Pressio externa* vero, cum convexae superficiei puncta B , b &c. a potentiis ipsis applicatis introrsum premuntur, juxta MB , nb , &c.

Ex ordine igitur literarum, quibus lineae potentiarum applicatarum repraesentatrices signantur, statim dignosci poterit utrum applicatae potentiae introrsum, an vero extrorsum premant. Nam si litera vel signum, quo punctum superficiei signatur, cui potentia applicata est, primum locum tenet, pressio est interna, externa vero, si secundum locum. Sic BM , bu &c. denotant pressiones internas, quia puncta





applicationis B, b & c . primum locum tenent, & MB, ub , & c . significant pressiones
externas.

VII.

67. Per *Basin solidi patientis* intelligo quodvis ejus planum vel sectionem planam
CBD horizonti aequidistantem.

VIII.

68. *Sectio vero recta solidi patientis* nobis est figura plana CAD solidi nostri, basis ejus
recta, cujus sectionis recta, cujus sectionis rectae vertex est punctum curvae CAD a sua
basi CD maxime distans, & linea AE ex vertice A ad CD perpendiculariter ducta dicetur
axis sectionis, & forte etiam ipsius solidi patientis.

72

IX.

69. *Scala potentiarum* solido patienti applicararum, est figura mixtilinea K_LrQP in plano sectioni rectae solidi patienis recto, atque per axem sectionis rectae AE, transeunti, cujus figurae ordinata quaelibet 3_{er} ejus axi KP, qui ipsi AE aequidistans ponitur, perpendicularis, exponit potentiam, qua unumquodque punctum curvae *cbd* sectionis *cbdc* basi CBDC parallelae, afficitur. *Origo* seu *vertex* hujus curvae O est punctum occursus curvae QrL cum suo axe KP.

X.

70. *Planum sublime* est GFI, quod basi patientis solidi aequidistans, per originem O scalae potentiarum transit.

XI.

72. *Planum humile* est, quod sublimi parallelum, per maximam ordinatam PQ, scalea potentiarum transit.

XII.

72. *Prisma redundans* est solidum GFBADIG, quod componitur Fig.19, ex solido patiente CBAD vertice ejus deorsum respiciente, & ex prismatico recto GFBDI super basi CBD & terminato ad planum sublime GFI.

XIII.

73. *Prisma deficiens* est solidum GFBADIG, quod remanet detracto Fig. 20 scilicet solido patiente CBAD, vertice ejus A nunc sursum converso ex prismatico recto GFBDI super basi CBDE, & terminato itidem ad planum sublime GFI.

XIV.

74. *Solidum patienti analogum* juxta scalam potentiarum est solidum Fig.19, 20 ; dum 2C2B2A2D, cujus quaelibet sectio 2c2b2d homologae sectioni *cbd* in solido patienti similis, aequalis & parallela intervallo 2H2e aequali respectivae ordinatae 3_{er} scalae potentiarum a plano humili 2G2F2I distat, dum homologa solidi ipsius patientis sectio *cbd*, intervallo He aequali abscissae O3e ad ordinatam 3_{er} pertinenti, distat.

XV.

75. *Solidum prismati redundanti analogum* est solidum, Fig.29, 2G2F2B2A2D2I, quod componitur ex solido patienti analogo 2C2B2A2D & ex Prismatico recto 2G2F2B2D2I, super basi 2G2B2D, utque terminato ad planum humile 2G2F2I.

XVI.

76. *Solidum prismati deficienti analogum* est solidum 2G2F2B2A Fig. 20. 2D2I, quod relinquitur ablato solido 2C2B2A2D patienti analogo ex prismati recto 2G2F2B2D2I ad planum humile 2G2F2I etiam terminato

XVII.

77. Fig. 19, 20. *Pseudo-cunius* sectionis rectae solidi patientis est solidum, cujus sectiones, plano humili vel sublimi parallelae, sunt rectangula, quorum bases sunt ordinatae figurae cujusdun similis & aequalis sectioni rectae solidi patientis; altitudines vero ordinatis modo nominatis respondentes ordinatae Scalae Potentiarum.

Circa axem KP sit figura 3C3A3D similis & aequalis sectioni rectae CAD solidi patientis, cujus ordinata quaelibet 3c3d proinde aequabitur ordinatae cd transienti per pundum e, quod respondet puncto 3c, per quod ordinata 3c3d ducta est, quae ordinata in scala potentiarum respondentem habet 3er. Pseudo-cuneus sectionis rectae CAD est solidum, cujus singulae sectiones plano GFI parallelae sunt parallelogramma rectangula, quorum bases sunt ordinatae 3C3d & altitudines ordinatae 3er scalae potentiarum, in eodem plano secante positarum, id est rectangula 3c3d.3er. &c.

Hi Pseudo-cunei hac etiam ratione generari intelliguntur super figuris 3C3A3D & PLQ, tanquam basibus, erecta intelligantur prismata recta, quae, ob bases suas ad angulos rectos sibi invicem occurrentes, se mutuo perforabunt. Solidum illud, quod a prismatis ad angulos rectos transversim se invicem perforantibus intercipitur, est Pseudo-cuneus, de quo agitur.

Si scala potentiarum est triangulum vel trapezium, tunc ex Pseudo-cuneo sit verus Cuneus.

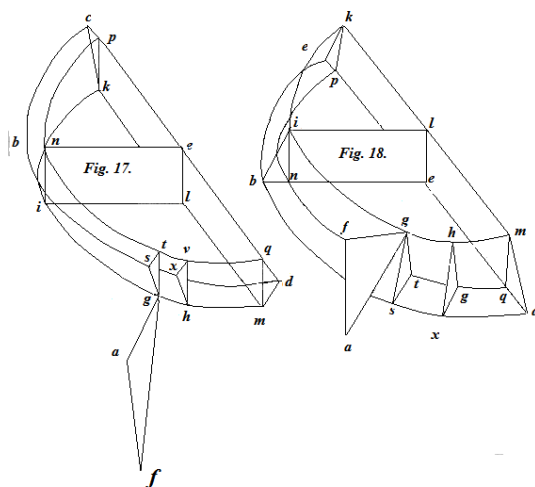
XVIII.

78. *Projectio* cujusque figurae in plano quodam, quod ideo etiam *projectionis planum* audit, est figura, quae in plano projectionis formatur, demittendo ex singulis figurae projectae punctis perpendiculares ad planum projectionis. Sic curvae *kim* projectio in plano projectionis 2G2F2I est curva 1p1n1q, quae a perpendiculis ex singulis punctis curvae *kim* ad planum 2G2F2I demissis, formatur.

PROPOSITIO IX. THEOREMA.

79. Fig. 17, 18. *Si in singulis punctis conicae superficiei kcbimd, duobus planis cbd & kim, ad distantiam indefinite parvam el vel ni aut pk a se invicem distantibus, interjectae, atque lineolis ck & dm terminatae, potentia fg ad angulos rectos applicata fuerit, pressio, quam universa superficies conica subibit, aequipollebit pressionibus, quos armilla pbq duobus curvis cbd & pnq (quae alterius kim projectio est in plano cbd) intercepta, & lineolis cp, dq terminata; & cylindrica superficies knm ad basin kigm & ad altitudinem el vel ni erectam terminatamque lineolis pk & qm, paterentur ab eadem*

potentia gf , in singulis armillae & ylindricae superficiei punctis perpediculariter



applicata.

Sumatur in curvae exteriori cbd quilibet arcus infinitesimus sx & per puncta s, x curvae perpendiculares ductae st, xy intelligantur curvae interiori pnq occurrentes in punctis t, y . Porro solidum $cbmd$ sectum cogitetur duobus planis per lineolas st, xy ductas & plano cbd rectis, eruntque solidi $cbmd$, & planorum secantium communes sectiones triangula rectangula sig, xyh ac solidum hisce triangulis interjectum txg prisma erit. Quibus intellectis, pressiones, quas subibunt facies $tghy$ & $styx$ (quae considerandae sunt, tanquam continuam superficiem in ty incurvatam $stghyxs$ basin $sghx$ habentem) aequipollent (§. 63.) pressioni, qua ab eadem potentia, gf afficietur basis $sghx$. Et cum hoc accadat, respectu cujuslibet alius $sghx$ insuperficie conica kbm , & homologarum areolarum $styx$ in armilla, & $tghy$ insuperficie ylindrica; perspicuum est, pressiones quas omnes areolae $sghx$ insuperficie conica subeunt, aequipollere pressionibus, quas sustinent omnes areolae $tsxy$, quae sunt in armilla, & quas sustinent omnes $tghy$, quae in cylindrica superficiei knm continentur. Ac propterea pressio, quam subit zona conica kbm ab omnibus ipsi perpendiculariter applicatis potentiis aequalibus gf , aequipollet pressionibus, quas ab omnibus perpendiculariter applicatis potentiis ipsis gf aequalibus patientur armilla pbq & superficies cylindrica knm Quod erat demonstrandum.

PROPOSITIO X. THEOREMA.

80. Si singulis punctis superficiei convexae aut cavae $CBAD$, qua Fig.19, 20, solidum patiens $CBADC$ ex parte terminatur, potentiae inaequales quidem & superficiei perpendiculares MB , ubi &c. vel BM , ubi &c. applicatae sint, sed quarum aea saltem omnes, quae singulis punctis cujusque sectiones bcd basi CBD parallelae applicantur, aequales sint & exponantur per homologam ordinatam. 3er scalae potentiarum ; Pressio, quam superficies curvae solidi patientis ab omnibus potentiis patietur, aequivalebit pressioni, qua urgetetur basis solidi patientis CBD juxta directionem basi perpendicularem $\alpha\beta$ per centrum gravitatis a solidi $2G2F2B2A2D2I$ quod prismati

redundanti (deficienti) GFBADI analogum est, ductam, quam pressionem idem hoc solidum analogum exponat; & simul etiam pressioni, quam exponit pseudo-cuneus sectionis rectae in solido patienti, seu solidum 3C3ALQ3D exserendae in sectionem solidi patientis rectam CAD juxta directionem et huic sectioni perpendiculararem, & per centrum gravitatis ω pseudo-cunei ductam.

Hoc est, si β designet solidum illud prismati redundanti (deficienti) analogum, & ω pseudo-cuneum memoratum in propositione; Potentiae β & ω in directionibus $\alpha\beta, e\omega$, in directonibus $\beta\alpha$ & ωe parienti solido applicatae, in aequilibrio consistent cum omnibus potentiis BM, bu &c. cavae superficiei CBAD, vel cum omnibus MB, ub convexae applicatis.

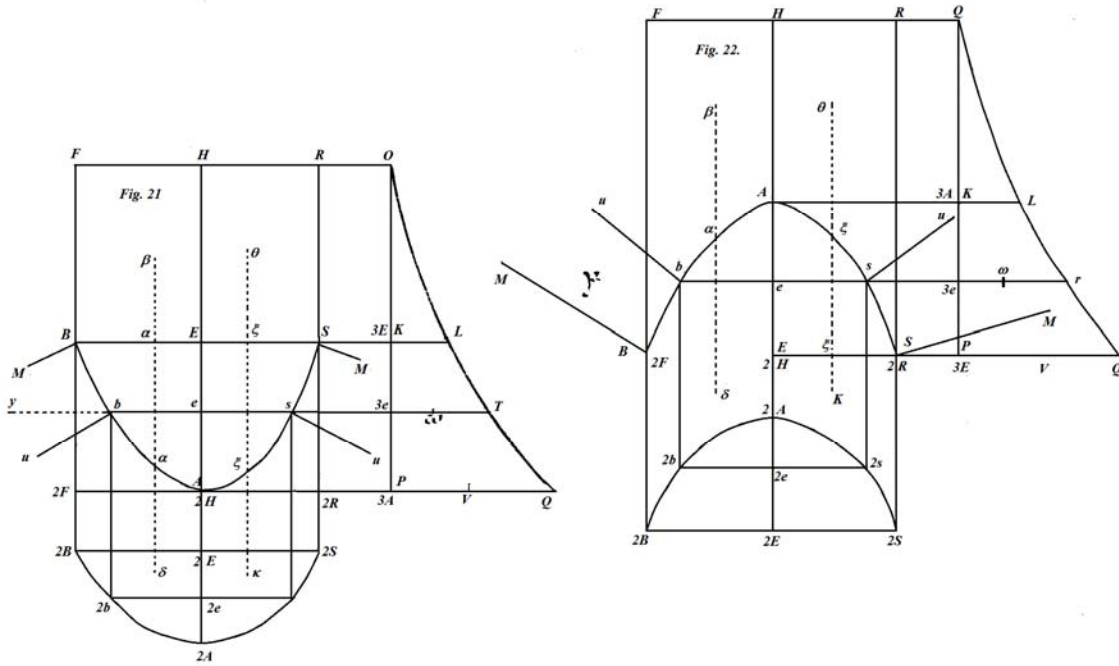
Solidum patiens sectum intelligatur duobus planis *cbd*, *kim* basi CBD aequidistantibus & indefinite vicinis. Portio superficiei curvae solidi patientis duobus planis secantibus intercepta, quam deinceps per *kbm* designabimus, est instar superficiei conicae duabus curvis *cbd*, & *kim* atque lineolis *ck*, *dm* terminatae. Curvae *cbd* projectio in plano humili 2G2F2I fit 2c2b2d, & curvae *kim* projectio in eodem plano humili 2p2n2q, in plano vero *cbd*, curva *pnq*.

Jam pressio quam superficies conica *kbm* subibit a potentia *bu* (secundum hypothesin) = 3er, in singulis ejus punctis perpendiculariter applicata (§. 79.) aequipollet pressionibus, quas paterentur armilla *pbq* (qualis in praecedenti propositione considerata) & Cylindrica superficies *knm*, si etiam singula armillae & cylindricae superficiei puncta eadem potentia *bu* = 3er perpendiculariter afficerentur; atqui potentia qua armilla *pbq* afficitur erit, ut haec armilla ducta in potentiam *bu* vel 3er, qua singula ejus puncta urgentur, id est, exponi debet per solidum *pbq3er*, vel quia armilla *phq* = sua projectioni 2p2b2q, ac 3er (§.74.) = 2H2e = 2b2b, per solidum 2p2b2q in 2p2b, id est per semitubum 2p2b2b2q2d, solido 2G2B2A2I inscriptum. Pressio vero quam superficies cylindrica *knm* ab applicatis potentiis pateretur, aequalis (§.63.) pressioni, quam subiret rectangulum *kpqm* quod cylindricam superficiem subtendit exponetur solido seu parallelepipedo *km.el.3er*, vel parallelepipedo *3k3m.3e β l.3 e r*, quod alteri aequale est atque pseudo-cuneo inscriptum. Unde, quia pressio, quam quaelibet alia zonula conica *kbm* solidi patientis ab applicatis potentiis patietur, similiter aequipollet pressionibus, quibus homologa armilla *pbq*, & homologa superficies cylindrica *knm*

ab eadem potentia singulis zonulae conicae punctis perpendiculariter applicata, urgentur ultra sequitur fore, ut pressionem, quas subibunt omnes zonulae conicae, quae in universa superficie curva solidi patientis continentur, id est pressio, quam ipsa haec solidi patientis superficies curva patietur, aequipolleant pressionibus, quas omnes armillae, & omnes cylindricae superficies zonulis conicis homologae ab applicatis potentiis patientur; atqui cum paulo ante ostensum sit, pressionem a qualibet armilla *pbq* exceptam exponi homologo semitubo cylindrico 2j2b2b2q cujus basis 2p2b2q duabus curvis 2p2n2q lineolisque 2c2p & 2q2d intercepta aequalis dt ipsi armillae *pbq*, cujus projectio est, altitudo vero 2b2b (§. 74.) aequatur ordinatae 3er scalae potentiarum, pressionemque cujusque superficiei cylindricae *knm* exponi parallelepipedo

3k3m.3e3l.3e3l Pseudocuneo inscripto; pressio omnium armillarum exponetur per omnes semitubos cylindricos 2p2b2b2q, qui in solido 2G2F2A2D2I prismati redundanti

(deficienti) analogo continentur, *id* esi per ipsum hoc solidum analogum, & potentia, quae ex omnibus illis quae superficies cylindricas *knm* in solido patienti urgent, nascitur, exponetur per omnia parallelepipeda Pseudocuneo inscripta; seu quia ejusmodi parallelepipeda collective sumta figurae, cui inscripta sunt, & in quam ultimo evanescent, (perinde ac omnes semitubi solido analogo inscripti, in hoc solidum prismati redundant vel deficienti analogum) aequantur, per ipsum Pseudocuneum. Ergo omnes potentiae singulis punctis universae superficiei curvae CBAD applicatae BM, *bu* &c. vel MB, *ub* &c. aequipollent duabus potentiis, quarum una, per solidum 2G2F2B2A2D2I exponenda, agat juxta directionem $\alpha\delta$ (§. 54.) basi CBD solidi patientis perpendicularem & transeuntem per centrum gravitatis δ solidi modo nominari, quod brevitatis gratia unica litera δ qua ejus centrum gravitatis signatur, deinceps indicabimus, vel, quod idem est, omnium potentiarum conspirantium planoque CBD perpendicularum, quae per semitubum 2p2b2q solido illa δ inscriptos exponuntur; altera vero per Pseudocuneum Q3C3A3D (unica itidem litera ω qua centrum ejus gravitatis signatur posthac indicandum) juxta directionem *re* vel *b γ* per centrum gravitatis ω Pseudocunei ductam, sectionemque rectae CAD solidi patientis perpendicularem, quandoquidem parallelepipeda Pseudocuneo ω inscripta repraesentant potentias conspirantes planoque CAD perpendiculariter applicatas, quarum media directio (§. 54.) per centrum earum gravitatis ω transibit. Propterea potentia δ in directione $\alpha\delta$ plano CBD perpendiculariter applicata, & potentia ω plano CAD normaliter applicata in directione *e γ* , simul agentes eundem essectum praestabunt ac potentiae BM, *bu* &c. atque adeo eadem potentiae δ & ω contrario sensu agentes, secundum directiones $\alpha\delta$ & *e ω* , quae omnino contrariae sunt directionibus $\alpha\delta$ & *b*, consistent in aequilibrio cum potentiis BM, *bu* &c. fingulis punctis B, *b* universae superficiei curvae CBAD applicatis. Sed potentiae eadem δ & ω agentes juxta directiones $\alpha\delta$ & *e γ* etiam in aequilibrio manebunt cum omnibus potentiis MB, *ub* &c. Quae erant demonstranda.



COROLLARIUM I.

82. Fig.22, 22. Si nunc loco solidi BAE superficie curva BbA duobusque planis AE & BE invicem rectis terminati, intelligatur solidum BAS superficie curva BbAsS unicoque plano BS terminatum, quod totius solidi (§.67.) basis fit, & planum EA (§. 68.) ejus sectio recta, fitque pro altera solidi parte EAsS eadem scala potentiarum, quae partis solidi patientis EAbD, ita ut potentiae bu , su singulis punctis sectionis bs parallelae BS applicatae inter se aequales sint & ordinatae $3er$ hujus scalae potentiarum; signentur, ut in propositione, in solido analogo omnes lineae iisdem litteris, quibus homologae lineae in solido patienti, sed praeposito binario tanquam nota diacritica, eruntque adeo solida $2F2B2b2A2H$, & $2R2S2s2A2H$ analogae prismatis redundantibus (deficientibus) $FBbAH$, $RSsAH$. Analoga illa solida notentur literis δ & κ , quibus eorum centra gravitatis signantur, & ut in praecedenti articulo, ω designet & pseudo-cuneum

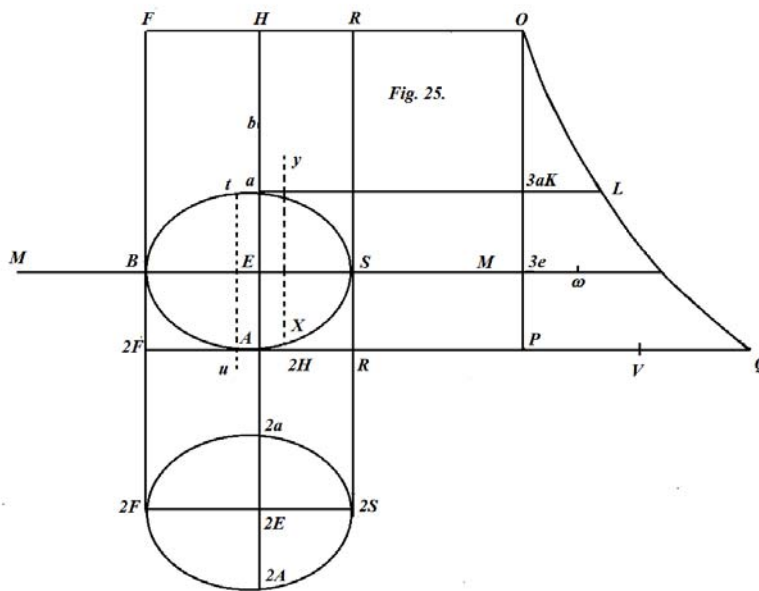
sectionis rectae AE in solido patiente BAS, ejusdemque pseudo-cunei centrum gravitatis quibus positis omnes potentiae SM , su solido EAS applicatae non aliam vim in solidum exferent, quam potentia κ trahens curvam AS secundum directionem $\xi\kappa$ sectioni rectae AE parallelam & per centrum gravitatis κ transeuntem, una cum potentia ω trahente solidum juxta $e\omega$ base BS parallelam, adeoque omnes potentiae BM , su , bu , SM & c. totius solido BAS applicatae aequivalent potentiis δ , & κ solido in directionibus $\alpha\delta$ & $\xi\kappa$ applicatis, cum ejus vertex A deorsum respicit ut fig. 22, vel in directionibus

$\alpha\beta$ & $\xi\theta$ cum solidi vertex sursum conversus est ut fig. 22; ac insuper potentiae ω solidi alis AbB , AsS hinc inde applicatae in directionibus $b\gamma$, $s\omega$ utroque casu. Sed si potentiae solidi patienti applicatae sunt exterius prementes, eae spectandae sunt tanquam agentes juxta MB , ub , MS , us &c. eoque casu hae potentiae aequipollebunt iisdem ac prius, δ , κ , ω , sed in directionibus, $\delta\alpha$, $\kappa\xi$, γb & ωs prementibus solidum BAS in casu figurae 21, & juxta directiones $\beta\alpha$, $\theta\xi$ in figura 22.

COROLLARIUM II.

82. Adeoque, si solidi BAS partes BbA & SsA circa verticem A converti atque instar alarum alicujus sollis ab invicem diduci atque iterum ad se mutuo constringi queant, erit $\gamma.Ae + \delta.\alpha E$ momentum totale potentiarum γ vel ω , & δ , quae aequipollent potentiis curvae superficiei AbB in singulis perpendiculariter applicatis BM , bu , &c. vel MB , ub &c. Et $\omega.Ae + \kappa.\xi E$ momentum totale potentiarum ω & κ solidi parti AsS applicatarum & aequipollentium potentiis SM , su &c. vel MS , us &c. singulis punctis superficiei curvae AsS applicatis.

COROLLARIUM III.



83. Fig. 25 ; Si nunc solidum patiens $aBAS$ unica superficiei curva terminarum fit, cujus singula puncta a potentiis ipsi perpendicularibus urguntur & quae potentiae determinantur scala potentiarum $KLQP$, ita ut supremum punctum a urgeatur a potentia $ba = KL$, infimum A a potentia $2A2H = PQ$, singulaque puncta circumferentiae sectionis BS a potentia $MB = MS = 3er$ & sic de caeteris respective : tale solidum $aBAS$ in altum urgebitur juxta directionem planis FO , vel $2FQ$ perpendiculararem, vi quam exponit solidum $2a2B2A2S$ quod (§.74.) patienti analogum est. Nam sicut supra (§.82.)

vis seu potentia aequipollens singulis potentiis toti superficiei curvae BAS (figuris 21. & 22.) perpendiculariter applicatis, constat potentiis contrariis atque aequalibus γ & ω perpetuo in eadem directione $\gamma\omega$ basi BS parallela in contrarias partes agentibus, & potentiis δ & κ conspirantibus atque juxta directiones $\delta\alpha, \kappa\xi$ basi normales agentibus; quarum potentiae γ & ω toties se mutuo destruunt, quoties totum corpus BAS firmum est, nec ejus partes BAE, SAE ab invicem deduci possunt, adeo ut solae potentiae κ & δ conspirantes supersint considerandae, reliquis γ & ω , quasi non essent, sepositis; jam hae potentiae, quae potentias efficaces significant, δ & κ simul sumtae significant totum solidum 2F2B2A2S2R quod prismati redundantanti (fig. 22) vel deficienti (fig. 22) FBAS analogum est. Ita etiam in solido nostro aBAS potentiae directe contrariae per pseudo-cuneum ω expositae se mutuo elidunt, ipsae vero, quae juxta directiones planis BS vel FR perpendiculares agunt, hoc loco considerandae veniunt; nam pars superior BaS solidi nostri deorsum urgebitur vi seu potentia 2F 2B 2A 2S 2R, & inferior BAS potentia, quam exponit solidum 2F2B2A2S2R, quod prismati redundantanti FBASR analogum est, in altum urgetur juxta directionem plano BS rectam. Adeoque (§. 38.) detracta potentia minore, quae exponitur solido 2F2B2a2S2R analogo prismati deficienti FBaSR, ex potentia majore, quam solidum redundantanti prismati FBASR analogum exponit, quandoquidem minor potentia corpus patiens perpendiculariter deorsum, major vero verticaliter in altum urget; & remanebit vis corpus in altum cogens exponenda solido 2a2B2A2S, quod patienti solido aBAS analogum est.