

THE  
THEORY  
OF  
HARMONICS:  
OR,  
AN ILLUSTRATION  
OF THE  
GRECIAN HARMONICA.  
IN TWO PARTS:

- I. As it is maintained by EUCLID, ARISTOXENUS, and BACCHIUS SENIOR.
- II. As it is established on the Doctrine of the RATIO: in which are explained the Two DIAGRAMS of GAUDENTIUS, and the PYTHAGOREAN Numbers in NICOMACHUS.

With PLATES, an INTRODUCTION to each PART, and a General INDEX.

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L O N D O N:

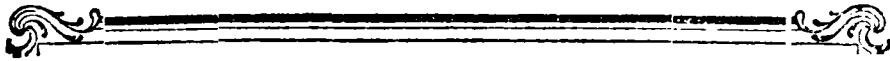
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## INTRODUCTION.

**I** HAVE no other excuse for troubling the Public on a subject that has already been treated of by so many learned writers, than the certainty of having made some discoveries, which may, in part, remove the great obscurity that has so long attended the most curious and diligent enquiries after the true principles of the ancient Grecian Harmonica. The advantages I have received in my profession from that doctrine, would never have been a sufficient temptation to appear as a writer, particularly on a subject so difficult to be explained, so little qualified as I am to do justice to it, were I not in hopes that the knowledge of it may be very useful to professors, by the great advantages they may receive from the true and unerring principles of

B

Science.

Science. To explain the ancient doctrine of the Harmonica, and the several parts into which it is divided, and to reconcile it to modern theories, is the great object that has engaged me in this pursuit; and however conscious I am of my own inabilities, yet truth, divested of ornament, cannot fail of meeting with indulgence from the Public, especially when it leads to useful discoveries.

Nothing contributes so much to the encouragement of Study, as the knowledge of some governing and leading principle; some visible and faithful guide, that will conduct us through the mazes of Science, and teach us to love and obey her laws. It is this that warms and animates our endeavours in the arduous pursuit, and in the end rewards our labours with success.

This governing principle shews itself in no part of human learning so much as in the various operations of numbers; whose powers, by a kind of magic, have greatly contributed to the many discoveries and improvements that have been made in all arts and sciences; nor can it be otherwise,  
while

while Truth is the great object to which those powers are directed.

Where the variety of proportion and its different relations constitute the beauty and harmony of parts, from which the great perfection of any art or science is derived, the application of some branch of the Mathematics is indispensibly necessary, since without its friendly aid we must wander in obscurity and uncertainty.

From this great source the first principles of Music were discovered ; and from the same fountain others must be drawn, if we would be led by the most unerring guide towards perfection, to which point the efforts of the professor, however weak, are directed.

The usual method of treating this subject has been such, that without a profound knowledge of the Mathematics, together with the finer and more sublime parts of Geometry, he is excluded from the advantage which the writings of so many learned authors would give him, and is left to the uncertain

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rules

#### 4      I N T R O D U C T I O N.

rules and examples of his practical predecessors, with what nature, and his own feelings, will dictate.

In this situation there remains no hope but that of uniting the practical with the demonstrative, the art with the science ; at least as far as a perfect knowledge of Harmonics is concerned ; leaving the more philosophical parts to others.

In order to effect this, the operations of numbers must be the most simple, and their application to sounds the most rational and easy to be comprehended,

This ease and simplicity will naturally engage the attention of the professor, who will readily give up some part of his time from that more severe and laborious practice, which all instruments now require, to a study that will not only lessen his labours, but at the same encrease his mechanical powers, and raise his reputation on the most solid and lasting foundation.

Among

Among the theories which have appeared at different periods, those of the Greeks seem to have all the advantages that can be wished to lead us to the true knowledge of Harmonics ; for as their principles are in nature, they must be fixed and immutable.

This appears in no part so obvious and certain, as in the formation of their Scales, which, from the impossibility of altering their original order, must ever remain in the same form and proportion in which they have always been ; as the different combinations of Time are naturally confined to the two Primes, 2 and 3, or their Multiples, from which there can be no variation.

Hence we may conclude, it is scarcely possible that this natural and absolute necessity in the order and disposition of Sounds should have long escaped the observation of the ancients ; or that they should not soon have discovered the principles by which Tune was so limited and circumscribed ; especially as their theories were not formed by mere practical musicians, but by men of learning, who taught them

## 6 INTRODUCTION.

them publicly in their schools, together with other arts and sciences, proper for the most finished education.

But that which confirms this opinion of their knowledge in the Theory of Harmonics, is the simplicity of the Tetrachord, which always continued their favourite system. For however they disagreed with respect to the necessity of measuring intervals by the Ratio, yet they were all unanimous in asserting the perfection of the Tetrachord, which is evident from its making so essential a part in all their Diagrams.

That a system, so simple as this, should continue for so many ages without the least alteration, is not only a proof of the perfection of its composition, but that it has its foundation in nature ; or in such laws as could not be altered without destroying the original harmonious constitution of the system, and, with it, the order and regularity of the scales.

By

## INTRODUCTION. 7

By what means it was laid aside in the tenth century, and succeeded by Guido Aretine's Hexachords, ( the next ancient system ) will be observed in page 13.

As to the particular time when the Tetrachord was first used, or by whom invented, it is not very material to enquire, since there can be no doubt of its being the most ancient musical system in the world, And if we would wish to know more with respect to the great antiquity of Music, the sacred and prophane writers will abundantly inform us: for as it was very early used in religious and other ceremonies, its improvement, in an equal degree with other arts and sciences, seems a natural consequence.

Aristoxenus gives it the highest antiquity; where in speaking of the three genders, the Diatonic, Chromatic, and Enharmonic, page 19, he says, " The Diatonic is to be placed as first and " most ancient, and it is that which human nature " first receives."

But

But without insisting on this, it must be granted, that the human voice is tuned by this scale or gender, which is universally the same, at all times, and in all countries; for as this is the scale of nature, there is sufficient reason to believe, that the principles of harmony, the number and quality of consonances and dissonances, are fixed and determined by certain laws, which, therefore must remain unalterable, whether they are understood in part, or in the whole. Thus, the blood circulated before the great Harvey was born, and the laws of gravity and attraction operated, before that great ornament of this country, Sir Isaac Newton, existed; nor could their discoveries make any alteration in those laws of nature, for they were constantly and uniformly the same, both before, as well as after.

The many wants to which the condition of man is subject, contribute by a kind of necessity to the improvement of all arts and sciences, except music; but in this, where pleasure is the end proposed, we are too impatient to be gratified, and the charm is too powerful to admit of delay from the cold and  
dry

dry investigation of science. This is perhaps one reason why we meet with so few theories.

Notwithstanding the great antiquity of Music, we have no writers come down to us prior to the Grecians; and, of these, the number is not very considerable.

In them there is mention made of earlier writers, of whom a large catalogue may be seen in the 3d book of Fabricius's *Bibliotheca Græca*.

The great and wonderful effects which the Grecian musicians are said to have had over the passions, have inclined the moderns to believe that the art has long since been lost, as we meet with no such effects now; and therefore they conclude we are entirely ignorant of the noble and sublimer parts, which operated with such an irresistible force.

Some have taken the contrary side, endeavouring to prove those extraordinary and wonderful effects to be mere fable, and the invention of the poets. Others have denied that the Grecians had

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## to INTRODUCTION.

any knowledge of harmony, allowing them melody only, and limiting their Chorusses to unison or octave ; by which they have put their music on a level with common ballad-singing.

This opinion is principally founded on their Scales or Diagrams, which, if they were such as have hitherto been explained, must extinguish every sense of musical feeling, by imposing a series of such inconcinuous sounds, as it is impossible for the voice to move in, or the ear to approve. The truth of this may be made very evident from tuning a harpsichord by perfect fifths throughout, by which the several degrees will be tones major and limmas. From this experiment it will be found, that not only harmony, but even a perfect melody, cannot be obtained from principles so opposite to nature.

These writers, in favour of the moderns, have not been aware of this very great objection ; they did not consider, that if the scale contained the true and natural degrees of voice proper for melody, (which they acknowledge the Greeks had improved to the greatest perfection) it must also have been proper

## INTRODUCTION.

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proper for harmony ; which, agreeably to the laws of science, is rather the Parent than the Offspring of melody, as will appear an undeniable fact in the course of this work.

If however the true Diatonic, which is the scale, or voice of nature, has not yet been found in their diagrams, it is not impossible, but they may admit of some other construction, some other interpretation, which may lead us to a more certain knowledge of their doctrine, and the true principles by which their Harmonica is to be understood.

But before I proceed, it may be necessary to take a slight view of the theories formed by the Latins, in order to recover music from the obscurity in which it lay buried for many ages.

This enquiry will be the more necessary, as it will lead us to a better knowledge of the Grecian theories, and enable us, in some measure, to judge how far the Latins deserve the honour of being not only the restorers of music, but also the inventors of harmony ; or, which is the same, of uniting

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two,

two, three, or more different melodies, agreeably to the laws of harmony : a discovery of the greatest importance, and which by many is said to have been unknown to the Greeks.

Musick must have been in a very weak and infant state among the Latins, if we may judge by their systems ; from which it appears, that the few tracts of the Grecians in their possession were of little service to them. For, in these tracts, the musical characters were numerous, and a great variety of distinctions was necessary to their right application. These, with many other difficulties, were not easily got over ; and it is not to be wondered, that the Latins chose to substitute a part of their own alphabet, rather than continue so great a variety of characters, as was used by the Greeks : for as the compass of their system was the same, it required only their first fifteen letters, from A to P, to be applied from grave to acute to compleat the Bisdiapason.

Hence it appears, that the Grecian doctrine must have been very ill understood, if a system of so little meaning could be adopted, whose great imperfections

fections required alteration. Accordingly, Pope Gregory, some time after, discovered that the whole was no more than a conjunction of two octaves, and that the return of the tones and semi-tones in the acute, was in the same position as in the grave; and therefore he reduced the whole system to a Heptachord, (a system of seven sounds) to which he applied the first seven letters of the alphabet, in the following order, to distinguish the Grave, the Mean, and the Acute.

A.B.C.D.E.F.G. || a.b.c.d.e.f.g. || aa.bb.cc.dd.ee.ff.gg.

This Gregorian scale continued without any material alteration till about the eleventh century; when Guido Aretine, a Benedictine Monk, formed a new system composed of three Hexachords, each consisting of six sounds, which he called by the names of the B quadro Hexachord, the natural Hexachord, and the B flat, or moll, Hexachord.

The gravest sound of the B quadro was our G, which is a tone lower than the lowest in the Grecian Diagram, and which he called for that reason by the name.

name of Hypoproslambanomenos, and distinguished it by the Greek letter Gamma, probably to perpetuate his improvement on their Diagrams.

The natural Hexachord had our C for its gravest sound, a fourth above Gamma or G, and the Moll, which had F for its gravest sound, was a fourth above C; and when either of these Hexachords was found one or more octaves higher, the same name was continued, as being only a repetition of the former; to which the seven Gregorian letters were added as before, except the Gamma, which was applied to the gravest sound G only. But as these letters were not sufficient to distinguish the mutations of the Hexachords, which were now to succeed in preference to the Tetrachords of the Greeks, something more was necessary to be done. For this purpose Guido added six syllables, taken from the first strophe of a hymn to St. John Baptist, beginning *Ut queant laxis*, &c. from which he selected the following syllables, and applied them to each Hexachord: Ut, Re, Mi, Fa, Sol, La. Of these, the Mi and Fa always distinguished the place of the semitone, and the other intervals, from Ut to

to Re, and from Re to Mi, Fa to Sol, and Sol to La, were always tones; but whether major, or minor, was not determined.

The Bb, or Moll Hexachord, is no new discovery, as it is formed on the same principles, and for the same purpose, in the Synemmenon Tetrachord of the Greeks.

The major mode, or scale, is principally described by this system of Guido; and the minor is rather implied than determined by any separate laws proper for a distinction of such great importance; and the whole seems calculated for the conduct of a voice in the first rudiments of singing, rather than a discovery of any one principle which can lead us to a knowledge of Harmony.

There is one discovery which must not be passed over, as it does Guido great honour, and has contributed much to the improvement of practical music.

It

It must be remembered, that the Gregorian letters, and the syllables which Guido added to them, were the only musical characters to determine the difference between acuteness and gravity. To prevent therefore the confusion which this must occasion, he is said to have placed several parallel lines at a small distance from each other, on which, and the spaces between, he marked little dots or points, by which the several degrees, from the most grave sound to the most acute, were soon discovered ; but as he found, that the extent of the natural voice could not be brought within the limits of four, five, or six lines, and that there were at least three species of voices, he applied the same number of lines and spaces to each, together with proper marginal characters or cliffs, to distinguish one from the other. Thus the gravest or bass voice was represented by the F Cliff, which confined it within the proper number of lines ; the mean, or C Cliff, was placed a fifth higher than the former, and its limits were equal to that species of voice ; and, lastly, the most acute or treble voice, which placed the notes a fifth higher than the mean, or C Cliff, was called the G Cliff.

By

By these improvements not only the acute, mean, and grave sounds were easily discovered, but also the species of voice to which they belonged.

Another part of his doctrine related to the mutations of the Hexachords, which depended principally on the position of the semitone. This interval was a material part in the Grecian Tetrachords, and by it they were conjoined to each other.

These are the chief parts of Guido's system; and, as they are to be met with in almost every writer from his time, and are so well known to professors, it is needless to be more particular.

He wrote a book, which he called *Micrologus*, to explain his doctrine; but, as I have never seen it, I can only observe, that if there had been any thing very material with respect to Harmony, the many writers in his favour would not have omitted to have taken proper notice of it; especially, as they have done him the honour of styling him the Restorer of Music, and the Founder of the modern harmonic system.

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But

But how three simple Hexachords, or their Octaves, unsupported by the fundamental basses, (for as the writers since Guido take no notice of them, we must conclude they made no part of the theory, notwithstanding they are applied by the moderns;) how, I say, these Hexachords, whose intervals were undetermined by the ratio, could form a sufficient number of scales necessary for the mutations, together with a series of flat and sharp Diesis, proper to determine the place and magnitude of every interval necessary to perfect the most remote transposed scales, either major or minor, seems no less surprizing, than that of limiting the theories of the Greeks to a simple melody only of two Tones major and a Limma to each Tetrachord.

The truth is, we are convinced by demonstration, that Harmony has been discovered for some centuries past, but by what means, or by whom, is not so easily determined.

Vicenzo Galileo, who wrote A.D. 1581, says, page 80, that the discovery of composing or uniting several melodies was not more than 150 years prior to that

that time, which does no very great honour in respect to harmony to the sufficiency of Guido's system, which was in general use for so many years before.

Now, if Guido's theory was not capable of demonstrating the true principles of Harmony, to what shall we ascribe them that will not give the Grecians as good, if not a better, claim to the discovery, especially if we consider their great ambition of carrying every art and science to the highest perfection ; amongst which, Music was highly favoured, and men the most eminent for their learning made it an object of their study, and a part of the most finished education of their disciples.

I have now gone through the principal parts of Guido's system of Hexachords, which would have made no part of this work, had it not been with a view to shew how far his claim to the invention and improvement of Harmony can be admitted ; or whether, by an impartial and candid examination of the doctrine of the Grecians, they may not be found to have a better claim. This will best appear by their theories, which, as it is not possible to explain

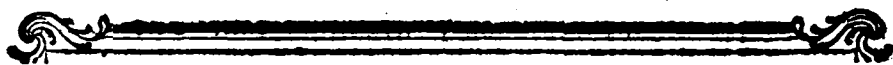
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without the assistance and application of harmonic principles, (the only criterion by which we can judge) we must have recourse to harmonic principles, and thence may conclude in favour of the Greeks, by having the only proof on our side of which the nature of the subject is capable.



T H E



THE  
THEORY  
OF  
HARMONICS.  
PART THE FIRST.

THE following treatise being designed to illustrate the doctrine of the Grecians relative to the Harmonica, and to prove their claim to the true Principles of Harmony, it will be necessary to mention the Authors who have professedly written on the subject. They to whom I refer have been collected by the learned Marcus Meibomius, who has given an edition in Greek, with a Latin translation, and notes. [Amst. 1652, Elzev.]

Aristoxenus,

Aristoxenus, a disciple of Aristotle, the most ancient of these, has written three books, which he calls The Elements of Harmonics.

Euclid, the author of the Elements of Geometry, has written an Introduction to Harmonics.

Nicomachus, a Compendium of Harmonics; wherein he promised to explain himself more fully in his Commentaries. If he performed his promise, the loss of that work, among the several losses of the works of antiquity, must be regretted.

Alypius has written an Introduction to Music, and gives the various signs or characters by which the different degrees of tune were marked, peculiar to the several modes.

Gaudentius, the Philosopher, a favourer of the Pythagorean doctrine, has written an Introduction to Harmonics; in which are contained the two Diagrams that make the foundation of the second Part of this work, in which the Ratio will be applied to the Intervals.

Bacchius senior has written a short Introduction on the Art of Music, by question and answer.

Aristides Quintilianus treats both of the Harmonica and Rythmica.

These seven authors may be divided into two sects; the one following Pythagoras, in determining the difference of sounds,  
in

in musical intervals, by the Ratio ; by which is discovered the exact magnitude of each interval, consonant or dissonant ; the other, in opposition to the Ratio, making the Ear the supreme judge, as being more immediately concerned in the perfection of all musical intervals, and their succession.

As the physical properties of sounds, and the ratio of intervals, make no part of the theory of Aristoxenus, it is no wonder that the majority of succeeding writers embraced his doctrine in preference to that of Pythagoras ; and it is to this cause we must ascribe many difficulties we meet with in their works.

It required the most judicious and masterly contrivance to unite the different parts into which Aristoxenus's theory was unavoidably divided, and which were mutually to depend on each other, so as to establish a Theory independent of the Ratio, and at the same time capable, from the first definition of a sound, to comprehend and describe all the vast variety of intervals, which the different Systems, Genders, Tones, and Mutations demanded ; for, a deficiency in either of these essential parts would not only have destroyed the whole, but at the same time have fixed the Pythagorean doctrine on the most solid foundation.

The success which attended this opposition to the Ratio will best be understood from the principal rules laid down by these authors, which I shall collect and apply, without any forced construction, to the harmonic parts they were intended to explain. I proceed, therefore, to the Seven Parts into which the Grecian Harmonica was divided.

Euclid

Euclid gives this definition of Harmony; [p. 1.] that "It is  
 " a science which contemplates the theoretical and practical  
 " nature of a modulated series." He adds, "A modulated  
 " series is compounded of sounds and intervals, keeping a  
 " certain order, and its parts are seven; of Sounds, Intervals,  
 " Genders, Systems, Tones, Mutations, and the Melopœia."

For the better understanding this general definition, I shall  
 explain each part separately, with such observations as the  
 several articles may require.

### I. OF SOUND.

"Sound is a pitch of the voice in one tension; [Aristox. p. 15.]  
 or, as Nicomachus says, p. 24, "An indivisible voice."

The ancients considered the voice as capable of two different  
 species of motion. Euclid, [p. 2.] calls one, "the Continued,  
 " proper for Diction;" the other, "the Diastematic,  
 " (divided by intervals) proper for Melody."

### II. OF INTERVALS.

Bacchius senior [p. 2.] says, "An Interval is the difference  
 of two sounds, dissimilar in acuteness and gravity." Aristox-  
 enus [p. 15.] describes it to be "the space between two sounds,  
 " not having the same tension."

Aristoxenus and Euclid give, though in a different order,  
 these five divisions of intervals:

The

“The first, by which they differ among themselves in magnitude; the second, by which the consonant differ from the dissonant; the third, by which the compounded differ from the un-compounded; the fourth, by gender; and the fifth, by which the rationals differ from the irrationals.” Aristox. p. 16.

“The difference by magnitude (according to Euclid) is when some of the intervals are major and others minor.” Thus, ditone has a greater magnitude than trihemitone, and tone than hemitone or diesis: but diatessaron is less than diapente or diapasen, and the like.

The difference by consonance and dissonance is, that “the consonant intervals are formed by an agreeable mixture of the acute and the grave, but the dissonant refusing to mix, hurt the ear with some harshness.” Euclid, p. 8.

This mixture of sounds, signifying as consonance or dissonance, is a proof at least of two melodies or voices being united in the same song.

The difference of the compounded from the un-compounded is, that the compounded are such intervals as may be divided into less parts, such as the tone into two hemitones in the chromatic gender, and the hemitone into two dieses in the enharmonic; but in the diatonic gender, neither the tone nor hemitone can be divided, and are therefore un-compounded intervals.

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There

There are, as Euclid observes, some intervals that partake of the nature of compounded and un-compounded ; as those from the hemitone to ditone : for hemitone in the enharmonic is compounded, because it may be composed of two dieses, but in the chromatic and diatonic it is un-compounded ; the tone in the chromatic is compounded, but in the diatonic is un-compounded. Besides these, there are other intervals which partake of these two opposite characters, by being applied to different genders.

“ The difference by gender is, when some of the intervals are diatonic, some chromatic, and others enharmonic.”

“ The difference of the rational and irrational is, that the rational are those whose magnitude can be given, as tone, hemitone, ditone, tritone, and the like ; but the irrational are those that vary these magnitudes, more or less, by some irrational magnitude.” [Euclid, p. 9.]

The preceding definitions of intervals are rather too general to give that satisfaction which so interesting a subject requires ; yet as they introduce the genders, in which the number and magnitude of all intervals less than the fifth or diapente, will be particularly described, they have their use, and prepare us to reconcile those judicious remarks and distinctions we shall meet with in the genders.

From these five divisions of intervals I shall proceed to the genders, to shew by what rule these intervals are to be discovered, and their different magnitudes determined ; for although the

the ear had before been declared the supreme judge, yet in some instances it was not sufficient; as Aristoxenus owns, in speaking of the enharmonic dieses, [p. 19.] where he says, "the ear scarce accustoms itself to this interval, even with much labour."

### III. Of the GENDERS.

The genders will be particularly described by the number of parts into which Aristoxenus and Euclid have divided the tetrachord; by which division the magnitude of each interval is fixed and determined, according to the gender in which it is placed. And thus, not only all the imperfect concords, but even the smallest dieses, are discovered to have their particular magnitude within the limits of the tetrachord; together with those intervals which are proper for combination, and those which can be used only in succession.

The Tetrachord, not only from its great antiquity, but from the simplicity of its parts, is the principal foundation of the genders. Its composition consists of hemitone, tone and tone ascending, and the contrary descending. [Euclid, p. 3.]

By this definition we discover, that the hemitone is substituted in the place of the limma, which necessarily alters the two tones major into minor and major, which is truly diatonic.

That this may be better understood, it must be observed, that the composition of the tetrachord is limited to four sounds,

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or

or three intervals, the extremes of which make the interval of a fourth or diatessaron, and are always in the diatonic gender fixed and immovable; but the two intermediate sounds are to be moved and altered, according to the gender that is to be formed. As these alterations will consist in making the middle sounds graver in one diagram, and acuter in the other, the modern characters of sharp and flat must be applied with the same effect, and in the same manner as the ancient dieses. These will be extracted in proper order from the several scales or diatessarons, which form the modes, and will be placed next each other as a scale, to distinguish and represent the species intervals, which though not proper for melody in that form, yet are absolutely necessary to divide the tetrachord into a certain number of parts, to determine the absolute magnitude of different intervals proper for each gender, and to form the different scales.

Euclid says, [p. 9.] that "Every melody is either diatonic; chromatic, enharmonic, common, or compounded of them." And he adds, [p. 10.] "The mixt is that in which are perceived the characters of two or three genders; of the diatonic and chromatic, or diatonic and enharmonic, or chromatic and enharmonic; and even of the diatonic; and chromatic and enharmonic; and that the difference of genders is made by moveable sounds." He multiplies these three genders into six, and adds, that "Colour is a special division of the gender, and that the rational and known colours are six; viz. one of the enharmonic, three of the chromatic, and two of the diatonic." And after enumerating the particular composition of each, he tells us, [p. 11.] "that these colours are all shewn in numbers. The tone is supposed to be divided  
" into

“ into twelve very small parts, each of which is called the  
 “ twelfth part of a tone; and by the same method as the tone,  
 “ the other intervals are divided; viz. a hemitone into six of  
 “ those twelve parts, the quadrant diesis into three parts, the  
 “ triental into four, and the whole diatessaron into thirty.  
 “ Therefore the enharmonic will be sung by the magnitude of  
 “ three of those twelve parts, and three, and twenty-four.  
 “  $(3 + 3 + 24 = 30)$ ”

“ The chromatic moll, by 4 and 4 and 22,” = 30.

“ The sesquialter chromatic, by  $4\frac{1}{2}$  and  $4\frac{1}{2}$  and 21,” = 30.

“ The toniæum chromatic, by 6 and 6 and 18,” = 30.

“ The diatonic moll, by 6 and 9 and 15,” = 30.

“ The diatonic syntone, by 6 and 12 and 12,” = 30.

In order to explain this part of Euclid's doctrine, I must refer to Plate IV.

In this Plate the several intervals in each gender are regulated, and their magnitude determined, by the number of spiss or small intervals, contained in each, of which the following is an explanation.

Plate IV. K is composed of two tetrachords conjoined, to which the note of disjunction, marked by a +, is added to compleat the diatonic octave.

L contains the spiss or small intervals, each being distant from the next, as directed by Euclid, three parts of twelve; which divides the tetrachord into thirty such parts.

N con-

N contains the enharmonic gender; in which, after the two dieses are taken out of the tetrachord, of the magnitude of 3 and 3, the uncompounded ditone (consisting of twenty-four such parts, which are equal to the major third, an imperfect concord) is added, and completes the tetrachord in its proportion of thirty parts.

P is the sesquialter chromatic; but  $4\frac{1}{2}$  and  $4\frac{1}{2}$  are intervals not to be found in a continued series of 3; the interval, however, of seven dieses, or twenty-one parts, completes the tetrachord, and is an incomposite interval equal to a deficient fourth; which is an acquisition of great consequence, where all species of musical intervals are to be discovered and limited to their several magnitudes.

R, the toniæum chromatic, is composed of a semitone major and a semitone minor, to which an uncompounded interval, equal to a minor third, an imperfect concord, is added, which completes the number of parts equal to the tetrachord.

S is the diatonic moll; in which the two first intervals form the system of the deficient third, and the last uncompounded interval is the superfluous second. This gender, directed by a skilful composer, is very pathetic and sentimental.

T is the diatonic syntone. This gender is the scale of nature, consisting of semitone, tone, and tone. The perfection of this gender is such, that all, who sing, even without any knowledge of music, will sing their different melodies in this gender.

gender. It is not only the foundation of all the others, but by a judicious mixture with them forms all the colouring, or light and dark shades of music ; and without this, viz. the diatonic syntone, the other genders would rather disgust than please.

W, the chromatic moll, is involved in the same difficulties with the sesquialter chromatic ; for as neither 4, nor  $4\frac{1}{2}$ , are aliquot parts of 30, so no interval of these magnitudes can be found to have a place among the spifs, which are in a continued series of 3 equal to 30 ; and as no additional note can be placed among the spifs without exceeding the number limited by Euclid, and increasing the number of modes, which would confound and disorder the whole of their doctrine, so no alteration whatever can be made in the present number and order of the spifs ; and as the ratio was not admitted in Euclid's Introduction to Harmony, the difference of a comma cannot be applied on the present occasion, consistent with principles in opposition to the ratio.

For these reasons I omit any farther enquiry concerning the intervals at P and W ; particularly as those already discovered supply us with all the variety which every species of musical composition can want, either in succession or combination. For succession, we have the enharmonic dieses, the semitone minor, and deficient third ; and for combination, we have three species of 2ds, three species of 3ds, three of 4ths, three of 5ths, three of 6ths, and three of 7ths, with their respective octaves. These intervals are expressed at X and Z, Plates IV and V, and we discover in each (the one being an inversion of the other) three intervals

intervals proper only for succession, and seventeen, including the octave, for combination.

Thus the great end of the genders is in all respects answered, which was to fix and determine the magnitude of all species of musical intervals ; which could not be found in diagrams, composed only of tones major and limmas. But exclusive of these very great advantages, the genders contribute in a wonderful manner to the pleasure of those at all acquainted with them, by distinguishing the phrase, and giving a certain colouring to heighten and support the sentiment, and adding to the powers of mutation, in a more extensive manner than the diatonic alone can ever arrive at.

I cannot leave this subject without observing, that the great objection made by all writers to the Grecian theories has been, that they had only the perfect consonances, but that the imperfect made no part of their doctrine, and consequently it was impossible for them to have harmony or music in two or more parts, moving agreeably to the laws of harmonic combination.

This objection was founded on the construction of their diagrams, which were composed of tones major and limmas, intended by the Grecians to be applied in a very different way than in later times has been imagined ; and while that application remained undiscovered, the objection was not only true with respect to harmony, but was also true with respect to melody ; for such an untunable scale cannot be applied to either with any effect.

This

This great and universal objection will be removed when I consider those diagrams, and is at present in part removed by a right explanation of the genders; an essential part of the Grecian harmonica, which can admit of no other construction, and has not only determined the magnitude of all the perfect and imperfect consonances, but of all the dissonances that can be discovered, or applied to musical composition.

#### IV. OF SYSTEMS.

It has already been remarked that any two sounds differing in acuteness and gravity form an interval; that intervals are varied according to the gender; and that genders are limited by the tetrachord to three intervals or four sounds; by which limitation the magnitude of each degree is determined.

Thus in the diatonic the natural degrees of voice proceed from grave to acute by hemitone, tone, and tone, or the contrary; and the whole composition of intervals forms the system of tetrachord.

Aristoxenus [p. 15.] and Euclid [p. 1.] agree "that a system is composed of more than one interval." As this seems to be a definition of the smallest system, it follows that the two species intervals of the enharmonic and the chromatic must be systems; and that the two species of thirds in the diatonic and chromatic may be made systems, by inserting a note between the extreme sounds of each third, as CDE, or DEF.

F

There

There are many species of systems described by the ancients, which, though they make a very considerable part, yet alone are not sufficient to supply the vast varieties of modulation. In aid therefore of this streight line of system, they have placed the uncompounded, or diatematic, intervals. By this method we discover two motions of voice; the first passing through the several degrees which form the system, and the other passing from one extreme of a system to the other, leaving out the intermediate degrees. Hence the diatessaron is of the same magnitude when a system, as it is when a diateme; and the only difference is that one is compounded, and the other uncompounded: therefore every interval in the diatonic, as diapente, diapason, and the like, which admits of a composition, may be a system with, or a diateme without, such composition.

Systems then being a modulated series from grave to acute, or the contrary, are distinguished by Euclid [p. 12.] in the following order:

“ There are seven differences of systems; four of them are  
 “ in the intervals; namely, the difference of magnitude, of  
 “ gender, of consonance and dissonance, of rational and irrational. There are three peculiar differences of the systems;  
 “ that of ordinate and preposterous, of conjoined and disjoined,  
 “ of immutable and mutable.”

The difference by magnitude and gender has already been treated of. The next difference, of consonance and dissonance, is distinguished by a comparison of the first sound with the last,  
 which

which terminates the whole, and may be sometimes consonant, and sometimes dissonant.

There are six consonant systems described by Euclid, [p. 12 and 13]. The first three are simple, as diatessaron, diapente, and diapasos; the next three are compounded of diapasos and diatessaron, diapasos and diapente, and bis-diapasos, which is the greatest and most perfect system, containing all the less. He also mentions a seventh and eighth system, viz. compounded of bis-diapasos and diatessaron, and bis-diapasos and diapente. Which shews that the limits of their scale were not so much confined as has generally been imagined; for it wants but one note only (the hypoproslambanomenos) to be of the same extent with that of Guido.

But Aristoxenus goes much further; for he says [p. 21.] that "A triple diapasos symphonizes, and a quadruple, and " even a greater magnitude."

Each of the first three systems is divided into different species or figures, by which, though their magnitude is the same, the parts of their composition are varied. This variation of the species is occasioned by the different position of the hemitone.

In the diatessaron it has three places, as  $\overline{BCDE}$ ,  $\overline{CDEF}$ ,  $\overline{DEFG}$ ; in the diapente it has four, as  $\overline{EFGAB}$ ,  $\overline{FGABC}$ ,  $\overline{GABCD}$ ,  $\overline{ABCDE}$ .

F 2

The

The several species or figures of the diapason are not only distinguished by the position of the two hemitones, but by the disjunction of the tetrachords.

Euclid [p. 14.] has been careful in distinguishing these different species. Thus from barypycni to the next barypycni above, or from B to E, is the first species of the diatessaron, [Plate I.] whose hemitone is in the grave. The second species of diatessaron is the mesopycni, or from C to F, whose hemitone is in the acute. The third species is the oxypycni, from D to G, whose hemitone is in the middle, consisting of tone, hemitone, and tone.

In this manner the three species of systems are explained; but as the extremes are all of the same magnitude, we can have but one diastem in the ratio of  $\frac{4}{3}$  to each, or from one extreme to the other. After these follow the four species of diapente, thus described by Euclid, p. 14, 15.

“ The first, that which is contained in the barypycni,  
 “ whose tone is first in the acute; and is from hypate meson  
 “ to paramese; the second, that contained in the mesopycni,  
 “ whose tone is second from the acute, and is from parypate  
 “ meson to trite diezeugmenon; the third, that contained in the  
 “ oxypycni, whose tone is a third from the acute; and is from  
 “ lichanos meson to paranete diezeugmenon; the fourth, that  
 “ contained in the barypycni, whose tone is a fourth from the  
 “ acute, and is from mese to nete diezeugmenon, or from  
 “ proflambanomenos to hypate meson. Also in the diatonic  
 “ gender, that is the first species (of diapente) whose hemitone  
 “ is

“ is first from the grave ; that the second, whose hemitone is  
 “ first from the acute ; that the third, whose hemitone is  
 “ second from the acute ; that the fourth, whose hemitone is  
 “ third from the acute.”

To these the seven species of diapaſon (mentioned by Euclid, p. 15 and 16 compared) ſucceed. They are called Tones or modes, very probably from the tone of diſjunction, whoſe different poſition makes one of the principal diſtinctions of the ſeveral ſpecies ; and as they are compoſed of the two foregoing ſystems of diateſſaron and diapente, the two ſemitones muſt have their place in each.

[Plate I.] “ The firſt is that contained in barypycni, from  
 “ hypate hypaton to paramefe, (or from B to B) whoſe tone of  
 “ diſjunction is the firſt from the acute, and whoſe hemitones  
 “ are firſt from the grave and fourth from the acute.”

“ The ſecond ſpecies is contained in meſopycni, (or from  
 “ C to C) whoſe tone (of diſjunction) is the ſecond from the  
 “ acute, and whoſe hemitones are the third from the grave,  
 “ and the firſt from the acute.”

“ The third ſpecies is contained in oxypycni, (or from D  
 “ to D) whoſe tone (of diſjunction) is the third from the acute,  
 “ and whoſe hemitones are the ſecond from the grave, and the  
 “ ſecond from the acute.”

“ The fourth ſpecies is contained in barypycni, (or from E  
 “ to E) whoſe tone (of diſjunction) is the fourth from the  
 “ acute.”

“ acute, and whose hemitones are the first from the grave, and  
 “ the third from the acute.”

“ The fifth species is contained in mesopycni, (or from F to  
 “ F) whose tone (of disjunction) is the fifth from the acute,  
 “ and whose hemitones are the fourth from the grave, and the  
 “ first from the acute.”

“ The sixth is contained in oxypycni, (or from G to G)  
 “ whose tone (of disjunction) is the sixth from the acute, and  
 “ whose hemitones are the third from the grave, and the second  
 “ from the acute.”

“ The seventh is contained in barypycni and apycni, (or  
 “ from A to A) whose tone (of disjunction) is first from the  
 “ grave, and whose hemitones are the second from the grave,  
 “ and the third from the acute.”

These seven species of diapason are without doubt of the greatest importance. Euclid is not satisfied with describing them by the tone of disjunction, but goes over them a second time [p. 16.] to fix the positions of the two hemitones peculiar to each species. See Plate II. where the hemitones are described by the black notes, and the tone of disjunction is marked thus †.

One tetrachord may succeed another by conjunction or disjunction.

The

The conjunction is made by a sound which is common to two tetrachords, and is the most acute of one and the most grave of the other.

The magnitude of this conjoined system is a heptachord; or seventh, as from B to A, and consequently is a dissonant system from one extreme to the other. [Plate I.]

The disjunction is made by a tone, which separates the most acute of one tetrachord from the most grave of the other, and its magnitude is diapason, as from E to E, Plate I.

“ There are (as Euclid observes, p. 17.) two perfect systems ;  
“ the one less, the other greater.”

“ The less is by conjunction, from proslambanomenos to  
“ nete synemmenon, containing three conjoined tetrachords,  
“ hypaton, meson, and synemmenon, besides a tone from proslambanomenos to hypate hypaton, (or A to B in the grave)  
“ and is finished by the consonance diapason and diatessaron :”  
From A to the second D, or from the first to the second apycni, which he has made the limits of this less system. [Plate I.]

“ The greater system is by disjunction, from proslambanomenos to nete hyperbolæon ; and it contains four tetrachords,  
“ disjoined by pairs conjoined together ; namely, hypaton,  
“ meson, diezeugmenon, and hyperbolæon ; and two tones  
“ besides, one from proslambanomenos to hypate meson, and  
“ the other from mese to paramese, and is included in the consonance of bis-diapason, (from A to the second A) or from  
“ the

“ the first to the third apycni, which are the limits of the “ great system.” [Euclid, p. 18.] The great extent of this system discovers, that the ancients were not confined to the limits of diapason, as has been imagined.

It appears by the doctrine of the ancients, that the voice had two principal motions ; one, the diastematic or harmonic, when the sounds succeed each other by an uncompounded interval, as a 4<sup>th</sup>, 5<sup>th</sup>, or 8<sup>ve</sup>, and the like, proper for the fundamental bass ; the other motion of voice was by degrees or systems, which being more natural and better adapted to the singing of the voice, from their great variety, required a more critical description.

The diastematic motions are the only perfect intervals that can be discovered in a scale or diagram of tones major and limmas ; and as it is impossible to form a tunable system out of such a scale, it follows, that these uncompounded intervals are the true and original fundamental basses, and cannot be applied to any other part of a theory. As to the second motion of voice, in which the degrees or systems are formed, they are the products of those fundamental basses, by whose harmonic power the tetrachord conjoined, or disjoined, and all other systems are formed and regulated. See Plate III. and VII. in which mese, or the tone of disjunction, is marked thus +.

I must observe, that all these systems are formed by the tetrachords, and not from the diagram of tones major and limmas ; for Euclid, p. 8, and Aristoxenus, p. 20, say, “ that all intervals less than the fourth, or diatessaron, are discords.”

This

This is very true if applied to the diagram of tones major and limmas, in which no consonant system can be found ; but with respect to these tetrachords, which are all truly diatonic, it cannot be applied to them. In order to remove this seeming contradiction of intervals, or even systems, less than the diatessaron being discords, and to give all possible satisfaction on this subject, I must observe, as before, that the Grecian theory was divided into two parts. The first and principal was the perfect system, or diagram of tones major and limmas ; from which system all harmony with its roots or fundamental basses arises. The second part related to melody, which is connected with, and inseparable from, the fundamental bass. Thus the scale of tones major and limmas appears to be expressly formed to regulate the fundamental basses, and cannot be applied as consonant systems, like the tetrachords, nor indeed to any other part of their theory.

The harmonic parts, from whence melody has its original, are found in the tetrachords, systems and genders, &c. and are the products of the fundamental basses. For a full explanation of which see pages 50, 51, &c.

In the scale of tones major and limmas, the only consonant sounds to be found are the diapason, diapente, and diatessaron. These three consonances have a very distinguished character by being applied to each tetrachord as fundamental basses ; for they not only discover their true and harmonic powers in forming the diatonic octave or scale, (all of which may be extracted from among the harmonics, and reduced to a system of degrees) but the name of the scale or key-note, when formed, will

G

have.

have the same name as the principal consonance, or diapason. This concord is not only the original of the other two, by including them within its limits, but is superiour to either by being the final note of cadence, and all other perfect terminations. The diapente is more limited in its powers as a fundamental bass in this respect, that it forms the imperfect repose, and is necessary to lead to the final by forming the cadence.

The Diatessaron is one of the primary fundamental basses, and contributes with the other two to form the scale, or diatonic octave, but is incapable of forming any termination; it has however one character peculiar to itself, that of being the first and original discord, as its relation to diapente is tone major; yet, with this application, it becomes an harmonic sound, as all original discords are, and not a fundamental bass.

Having discovered the great importance of the scale of tones major and limmas, in being the original of the three fundamental basses applied to each tetrachord, I shall now distinguish them in another point of view, in order to facilitate their use. I shall place them as antecedent, mean, and consequent, or as first, second, and third terms. In this form, the diatessaron, or fourth, will be the first; the diapason and most perfect consonance will be the key note, or second term; and the diapente, or fifth, will be the third term, attended with their respective harmonics of an octave, fifth, and third. It must be remembered, that all systems have their foundation in these three terms, and that as they will be frequently applied in explaining the tetrachords, it is unnecessary to add more here. I will therefore proceed to explain one of the most curious and interesting

interesting particulars discovered in the whole theory of harmonics, namely, an inverted system, which will be found of the greatest consequence; for, by the inversion of the first diagram a second is discovered.

Euclid [p. 15 and 16.] places his seven modes in the following order, and Bacchius Senior [p. 12.] places his seven in a contrary or an inverted order, and the same names are applied to each.

|         |   |   |   |   |              |   |   |   |   |                    |
|---------|---|---|---|---|--------------|---|---|---|---|--------------------|
| Euclid. | { | B | — | — | Mixolydian   | — | — | — | F | } Bacchius Senior. |
|         |   | C | — | — | Lydian       | — | — | — | E |                    |
|         |   | D | — | — | Phrygian     | — | — | — | D |                    |
|         |   | E | — | — | Dorian       | — | — | — | C |                    |
|         |   | F | — | — | Hypolydian   | — | — | — | B |                    |
|         |   | G | — | — | Hypophrygian | — | — | — | A |                    |
|         |   | A | — | — | Hypodorian   | — | — | — | G |                    |

Though this inversion of the diagram may appear at first sight to be of little consequence, yet it has engaged the attention of many writers, and occasioned as many different opinions, without producing any thing decisive; for whether the diagrams are to be read backwards descending, or the contrary, or whether the grave sounds are called acute in one age, and the contrary in another, is of very little consequence, where an art is to be improved, or the secrets of science to be discovered. But I shall pass by these and other favourite opinions, which have only served to perplex the subject, without leading us to any useful discovery.

To procure, however, all the advantages this doctrine can give us, a musical construction, supported by other parts of the Grecian harmonica, will shew its real importance. In order to demonstrate this I shall observe, that the seven species of diapason [Plates III. and VII.] include all the tetrachords in their different positions, whether conjoined or disjoined, and also the hemitones, tones, and all other necessary distinctions proper for harmony or melody; which will be explained in the next or fifth part, in which the modes or tones will be particularly considered. At present, therefore, I will offer a few observations, previous to my entering on so interesting and difficult a subject.

I observe first, that every system of diapason is called a tone or mode by Euclid and Bacchius, and that the first letter of that diapason is placed near the name, which is given to the mode.

Secondly, That Euclid places the mixolydian at hypate hypaton, (or B) in the grave, and ascends by degrees till he arrives at mese, (or A) in the acute; whereas Bacchius places his first, or mixolydian mode in the acute, (or at F) from whence he descends by degrees to the grave (or G); and each author distinguishes every succeeding mode by the very same names, but in an inverted order; as in the preceeding example, at page 43.

Thirdly, That in comparing Euclid's diapasons with those of Bacchius, the intervals are all the same; that is, the tones hemitones, and tone of disjunction, are in the same position in each, except being in a contrary direction. See Plate VI.

Fourthly,

Fourthly, That Euclid, [p. 19.] (according to Aristoxenus) affirms, that "the number of tones or modes are thirteen," but that number cannot be supplied, unless they are distinguished as two separate scales; the first ascending from grave to acute, the second descending from acute to grave.

Fifthly, That, agreeably to the position of the seven species of diapason, at Plates II. and III. we discover all the sharps in a numerical order from one to six.

Sixthly, That the same seven species of diapason being inverted, we have all the flats from one to six. See Plates VI. and VII.

Seventhly, That these sharp and flat dieses form the spiss intervals, the minor and major scales, the genders, the mutations, and divide the tetrachord into thirty parts, as directed by Euclid.

Lastly, That the fundamental basses, or three terms before mentioned, applied to each tetrachord, form a most sweet and perfect harmony.

These are the great outlines of a most extraordinary theory, which will be explained and applied in the next or fifth part of the harmonica; in which the tones and number of modes will be more particularly considered.

## V. Of the TONE or MODE.

This part of the ancient doctrine has been found the most obscure and mysterious. The many learned writers and commentators, who have attempted to explain it, have disagreed in many essential parts, whilst others, from the variety of opinions, despaired of success, and gave up the enquiry. The truth is, the wonderful and extraordinary accounts we meet with in ancient and modern writers, of the surprizing effects produced by the mutations of mode and gender, and other parts of the Grecian harmonica, have raised expectations in many, not to be satisfied with any thing less extraordinary than the effects said to be produced. Whereas, had they first considered what were the essential parts of practical music, and adapted them to the ancient theories, without expecting any thing almost supernatural, they would have found the agreement between the ancient and modern methods of treating the same subject so perfectly corresponding in the most essential parts, that nothing could be discovered in one, that would not be wanted and absolutely necessary in the other.

If a practical musician were to be limited to the four parts already explained, he would find a great deal wanting to enable him to exercise the full powers of his art. He would be greatly distressed in his operations, whilst confined to those sounds only which have been discovered. It is true, the genders have explained the division of the hemitone and tone, and other musical intervals, but have given no rule to apply those divisions to any other part than the two moveable or middle sounds of the trachord.

trachord. Something more therefore is necessary for a general application of those divisions, by which modulation may be carried to its greatest perfection, without any alteration or injury to the tetrachord. And if that which is so indispensably necessary for modulation can no where else be found but in the doctrine of the tones and modes, and if there is no other part of practical music to which they can be applied, let us not reject so interesting a part of ancient theory, without at least trying how far it will carry us.

Euclid [p. 19 and 20.] gives four different definitions of the word *Tone*.

He says, "It is taken for sound;" as a sweet or harsh tone or sound.

Secondly, "It is taken for interval; as from mese to para-mese (or from A to B) is the interval of a tone."

Thirdly, "It is applied to the place of the voice, when we mention the dorian, lydian, or any other tone."

Fourthly, "It is taken for intension or remission, when a person uses an acute, or grave, tone of voice, or between both."

The third definition, or "the place of the voice," is the subject of the present enquiry.

All

All authors agree, that the seven species of diapason are the foundation of the doctrine of the tones or modes, and that the tones to which they refer are not to be understood in the limited sense of sound, interval, or intension, but as a certain composition or constitution of harmonious sounds, proper for the place of the voice in the most extensive sense; provided the two hemitones and the tone of disjunction are placed according to the method described by Euclid in treating of the system of diapason.

That this method may be more clearly understood, I have (at Plate II. fig. 1.) formed the seven tones or modes, agreeably to the different species of the diapason, in which the positions of the two hemitones are distinguished by being black notes, and the tone of disjunction by the cross, marked  $\times$ , between the two disjoined sounds. The tetrachords are also distinguished by the curve line, which encloses the whole, with the extreme sounds, and the provincial name is annexed to each tone, as mentioned by Euclid.

In examining these constitutions, which are the foundation of the modes, we find the position of the two hemitones and the tone of disjunction the same in each with respect to acuteness or gravity; that is, from B to C, and from E to F, are, in all of them, the places of the hemitones, as from A to B is also that of the disjunction. Now, as the different position of these intervals should make the essential parts of modulation, which notwithstanding are the same in each, to which of the remaining parts of these compositions must we turn our attention to discover something characteristic, and expressive of the different

different relations, in which one tone or mode stands to another? Not in the several limitations of the diapason; for that will be a positive contradiction to a former part of the doctrine, which gives a minor system, consisting of a diapason and diatessaron, and also a major system, composed of twice diapason. Now, if a voice has liberty of motion through all, or a great part of this major system, what end can it answer to confine it to half that compass? or, what new variety of modulation will it procure, which we had not in a much greater perfection before? But this is not the only objection; for the synemmenon tetrachord, which gives a hemitone from mese to trite synemmenon (or from A to B $\flat$ ) in a different position from the two former, is absolutely excluded from a place in each of these seven species of diapason.

However great these contradictions may seem, they ought not to be imputed to the ancients, unless it shall appear that these seven species of diapason can admit of no other construction.

But the contrary of this will be evident, if we attend to the definition which Euclid has given; [p. 2.] wherein he tells us, that "tone is a place of the voice, capable of system without latitude."

This definition is very important; for it not only determines as many different places of the voice as there are systems of the diapason, but also that each system is without latitude. Thus it appears, that the diapason is no otherwise necessary with respect to the limits of the system, than its first formation,

H

by

by which the two hemitones and the tone of disjunction, have each their particular positions; for these are the only essential differences of the seven systems or places of the voice, after which the systems are without latitude; that is, the systems may extend beyond the limits of diapason, either to the grave or the acute, provided that the positions of these intervals are the same in the respective diapasons, above and below; for the order and composition of the first diapason being once fixed, determine all others belonging to the same tone, and are properly that which Euclid calls "the tone or place of the voice" "capable of system without latitude;" because every tetrachord found in this composition refers to one particular sound or tone, which forms one scale only, and no other.

After this explanation it remains to shew, how these seven places of the voice may be discovered agreeably to these directions; and how such discovery can encrease the powers of modulation, by disposing of the two hemitones, and tone of disjunction, in seven different places with respect to acute and grave.

To make this easily understood, I have, at Plate II. fig. 2. placed the seven species of diapason in the upper stave, each species rising a degree in the order as before, at fig. 1. in the same plate, and immediately under these I have placed seven others, at H, I, K, L, M, N, and P, whose compositions are in all respects the same, except that of being formed from one common sound, instead of rising a degree, as those above.

This

This common sound is B, the first of the gravest tetrachord, and in that respect has a preference to any other. From this construction we have a variety of hemitones distributed throughout, not discovered before; and as these intervals are the same with those in the stave above, the sharp, which is a modern character, is applied not only to fix the positions of the tones and hemitones peculiar to each mode, but also to shew their relation to each other, in order to perfect their mutations; without which the whole theory would have been imperfect.

Thus after the natural system at H, we have an increasing series of sharps from one to six. The first sharp is F, the fourth species of Diapason at I; we have two sharps, F and C, in the seventh species at K; and at L we have three sharps, F, C, and G, which is the third species; and at M, the sixth species, we have four sharps, F, C, G, and D; also at N, the second species, we have five sharps, as F, C, G, D, and A; and at the fifth, or last species of diapason, at P, we have six sharps, viz. F, C, G, D, A, and E.

In comparing these seven species of diapason, ascending from one pitch or common sound, with each other, we have five tones, each of which is divided into a greater and less hemitone; also one hemitone, divided into a hemitone minor, and an enharmonic diesis; the first from E at H, to E # at P; and the second from E # at P, to F at H, in the first species. With respect to the other hemitone, from B to C, it could not, according to this method, be divided, because B is the pitch, or common sound; but this is not material, as we shall, in another place, find it divided in the enharmonic gender, Plates IV. and V.

H 2

These

These different positions of the two hemitones and the tone of disjunction give every diapason a new and distinct character; which, with the other advantages before enumerated, explain the ancient doctrine of the tones or modes in the most satisfactory manner; and, by their agreement with Euclid's definition, leave no doubt that "the tone, or place of the voice, capable of system without latitude," is found in each of these seven species of the diapason; because, the constitution of the diapason being once formed, may be extended to the acute or grave in the same order, without latitude.

By this construction the most important part of the harmonica, with respect to modulation, is reconciled with the doctrine of the ancient modes, in a manner the most natural and useful to a practical musician.

The Tone of Disjunction, so often mentioned by the Grecians, and found in every diapason, comes next to be considered, in order to discover its character and use in this theory.

\*If we examine the seven species of diapason, where this interval is marked with a cross, we discover very little that can make this distinction necessary, except that of fixing the limits and order of the tetrachords; but as this seems of too little consequence for such a distinction, if confined to a simple melody, we must look for its true character and importance in harmony.

To shew this in the clearest light, I have (at Plate III.) repeated the seven species of diapason, formed from one common  
found,

found, as before; and have placed under each note its principal or fundamental bass at R and S.

I observe at R, the first species of the diapason, we have two tetrachords conjoined, and the tone of disjunction succeeds. That each of these tetrachords has three basses; the first has E, A, and D; the second has A, D, and G; and finishes at D bass. After which we are prevented from advancing to the tone of disjunction, unless the bass, and its harmonics at D, move first to those of A; then the tone of disjunction and its bass succeed. It appears also, that every bass in this diagram must have a minor third for one part of its harmony; in consequence of which, the B, in the first tetrachord, will be of a different species from that in the second; for the first tetrachord begins with a B natural, and the second demands a B flat, to be a minor third to G, the third bass.

From this observation we discover that these tetrachords are formed so as to partake of two distinct scales; as it is impossible to have a natural B and a flat B in the same diapason. Thus by applying the fundamental basses and their harmonics we discover that the true character of the tetrachords has its origin in harmony, by which their formation and connection appear rational and agreeable to the best theory; which will be more fully explained when we consider the Mutations.

The tone of disjunction distinguishes the termination of the last tetrachord, in which B was necessarily flat, and restores us to the first scale, in which it was natural; but in order to prevent the disagreeable effect, from too quick a succession of a B flat.

flat and a B natural, the intervention of an additional bass and its harmony is required, as marked at 8, (between the two fifths at R) previous to the disjunction ; this not only conducts us to the alteration of the scale, which is here intended, but prevents the consecution of two fifths, which in no case can be admitted. See Plate III. at R.

From these observations we discover the real importance of the tetrachords ; that they are not to be considered merely as simple systems, but as systems connected with, and dependent on, the laws of harmony.

I shall now compare the same species of the diapason at R with those at S, in the same plate.

In these two examples we find no alteration whatever in the systems ; for the two hemitones, the conjoint tetrachord, and the tone of disjunction, are the same in each ; and the only difference discovered is in the basses, which is undoubtedly a very material one with respect to modulation.

To understand this it must be observed, that the systems now before us at S are not tetrachords, but are a species of the diatessaron ; for the true position of the tetrachord, in a major mode, must be in all respects an inversion of the minor ; which Euclid has before told us is hemitone, tone, and tone ; agreeably to which, the hemitone must be in the acute in the major scale, as it is in the grave in the minor.

If

If we compare these systems at S with the tetrachords at R in the same diagram, or those at U with T, Plate VII. we shall find that two fundamental basses are sufficient for these systems, but that the tetrachords require three.

This distinction would have been of very little consequence, if the formation of the scale did not depend on the harmonious constitution of the tetrachord, which cannot be extracted from the harmonics of two fundamental basses, but is the product of three; for they are placed in this order not by accident, but to shew that the same melody may have two distinct characters, by being formed from different fundamental basses; and that without their application the minor and major scales must remain undetermined.

It has before been observed, that the basses at R required minor thirds; but these at S, on the contrary, do demand major thirds. This variation of the thirds occasions one of the most remarkable revolutions to be found in harmonics, and is the original cause of the mutations from minor to major scales, or the contrary. As to the difference of tone major and minor, which may be found in these systems, the ratio making no part of the Aristoxenian doctrine, I shall reserve my remarks on that subject for the Second Part of this work.

With respect to the systems now before us, we find but two basses to each at S, instead of three, which are necessary at R; likewise that the tone of disjunction at S requires an intermediate bass, D, to prevent the succession of two major thirds, equally to be avoided with the two fifths at R; besides which, the

the major third applied to this, and to the other intermediate basses at S, marks the change of the scale occasioned by the disjunction.

I have now gone through my observations on these seven species of diapason, so far as they relate to the explanation of the tones or different places of the voice. Their connection with harmony, and their mutations are supported by the fundamental basses, and an increasing series of sharps; and the distinctions of tetrachords conjoined, or disjoined, are made manifest, and supported by the most unerring laws of harmony.

The advantages arising from an increasing series of sharps are, it must be owned, very great; yet a Theory would be very imperfect if it should stop here and leave out the flats, which are equally necessary for the completion of the whole.

The flats being of a contrary nature to the sharps, require the whole order of systems to be inverted; but nevertheless the place of the tetrachords, the hemitones and tone of disjunction, must be preserved in the same positions before described, except that of moving in a contrary direction: but as this will best be understood by the musical notes, I shall refer my reader to Plate VI. fig. 1. and 2. in which the two diagrams are formed on this principle of inversion.

The first, which has already been explained, moves in an ascending, but the second in a descending, direction; placing the hemitone in the acute, from whence the tetrachord begins in a major scale.

In

In comparing their intervals, and other parts, we find a mutual agreement throughout the whole.

The seven species of diapason, extracted from the second diagram, have their beginning at fig. 3. Plate VI. and are continued till all the seven are formed.

Each diapason in the upper stave descends a degree to the next; but those immediately under are fixed to one common pitch at F; but in all other respects the intervals are the same with those above, and move in the same direction. Hence we have an increasing series of flats from one to six, which fixes and determines the positions of the hemitones, tone of disjunction, and the tetrachords, extracted from this second diagram; and the whole agrees with the first diagram before explained, whose common pitch was B, from whence this inversion is taken. See fig. 4. Plate VI. which exhibits the seven species of diapason belonging to each of the diagrams, above and below; the flats succeed and increase in the same order below, as the sharps do above. Example.

|         |      |      |      |      |      |      |
|---------|------|------|------|------|------|------|
| Sharps. | F #, | C #, | G #, | D #  | A #, | E #, |
|         | 1,   | 2,   | 3,   | 4,   | 5,   | 6,   |
| Flats.  | B b, | E b, | A b, | D b, | G b, | C b. |

The reason why B # and F b are not placed here will be given towards the latter end of the Mutations.

From the great advantages procured by the construction of these two diagrams we have all the materials necessary to complete the seven parts, into which the Grecians divided the har-

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monica:

monica: at the same time the powers of harmony operate so universally throughout the whole theory as to explain many parts which could never be understood upon any other principle. Among which are the conjunction and disjunction of the tetrachords, with the major and minor harmonics applied to each tetrachord, or system, of the second diagram, at Pl. VII, fig. T and U. The seven species of diapason, formed under each of these letters have their fundamental basses to which they severally belong, with the major thirds to the first at T, and the minor thirds to those at U. The tone of disjunction is distinguished in each by an additional bass as a temperament, not only to prevent the consecution of two eighths at T, and two minor thirds at U, but also that the fundamental bass should not move by degrees or partake in any respect of the nature of the system, which is set apart, and appropriated only to the melody. The additional bass at U generates a discord at the disjunction with the upper sound: but this shall be explained in the Second Part, when I treat of Discords.

From the principles already explained of the harmonica of the Grecians, it appears that the great variety of intervals, their relation, and the different orders of their succession, together with their particular application to musical composition, are the principal objects intended to be regulated by these several parts of their doctrine.

With these materials I proceed to the Mutations, which is the sixth, and most interesting, as well as the most difficult, part of the harmonica. The principles laid down by  
Euclid

Euclid will conduct us through all their charming varieties: at the same time the formation and number of modes will be discovered, which could not be explained so well before, as they depended much on the mutations. We shall then be convinced, by the most indisputable evidence, of the agreement between the several parts which compose and perfect this wonderful theory.

## VI. Of the MUTATIONS.

To explain how the Mutations are to be made, it will be necessary first to discover their object, and then the method by which they may be effected, so as to produce what Aristoxenus declares mutation to be, "A kind of pathos, occurring in the " order of modulation." [p. 38.]

In explaining the tetrachord on harmonic principles, it has been found that the several sounds, selected from the harmonics of a tetrachord and placed as a system, form the diapason; that they are a scale of sounds, peculiar to that tetrachord, and are not found in any other; and that whatever flat or sharp dièses are used in the tetrachord, the same are absolutely necessary for the formation of the system of the diapason, or scale.

The ancients very judiciously placed the mutations in the change of the tetrachord; not only because the several genders would be better understood, when applied to that small system, but because of its harmonic principles, (before explained); which they knew comprehended the whole scale or diapason, by

selecting its several harmonic parts, and forming them into one system.

By this harmonic construction of the tetrachord its great importance is discovered: and if compared with that description which we meet with in all authors who have written on this subject, we shall find every part unlike, except a simple system of four sounds, unconnected with harmony, and independent of every principle that can lead us to the knowledge of it. No wonder therefore that their diagrams have been exploded and their theories condemned.

If we examine the two tetrachords in the first species, and those which make a part of the other species of diapason, (at Plate III. fig. R and S) they, by having major or minor thirds to their respective basses, contribute to form different scales. By these different species of thirds we discover their genius and character to be very unlike. The first at R gives two scales with minor thirds, because each tetrachord forms a scale from among the harmonics peculiar to itself. Notwithstanding the intervals of the diapason are in appearance the same in each at R and S, yet the harmonics are in all respects very different by the application of the fundamental bass, which not only forms two scales of different species of thirds, but also fixes their distance from each other to be a minor third, with the same number of dieses, either sharp or flat.

With respect to these scales, every one has a principal, or governing note, to which all the other sounds refer, and stand in a particular relation.

This.

This principal note regulates the intervals, position, and motion of all the other sounds in that scale; and, by a kind of attraction, draws them to itself, or its harmonics, to finish the termination on the final place of repose.

This principal note is distinguished by the moderns as the key-note, or the tone; and sometimes in a more enlarged sense, both by the ancients and moderns, the Mode. But in order to be better understood in this very difficult part of the harmonica, I shall apply the words key-note, or tone, to the principal note of a simple scale; and the word *Mode* to comprehend several scales, whose situations, or relations, to the first or governing scale are such as open a ready and easy communication for the mutations; which are made by substituting one scale in the place of another, or by moving from the principal to any of the relative scales, or the contrary: by which means the sentiment is continually varied, and the same thought appears in a new light by being removed into a different scale.

Thus these additional scales, subject to certain limitations with respect to the principal or governing scale, constitute a mode, in the most extensive sense of the word; for as all the sounds in a scale refer to, and depend on, the principal sound, so all these secondary scales in like manner refer to, and depend on, the first, as a principal or governing scale. And in this sense every principal sound of the different scales in each diagram may, by a like addition of secondary scales, be formed into a mode; and although this mode is described by the principal sound of the first scale, as D major, D minor, &c. yet all the secondary  
scales

scales are understood to be connected with it, in as ample a manner as if they had been severally named. In order therefore to discover this principal note, the following observation will serve as a rule :

The fundamental bass to the last note of each tetrachord is the principal note: and the moderns call the scale by its name.

Thus in the first diapason at R, (Plate III.) we finish the first tetrachord at A, the fundamental bass; and therefore the whole composition of that tetrachord forms the scale of A, as the next tetrachord does that of D scale.

The disjunction, or conjunction, makes no other difference than that of obliging us to continue the names of the notes in the same order till the whole tetrachord is completed; and under the last note we find the bass, which gives its name to that scale.

Thus at the second species of diapason at R, the note after the disjunction begins the tetrachord at A $\sharp$ , and though B is the last note of that diapason, yet it is not of the tetrachord; and therefore to find the principal note we must return to C $\sharp$  and D $\sharp$  in order to finish the tetrachord; and the bass G $\sharp$  is found under the last, or D $\sharp$ , and is the principal found which gives its name to that scale.

But

But that this may be better understood, I will examine and make some observations on the harmonic constitutions of the tetrachord in the minor and major scales, as formed by the fundamental basses in each diagram.

|                                                                                                                                                               |                                                                                                                                                              |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|
| W. $\left\{ \begin{array}{l} \overbrace{B, C, D, E,}^{\text{1st Tetrachord.}} \\ G, A, A, A, \\ E, E, F, E, \\ \text{Roots } E, A, D, A. \end{array} \right.$ | X. $\left\{ \begin{array}{l} \overbrace{F, E, D, C,}^{\text{2d Tetrachord.}} \\ C, C, B, C, \\ A, G, G, G, \\ \text{Roots } F, C, G, C. \end{array} \right.$ |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|

In each of these tetrachords we find that in four combinations the fundamental bass, which gives its name to the key or scale, is used twice in each, viz. the second and last bass at A in the first, and the same at C in the second tetrachord; besides which, a repetition of the same key-note is discovered among the harmonics in three combinations out of four, in order that the impression of the principal sound and its harmonics, with which the ear should be most familiar, may predominate, and be preserved without any improper mixture of other sounds.

From these two tetrachords B, C, D, E, and F, E, D, C, the minor and major scales are extracted: and, except the difference of the pitch, the intervals are the same in each, without any sharp or flat diesis, or any distinction to support the idea of minor and major scales, till we apply the fundamental basses as above; which is the only rule or criterion by which we can judge of the difference: for the same melody or system may be formed in either scale, by applying different roots. See Plate III. at R and S, where the roots at R have minor thirds, and the same system at S has major thirds to each root. This, together

together with other instances, proves that the theories of the Grecians were founded in harmony; without which their minor and major scales must have remained undetermined. The two tetrachords at W and X are in their harmonic parts most unlike; for, exclusive of the observations already made, it appears that A, the principal or governing note of the first tetrachord at W, is used but once at X in four combinations, and is then a major third. In like manner it appears that C, the principal or governing note of the second tetrachord at X, has only one place at W in four combinations, and is then a minor third. These alterations preserve the character of each, and remove every possibility of mistaking one for the other; and at the same time discover the great order in which these operations are conducted, and the different effects they produce. If the Aristoxenians had not excluded the ratio from their theory, the difference of the major and minor tone, and other intervals, would have been explained by them. But this will be considered when we come to treat of the Ratio in Part II.

All tetrachords are formed on the same principles as those above, and their several positions, as at Plates III. and VII. discover the increase of the flat and sharp dieses, necessary to perfect every scale, even in the most transposed position. And hence we may conclude that the seven species of diapason, which are the foundation of the modes, could not be intended, as some imagine, to discover new scales of a different constitution, but only to change the position of those already existing, in order to improve and perfect the mutations. It is true that the enharmonic and chromatic dieses are described in their genders; and by particular divisions of the tetrachord their  
magnitude

magnitude is ascertained; but as this alone was not sufficient, where so extensive a distribution of intervals is wanted for the formation of so many scales, something more was necessary to increase those animating powers, essential to the mutations.

The different species of diapason, as before explained, answer this great end in every respect; for the formation of all scales out of the natural not only depends on the application of a flat or sharp diesis to alter some note a semitone higher or lower, but as we proceed with the diapasons, the number and position of the dieses, or flats and sharps, are fixed and determined from one to six in each diagram. By this method the major or minor scale is known the instant the number of dieses is discovered, and the absolute pitch of the key or tone naturally follows. But that this may be attended with as little difficulty as possible, I have formed all the scales from the two natural at A and C to the greatest number of sharp and flat dieses that can be wanted to complete the modes, even in their extreme position, by which all possible mutation is perfected. See Plates VIII. and IX.

In examining these scales we discover that both the sharp and flat dieses increase in a numerical order by a series of fifths from the natural scales A and C, extracted from the first species of diapason; and that as the sharps ascend by fifths from these natural scales, so the flats descend in a contrary direction by fifths; and as there are major and minor scales peculiar to each diesis, they are enclosed in a bracket in the margin, together with the names of the scales, the number of dieses necessary for their formation, and the species of diapason from whence they are extracted. The three terms, or fundamental bases are also

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placed

placed at the end of the scales, as being the foundation from whence they are formed. The composition of the tones and semitones is the same in all the major scales. The minor scales compared with each other have the same composition; but if compared with the major, there is a very essential difference in the position of the tones and semitones: the first is distinguished by being open notes, and the second by being black in all these scales.

There is also another difference with respect to the ascending or descending of the major or minor scales. In the first there is no alteration in moving either way; but in the second there are two accidental major sounds ascending, which are not necessary in descending. The preserving of a diatonic succession occasions this difference in descending and ascending in the minor scales.

These scales being explained, I proceed to their use and application, by which that most captivating part, the mutations, will be better understood.

The words tone, and mode have by some modern authors been used as synonymous terms; but, as I have observed before, Euclid tells us, [p. 2.] that "tone is a system without latitude;" that is, that the same system may move to the acute or the grave, without any limitation, provided the tones and semitones succeed in the same order, as in the first diapason or original scale.

From

From this definition it appears that every scale may be a tone, and have its principal note or key as marked in the margins of Plates VIII. and IX. but with respect to a mode, it is formed of several of those scales, subject to certain laws and limitations, as has been observed before ; without which it would be impossible to have either a consonant or dissonant mutation, or any other, agreeable to the directions of Euclid ; who, after enumerating four sorts of mutations, says [p. 20.] “ the mutation by tone is when we make a change from dorian “ to phrygian ; or, in general, from any one of the thirteen “ tones into any other ; and these mutations are made beginning “ from hemitone to diapason.”

From this description of the mutations, proceeding from hemitone to diapason, it appears that when the diapason, or principal tone of the mode, is formed, the mutation is made by an addition or subtraction of one diesis from the original number. Hence we discover that the mutations depend on the increase or decrease of the diesis. If we examine the scales at Plates VIII. and IX. it appears that any number of diesis forms two scales ; one with a major and the other with a minor third. In consequence of this we have a choice in fixing the principal tone of a mode in either of these scales ; and though the number of relative scales will be the same in each, yet those of the same species of third with the principal tone of the mode must, as being primary scales, have a preference in the mutations to those of a different species, which are of a secondary nature, and are used in a transient way, to give a colouring, and vary the lights and shades of the mutations

As three terms in the major scale have the same species of thirds, they determine the names of the three primary scales of the mode, one of which is a fifth above, and another a fifth below the governing or principal tone. The other three fundamental basses, which have a different species of thirds, describe the names of the secondary scales of the same mode. We discover from hence that every mode is, or may be, composed of six scales; viz. three with major, and three with minor, thirds; and that the pitch of the governing or principal tone is the name of the mode, and includes its five auxiliary scales, from which the mutations are made.

The harmonious constitution of a mode being thus explained, it is proper to observe that from the number of scales we may fix the principal tone or pitch of the mode higher or lower, in order that every species of voice and instrument may be accommodated to move within the limits best adapted to its natural powers. But this makes no alteration in the composition of the mode, whose harmonic parts, as well as the auxiliary scales to the principal tone, are the same.

The inversion of the tetrachords in the second diagram, Plate VII. being explained in a former part, and several observations made on the conjoined and disjoined tetrachords, I proceed to examine the extensive powers of the fundamental basses taken from the great and unchangeable system, which is the foundation of the tetrachords, and all other systems.

The principal motion of the fundamental basses is the interval of a fifth, of which the fourth is the inversion; it being the same

same whether we move from C to F five descending, or four ascending, as shall be most convenient in writing to bring them to their nearest degrees.

It has been found that every system of the tetrachord is the product of three terms or fundamental basses; and on examining the motion of the basses it appears that two of the harmonics move with the bass, and the third harmonic is common to two basses, and is a ligature; but of the two sounds that move, one is always a tone, and the other a semitone. See p. 63.

This is the harmonious constitution of the tetrachord; and the diatonic scale, or system of the diapason, peculiar to that tetrachord, is formed by extracting from the harmonics such sounds as are proper for a scale when brought to their nearest degrees.

The name or pitch of this scale is next to be discovered. In every tetrachord there are four combinations which occasion a repetition of one of the basses; the name of the scale is always the same as the note repeated.

I must here observe that the inversion of the first diagram has not only discovered a new creation of sounds, but that the symmenon tetrachord, from whence Guido formed his moll hexachord, is found in the fourth species of the diapason at the second tetrachord, formed by the first flat diesis. [Plate VII.] As this tetrachord had no place in the first diagram, it confirms the necessity of the inversion, without which the mutations in C and B $\flat$  modes would have been imperfect, by not having the auxiliary scales of a fifth lower or higher to move to. The same defect would have precluded this tetrachord from being a principal

principal tone of a mode, supported with all its relative scales proper for the mutations. Notwithstanding these great discoveries, we find only two original scales, the one with a minor third, and the other with a major third next the principal fundamental bass or tone; and that this original harmonious constitution of the tetrachord may be moved higher or lower in the mutations, by the use of sharp or flat diesis, in order to support every musical expression proper for every species of voice; so that the solemnity of the grave, the sprightliness of the acute, together with the sweetness of the mean between the two extremes, may move within the compass of their natural powers. For that these constitutions or scales were distinguished by Euclid [p. 2.] as tones, or "a place of the voice capable of system " without latitude," cannot be doubted: and when these scales are used successively, according to the order of the mutations, which admits of the increase or decrease of one diesis, the mode is then formed, with its five auxiliary scales added to the principal tone. For as every scale or tone has its principal or governing note, so every mode has its principal or governing scale, to which the other five auxiliary scales refer. And though we are not obliged to use them all, yet it is necessary to know how far their powers extend in the mutations, that a proper keeping with the governing scale may preserve its first impression on the ear. For this purpose one diesis more or less than the original number of the governing scale is the limitation of a simple mode.

From these principles we may form a perfect consonant mutation of a fifth higher or lower, or an imperfect mutation of a third higher or lower than the original pitch of the mode; or

we

we may, from among the different scales, form a dissonant mutation of a tone higher, or lower. These mutations are not difficult, if we attend to a very simple rule, which is, that after the pitch of the governing scale is known, we proceed to form a system of six degrees from that pitch or tone; and if it has a minor third, we must descend from the tone, but if a major third we must ascend in a contrary direction: as for example, in the minor mode of A the system will be A, G, F, E, D, C; but contrary to this in the major mode of C, the system must ascend, C, D, E, F, G, A. These are the names of the scales which may be applied to regulate the mutations in this natural mode. The rule is the same with respect to any number of sharp or flat dieses. As to the distinction between the primary and secondary scales in the major mode, the first, fourth and fifth scales ascending, by having the same species of thirds, are the primary; and the second, third, and sixth ascending, whose thirds are different from the other, are secondary: but in the minor mode, the first, fourth and fifth scales ascending are the primary; and the second, third and sixth descending are the secondary scales.

It will now be proper to turn to Euclid on this very important subject, whose decisive authority will confirm all the foregoing observations.

In page 20, he says, "Mutation is described four ways. By Gender, System, Tone, and Melopæia." He explains each, of these, but rather too concisely for so interesting a subject,

The

The mutation " by Gender (he says) is when there is a  
" transition from diatonic to chromatic, or enharmonic, or  
" from the chromatic or enharmonic to any of the rest.

The number and variety of dieses discovered in the seven  
species of the diapason in each diagram, and applied to the  
genders, are different to illustrate this mutation.

The second " by System (he tells us) is when the mu-  
" tation is made from conjunction to disjunction, or the con-  
" trary.

This mutation by the alteration of one diesis is a fifth lower  
by conjunction, or a fifth higher by disjunction. See the first  
and fourth species of diapason, Plate III. fig. R and S in the first  
diagram, and the contrary in the second, plate VII.

The 3d " by Tone (he says) is when we make a change  
" from the dorian to the phrygian, or from the phrygian to  
" the lydian, or hypermixolydian, or hypodorian; or, in gene-  
" ral, from any one of the thirteen tones into any of the rest;  
" and mutations are made beginning from hemitone even unto  
" diapason."

If these changes of the tone proceed from the interval of  
diesis to diapason, we have already seen how the system of dia-  
pason, being formed as a scale at any given pitch, is changed,  
and the whole constitution of its parts altered by the introduction  
of a sharp or flat diesis. By this construction, which is a very  
natural

natural one, we come to have a knowledge of the ancient tone or mode, whose natural position can receive no mutation but by a sharp or flat diesis.

With respect to a change from dorian to phrygian, &c. or, in general, from any one of the thirteen tones into any other, the spiss intervals, or genders, sufficiently supply whatever is wanted in compositions the most extensive and elaborate; such as operas, oratorios, and the like; in which a kind of wandering modulation is required to express the recitatives, agreeably to the great variety of sentiment.

These fugitive mutations are transient, serving to diversify and support the several incidents which must arise in long performances, in which the thirteen tones (mentioned by Euclid) may be used with the greatest success.

Euclid, in addition to his former definitions, says [p. 21.]  
“ Some mutations are made by consonant intervals, others by  
“ dissonant; and some of these are less concinnous or inconcin-  
“ nous, some more. Those that have a greater communion  
“ among themselves are more concinnous, and those that have  
“ less are more inconcinuous; for in every mutation there must  
“ be something common, either sound, or interval, or system:  
“ and this communion is taken according to the similitude of  
“ sounds; for when similar sounds fall mutually in the muta-  
“ tions, according as they partake of the spiss the mutation is  
“ concinnous; but when dissimilar, inconcinuous.”

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The

The "consonant mutation" has already been described by the alteration of a diesis, and the mutation is a fifth higher or lower with the same species of third. Therefore these mutations are more concinnous, by having a more natural communion with each other, than a dissonant mutation, which has nothing common; as for instance, if the change is made from the fifth above the governing scale to the fifth below with the same species of thirds, or the contrary, we have a dissonant mutation, in which the necessary alterations are two dieses; and if one scale is compared with the other, the difference will be a tone major in every part, and consequently not only less concinnous than the other described above, but in every part a dissonant mutation.

With respect to the next part of his definition, wherein he tells us that "in every mutation there must be something common; either sound, interval, or system;" "Sound," may be the pitch of a scale with a minor third, and afterwards move into another with a major third, with the same name; as from A minor to A major, or from major to minor, in which three dieses make the difference between the two scales.

In this mutation the thirds to the fundamental basses are altered from minor to major, but the fundamental basses are the same; and the whole forms a mixed mode in A, by having two species of thirds. See Plate VIII. In all mutations of this sort there is a difference of three dieses between the two scales.

"Interval"

“Interval” is the next thing which may be common.

The most remarkable interval which distinguishes the character of the scales in their most essential parts is that of the minor or major third; and therefore if the mutation is regulated by either of these intervals, it must be in changing from one minor scale to another, or from a major to a major third.

These mutations, as before observed, will be a fifth higher or lower, which are the perfect consonant mutations; or they may be a tone higher or lower, which have been before distinguished as dissonant.

As to the mutation in which the “Systems” may be common, the seven species of diapason in each diagram are so many examples of this change; for the systems at R are the same as at S, [Plate III.] and yet, by the application of the fundamental basses, we have minor thirds at R, and major thirds at S. The same two species of thirds are also discovered throughout the second diagram at T and U. [Plate VII.] Though the systems are the same in each species of diapason, yet the difference of the pitch of these systems, by the application of the fundamental basses, is a minor third; and the same number of sharp or flat dieses applied to one will be equally necessary to the other. See Plates VIII. and IX.

“ This communion (Euclid says) is taken according to “ the similitude of sounds.”

Thus far Euclid has directed the great variety of modulation; and taught us to vary the sentiment by consonant and dissonant mutations, together with a mixed mode, and the imperfect mutation of a minor third higher or lower; but the other imperfect mutation of a major third higher or lower is discovered in Euclid, [page 2.] where he says “ Mutation is a “ transposition of a similar into a dissimilar place.”

This description is much more applicable to the mutation of a major third higher than to any of the others; and where the alterations are so much greater they must necessarily be more dissimilar; as for instance, if we change from C major to E minor, the tone or pitch is removed a major third higher, with the addition of one diesis, and a different species of third to each scale.

If we compare this mutation with the consonant one, by disjunction or conjunction, and the imperfect one of a minor third, we discover that in the first it is made by the alteration of one diesis, but the thirds are of the same species; and in the second there is no other difference than in the species of thirds. Hence it appears that the alterations of a diesis, or of the same species of thirds, which in the former mutations acted separately, are now united, and concur to make this a dissimilar mutation. As to the dissonant mutations, they have been distinguished as such by Euclid, and cannot therefore be applied in the present case.

I must

I must observe, that all the mutations are found in the tetrachords, Plates III. and VII. and also Plates VIII. and IX. But so respectable an authority as that of Euclid is not to be neglected, especially as he has explained himself fully on the subject of the mutations, and as his doctrine is conformable to the most established rules of the best modern composers.

Having discovered the laws of the mutations, and the limitations of each mode, which are confined to six scales only, from whence we have an almost endless variety of modulation, which is not only the most animating part of musical composition, but prevents that tiresome monotony of cadence, which is the constant attendant of moving in one scale only ; I shall now consider how the number of modes is fixed and determined by the seven-species of diapason in each diagram, agreeably to the Aristoxenian writers ; by which the agreement and harmony of the several parts of this theory will be united and confirmed in a satisfactory manner, and remove every objection that has been made to this part of the ancient harmonica.

There are different opinions among the ancient writers with respect to the number of modes. Aristoxenus, Euclid, and Aristides Quintilianus, mention thirteen. Alypius gives the names and characters of fifteen.

These two opinions are combated by Ptolemy, who has written expressly in favour of seven only ; of whose work the learned Dr. Wallis has given a translation in Latin with notes.

As

As I do not intend to become a party in the dispute, I shall only assign some probable reason for this difference of opinion, which will be supported by the whole theory, and will lead us to the knowledge of their use and application; but with respect to their names, as they can be of no use to the modern musician, I shall leave them for those who may think them of more consequence.

It has been observed before that Euclid, in describing the seven species of diapason [p. 15.] gives at the same time the names of seven modes, by which, he says, they were called by the ancients, and which I have placed accordingly, in Plate II. fig. 1. But (in page 19.) he tells us that Aristoxenus mentions thirteen tones or modes, and he gives their names, distinguishing some as acute, and others grave.

To reconcile these different opinions will not be difficult, if it be granted (and I think there can be no doubt) that the inversion of the diagram, already explained page 43, was so universally understood as to make a particular description of it unnecessary; and the more so, if we consider that the intervals, tetrachords, hemitones, and tone of disjunction, were the same in each, except the difference of the thirds, and their motion in a contrary direction; and therefore an explanation of the first could be equally applied to the second diagram by those who were acquainted with the doctrine; for the flat dieses were as necessary to perfect this theory as the sharp, and the want of either would have destroyed the whole.

This

This observation is so true, that no practical musician can refuse his assent to it; especially if it be considered that the formation of the spiss intervals and genders in each diagram, which correspond so exactly with the method prescribed by the Aristoxenian writers, could not be discovered on any other principles.

The construction of the several species of diapason, already explained, has proved the necessity of six sharp and six flat dieses. Therefore, if to these twelve dieses we add the natural scale, which is the medium between the two species of sharp and flat dieses, we have exactly the number of thirteen tones or modes mentioned by Euclid.

By this double doctrine of the diagrams (if I may call it so) the ancients are vindicated from the charge of inconsistency, or of giving contradictory accounts of the modes. Some modern writers of reputation have observed that the more ancient of the Greek writers looked upon grave sounds as high, and acute ones as low; and that this distinction was afterwards changed to the contrary, and has since prevailed universally.

If Ptolemy was not acquainted with this double doctrine, it is as easy to account for his limiting them to the seven species of the first diagram, as it is for Alypius to make them fifteen by the addition of two more dieses. It was very easy to fall into this mistake from not observing the limitations prescribed by the diapasons in the two diagrams.

If

If we examine the mutations of the thirteen modes [Plates VIII. and IX.] it will be found that the extreme modes at the sixth sharp and sixth flat cannot be equally perfect with the others, unless these additional dieses are continued, to give a latitude of mutation, by rising a fifth, from a scale of six sharps to one of seven; or to fall a fifth, from a scale of six flats to another of seven.

Yet these auxiliary scales must not be considered as principals, but only as increasing and perfecting the powers of mutation in these extreme modes, in common with the others.

Without attending to this observation, and seeing how much the number of modes depends on those of the dieses, we should determine in favour of the fifteen, mentioned by Alypius, rather than the thirteen of the Aristoxenians, until we recollect the first limitations of the modes, which confined them to the species of the diapason, in which neither of these additional dieses is found; because the notes to which they are applied are the natural and fixed points, from whence the diapasons in each diagram begin their progression. Thus B natural in the first diagram, and F natural in the second, could receive no alteration in their first application; but from the necessity of forming two auxiliary scales to perfect the mutations of six sharps and six flats, the seventh sharp and flat diesis became absolutely necessary; though neither the one nor the other can be a principal note, or pitch of a mode.

These

These extreme sounds are a part of the sesquialter chromatic gender, and form an interval of twenty-one parts of the tetrachord, equal to a deficient fourth peculiar to this gender.

Hence we may conclude that if the spiss intervals within the limits of diapason (as described at Plate IV.) are to determine the number of modes, whose mutations are fully complete, there cannot be more or less than thirteen: for it appears, that if the minor and major scales were each to be formed into modes in separate and distinct classes, and be brought to the account, we should have an even number; because the two natural scales being added to the two belonging to each diesis, would make them continually even numbers. But as no such is contended for among the ancients, we may conclude that the six sharp and six flat dieses, added to the natural mode, complete the whole number, which Euclid and others have mentioned, and which must therefore be limited to thirteen.

I have before taken notice of the Grecian diagrams, which, being a composition of tones major and limmas, have occasioned one of the principal objections by the moderns to their doctrine.

This composition of sounds has always been understood by the moderns to be the scale, or natural degrees of voice, in which the eighths, fifths, and fourths only could be discovered; but as the imperfect concords of thirds and sixes major and minor were not to be found in the diagrams, and as they make so great a part of musical composition, they have con-

M

cluded

cluded it was impossible for the ancients to have any knowledge of harmony, by uniting two or more melodies together.

They did not consider the great advantage of a general arrangement of the seven species of the diapason, the nature and formation of the tetrachords, the genders, and other parts of the harmonica, which would so easily have removed their objections, by supplying them with the imperfect concords, and every other musical interval: nor did they consider the nature and extent of their objection; for if an untuneable, or inconcinuous series of sounds was improper for harmony, it must for the same reason be improper for melody, even if attempted by the most eminent professors.

This last, namely, melody, the advocates for modern improvements in harmony allow the Grecians to have had in very great perfection; but yet deny the possibility of their having any knowledge of combination or harmony, from the very nature and constitution of their scale, or diagram.

As this objection has already been removed by the discovery of the imperfect concords, together with the tone major and minor, semitone major and minor, and many other intervals, which could not be found in a scale of tones major and limmas, but which, according to the rules laid down by Euclid, have been discovered in forming the genders; one may conclude that the scale of tones major and limmas was originally intended for a very different purpose than that to which it has been applied

applied by the moderns. But that I may not draw a conclusion without good and sufficient authority, I shall in support of it shew the original of this scale, its application and importance in a musical theory.

The seven species of diapason, as described at Plate III. fig. R, are regulated agreeably to this scale. In order to understand these properly, we must discover some particular sound, not only the most characteristic in the diapason, but common to each species.

This sound was distinguished by the ancients in every diapason by the name of Mese, and its position was the next sound to the tone of disjunction, which is marked by a cross † in each species of diapason. Plate III.

This mese determines the utmost limits of the tetrachord to which it is applied; and if we proceed to the sound above it, the principal sound, or key, is altered a fifth more acute, which requires an alteration of one diesis to complete the whole.

As the mese was the only sound in every diapason, that required two basses to distinguish and mark the disjunction, its position was regulated by the order in which the seven species of the diapason were placed. By this the place of the principal sound, or key-note, was easily discovered, as well as the number of diesis necessary to form a new scale.

M 2

Thus

Thus the mese was always found to be a fifth to the principal note of the scale; but when the temperament, or second bass, was applied, it was an eighth to it, and the principal sound or key-note of the succeeding tetrachord.

This distinguished character of the mese, supported by the application of the fundamental basses and harmonic principles, prevented the disagreeable effects of three tones in succession; and the sounds that formed the scale are extracted from the harmonics of each tetrachord.

Thus the knowledge of the ancients with respect to harmony appears to have been very great, in not admitting the imperfections of the system of the diapason: a system incapable of harmonic principles without introducing a new scale, or destroying the whole theory by having three tones in succession.

If we examine the seven species of diapason at R, Plate III. we discover that the mese descends in an alphabetical order from the first to the seventh species of diapason thus, A, G#, F#, E, D#, C#, B.

This system, at first sight, has the appearance of being diatonic, but will be found to be tones major and limmas in the Second Part of this work, where the magnitude of all intervals will be determined by the ratio. Thus from A to G#, and from E to D#, will each be found a limma, and the other intervals will be tones major; all of which are formed by the mese, or note of disjunction.

If

If we attend to the regular order in which the seven species of diapason succeed each other, and to the position of the mese, together with the number of dieses necessary to form the several tetrachords, on which the mutations so much depend, we must own this to be a theory deserving our greatest admiration ; not only from the wonderful agreement discovered among the several parts, but as it contains more musical knowledge with respect to harmony, than any theory formed by the moderns.

I shall end this part, which has already been longer than I intended, with one observation, which may put an end to the dispute between the advocates for ancient, and those for modern, harmony ; and which I have never yet met with in any author. This is, that the great, the perfect, and immutable system of the Grecians, so little understood, and almost universally condemned by the moderns, is a very essential part of our own theory : for if the principal sounds, or pitch, of the several modern scales are placed in an alphabetical order, in the same manner as the mese, the relation of each principal sound to the next will be either a tone major or limma.

In consequence of this observation it appears that the ancients had as good a claim to the knowledge of harmony as the moderns. The truth is, the principles of science have always been the same, and are governed by the same fixed and unerring laws, though they are not always equally understood : and I wish to have any concord, or other musical interval, pointed out, which is not to be found in the harmonica of the Grecians.

As

As I am desirous of removing every objection to the Grecian doctrine, I must not pass over a very material one."

Aristoxenus says [p. 20.] "that many intervals are modulated less than diatessaron, but are all dissonant."

If this were true, all systems must be discords, which would destroy the harmonious constitution of the tetrachord; but as he cannot be understood to have any such intention, it must relate to the movement of the roots or fundamental basses, which move by fifths, or fourths: the inversion of the fifths: this will be easily understood; for in this theory we have no instance of the roots moving a tone, and the third is the only motion that remains; the consequence of which will be a change of key from major to minor, or the contrary, and will occasion a temperament that always attends a fourth term; which, in the Second Part, in treating of discords, will be more particularly explained. At present it is sufficient to observe, that no such motion of falling or rising a third can be found among the fundamental basses of the tetrachords, except at the disjunction at S, Plate III. which changes the scale a fifth higher, and is not one of the primes, but is a secondary fundamental bass, or fourth term, and moves immediately into one of the primes of the next tetrachord.

This explanation discovers that this part of Aristoxenus's doctrine must be applied to the roots, and not to the systems or melodies.

Euclid

Euclid [p. 8.] mentions something similar to what Aristoxenus has observed. He first enumerates the species of consonances, and then says that "all intervals less than diatessaron" are dissonant," and tells us what they are; such as "diesis, hemitone, tone, trihemitone, and ditone."

In this account of dissonant intervals there is no mention made of the limma; but this very account furnishes a proof of Euclid's being well acquainted with the imperfect consonances; notwithstanding they are excluded from a place in the diagrams, in the same manner as the limma is in all other places of the harmonics.

As the same objection to all intervals less than diatessaron being admitted among the primes, or fundamental basses, must remain, as before mentioned, we may observe that the diagrams are not disordered by introducing the imperfect consonances, but are preserved in their first and original state of tones major and limmas, as fundamental basses.

It may be expected that the discords which make so essential a part of a musical theory should be treated of next, but as they will be better understood when explained by the Ratio, I shall reserve them, together with their preparations and resolutions, for the Second Part.

These observations do not so properly come under the sixth general head of the harmonica, according to Euclid; yet as the knowledge of the primary fundamental basses is so essential  
to

to the mutations, it was necessary to place them in that light which their importance demands.

I now proceed to the Melopæia, or Seventh and last Part of the Ancient Harmonica.

## VII. Of the MELOPÆIA.

The Melopæia is that part of the harmonica which determines the motion, or succession, of sounds proper to produce air, or melody; and, according to Euclid [p. 22.] consists of Ductus, Nexus, Petteia, and Extensio.

“Ductus” is a gradual succession, either ascending or descending.

“Nexus” directs the succession to be by harmonic intervals, such as a 3d, 4th, 5th, or 8th, and the like.

“Petteia” is a repetition of a sound in the same tension.

“Extensio” is a continued fingering of a sound in the same tension.

Aristides Quintilianus [p. 29.] has given many divisions and subdivisions of the Melopæia; but if it means no more than simply to determine all possible motion of sounds, without  
any

any regard to the various combinations of time, the four parts mentioned here by Euclid give every thing that the subject demands with respect to a simple melody ; which, notwithstanding, may be applied to two or more, with equal success, in the following order :

“ Ductus,” or a succession of degrees :

Two, and sometimes three, melodies may move in similar motion ; and two in the contrary, either ascending or descending.

Four melodies may move in succession by uniting similar and contrary motion.

“ Nexus,” or the motion of harmonic intervals :

Two, and sometimes three, melodies may move by harmonic intervals in the similar motion ; or three, and sometimes four, in the contrary.

The two motions of the Ductus and Nexus may be variously changed, and applied to one, two, or more melodies.

“ Petteia,” or the repetition of a sound in the same tension :

By this the various combinations of time may be described, but nothing relating to tune : or it may be applied as a basis to every sound in succession in the melodies ; for in this last  
N instance

instance, as the number of pulsations, or repetitions to divide the time, is not limited, such an application of the Petteia to sounds in succession will constitute air, or melody, and when united with the other parts of the Melopæia, in two or more melodies, is capable of producing great and noble effects.

“ Extension :”

To know how to apply this part of the Melopæia, it will be necessary to discover if a sound, continuing in the same tension, can be common to two or more fundamental basses.

Of this we have an example in all basses moving a fifth; as in the disjunction of the tetrachord at R, Plate III. where the last note of the tetrachord is a fifth to the first bass, and an eighth to the second, and continues in the same tension during the change of the bass and two of the harmonics.

All ligatures, syncopation, and the preparation of discords, are so many applications of extension to two basses, and are in no respect contrary to the laws of harmonics. But if we consider extension in its greatest latitude, it will be found to be an application proper for three, four, or any number of basses or melodies in succession: for as extension, when confined to a simple sound, has not in itself any of those varieties we meet with in tune and time, it follows, that the motion of other sounds placed in opposition to it, according to the three first parts of the Melopæia before described, must procure us this effect. And if any discords should be generated by the sounds in motion, when compared with the note of extension, they

they are rather accidental than essential, because they require no other resolution than that of continuing the motion of those accompaniments, till they finally terminate in a consonant proportion.

Thus extension being applied to C, and the bass moving C, D, E, or E, D, C, at the same time, we have in this case the discord of the seventh at D, the second sound, which is resolved by the motion of the third sound in the bass at E, without any alteration in the extension; and it would be the same if the thirds to the basses and other harmonic sounds were added. But if any discords are generated among the melodies in motion, as frequently happens, they must be regularly prepared and resolved, according to the rules of discords, but without interfering with the extension, which may be continued any quantity of time that the composer pleases.

Many examples might be brought from the best writers, if what has been already said on this subject were not sufficient to explain this important part of the *Melopæia*; without which harmony would lose a great part of its beauty, and be robbed of its principal and most noble effects.

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I have now gone through the Seven Parts into which the Ancients divided the Harmonica. I have given each part a musical construction, by which their mutual relation and agreement have been explained, and reconciled to the laws of harmony, beginning with the first idea or definition of a musical sound, and proceeding to the knowledge of intervals, or the distance to be observed between one sound and another in the

the formation of genders, systems, tones, or modes; from which we have been able to form and regulate the several scales, and discover the relation which every sound in a scale has to its principal or fundamental sound; likewise how any given scale, taken as a principal one, is connected with others, by which, under certain limitations with respect to the dieses or position of the hemitones, the consonant and dissonant mutations are regulated; the number also of sharp and flat dieses necessary to perfect every scale, has been collected and disposed in a particular order, proper for the discovery of the genders, as well as the spiss and all other intervals, which can be wanted in the most elaborate compositions.

I was induced to, and encouraged in, this extensive and arduous task by some discoveries which appeared to be of the greatest importance towards the undertaking and explaining of a theory, which had for many ages been only a subject of dispute; neither party being able to determine any thing conclusive in support of their different opinions, either for, or against, the harmonic principles of the Grecian doctrine. Nor could I have flattered myself with better success, had not the inversion of the first diagram offered something the most interesting and agreeable to my wishes. To this succeeded the order of placing the seven species of diapason in each diagram, which encouraged me yet more to proceed; but when the tetrachords in their various positions could not be formed without the sharp and flat dieses, and the conjunction and disjunction could not be explained without the application of the harmonic principles, I remained no longer in doubt, but was fully convinced that without a perfect knowledge of harmony it must have been  
been

been impossible to have formed a theory so expressive and curious as the Grecian in all its parts ; nor can it be understood, unless explained by the same laws, by which it was originally formed.

However, that I may give some satisfaction to those who object to all theories not demonstrated by numbers and supported by the Ratio, I have determined to try how far the power of numbers will carry me in a theory of harmonics, agreeably to the *Pythagorean* doctrine, which will be the subject of the Second Part of this Work.

END OF THE FIRST PART.

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INTRO-



# INTRODUCTION

T O

## THE SECOND PART.

**T**HERE are various applications and definitions given by the Ancients of the word *Musick*; but as they do not fall in with the plan proposed, I shall consider Music in a less extensive sense, and apply it only to Harmony and Melody.

Harmony is the science of sounds, of which melody is a part.

Air

Air and motion, under certain regular measures and proportions, are the efficient causes:

To discover what this motion is, and how to apply it to musical sounds, so as to be able to know their agreement, difference, or mutual relation, is the object of Science.

It will be proper, as a preliminary, to explain the nature and properties of musical strings, with respect to their different lengths and vibrations, whose agreement or difference is the object of the Ratio.

On these a foundation may be laid, from which a theory, the most rational and agreeable to harmony, may be formed; one that will not only determine the proportion, agreement, or difference of sounds, with respect to acute and grave, but will teach us their use and application, so as to answer all the demands that harmony or melody can possibly make. But to arrive at this point of perfection, some original leading principle, some universal character must be discovered, whereby not only con-

cords

cords and discords, and their various combinations may be known, but their progressions and successions; for unless this very useful part of musical knowledge can be determined by, and reconciled to, the laws of science as well as of practice, we should still want a faithful guide, an unerring principle.

It is not sufficient to know the several proportions of harmonical combination, we must endeavour to discover some principle that will not only teach us how to form the mass, but how to give it life and motion.

All sounds are formed or generated by motion; that is, by percussion or collision; and as the air is the medium that conveys the sound to the ear, if the motion cease, the sound will also cease; and to whatever distance the sound is carried along this medium, so far the motion passeth, as it were in a sphere; according to the force with which the sonorous body, the centre, is acted upon.

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The parts of a sonorous body being thus put in motion, the trembling or vibration is either equal and uniform, or else unequal and irregular; and according to the constitution of the sonorous body is swifter or slower: from whence arises the difference of sounds.

Sounds are concinnous, or inconcinnous; that is, such as are proper, or improper, for music.

Again, they have the difference of acute and grave; which, like great and small, are but relative ideas; each of them has a certain magnitude, but one only is great, and the other small: thus if two sounds of different dimensions be compared, one is acute and the other grave.

This relation of acuteness and gravity is one of the principal and most interesting parts of a musical theory, and is properly the object of mathematical demonstration.

The

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The degrees of sound may be infinite ; yet with respect to those which are proper for music, they are limited.

All sounds differing in acuteness and gravity constitute an Interval.

This interval may form a concord or a discord, according to the relation of inequality, or particular difference of tune. But in order to fix the degrees of tune, and measure their relations by certain determinate qualities, we must have recourse to the *Vibrations* of elastic bodies and their different *Lengths*.

If two strings of the same length, thickness, and tension, be put in motion, they will vibrate in equal times ; that is, they will move in the same measure, joining and uniting in every course and recourse, keeping still the same equality of motion and identity of tune, having no interval or space between. Therefore the habitude of these two strings is called unison, and has the same relation

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to

to sounds that unity has to numbers, and is in the ratio of 1 to 1, or of 2 to 2, &c.

If two strings are in all respects the same, except their lengths, which are in the proportion of 2 to 1, that is, when the grave string 2 is as long again as the acute 1, the gravest sound, or longest string, will make but one vibration in the same time that the acute, or shortest string, makes two; and when compared together will make the interval of an octave, whose ratio is as 2 to 1.

If six strings are taken, whose *Lengths* are in the following proportion, 1, 2, 3, 4, 5, 6, the next of any two next numbers will be the acute term, and the other the grave. But it will be the reverse of this if the same numbers are applied to the *Vibrations*; for then the less of any two next numbers will be the grave, and the other will be the acute.

Hence we may observe, that the different lengths of chords are inverse or reciprocal, as their different vibra-

## TO THE SECOND PART. 101

vibrations ; and we may equally apply to one what is found true or agreeable to the other. For as their lengths are increased, so their vibrations are in the same proportion decreased, and vice versa.

Here then we discover *two distinct characters* in musical strings : the first gives the ratios, which express the intervals from the acute term to the grave, by comparing a shorter string with a longer ; the other gives the ratios, by comparing the vibrations of a grave with an acute sound ; and each of these operations begins at unity, and moves in a contrary direction.

As the ratios are the same in each, it may seem very immaterial whether we descend or ascend from a given sound or pitch ; but when it shall appear that the formation of *the minor and major scales* depends on these two opposite properties in musical strings, the importance of the application will sufficiently justify the distinction.

For the information of those who are unacquainted with the nature of these operations, I  
have

have at Plate X. fig. K, placed six lines, which represent so many strings, whose different lengths from unity shew the different ratios of their intervals. The figures placed on the left of these lines 1, 2, 3, 4, 5, 6, indicate their different lengths; and those on the right shew the musical interval, as an octave, 12th, 15th, &c. with a letter annexed to each, the more readily to discover their relation to unity, whose pitch is fixed at B.

Thus from B to B is an octave in the ratio of 1. 2.

From B to E is a 12th, or compound 5th, in the ratio of 1. 3.

From B to B a 15th, or compound octave, in the ratio of 1. 4.

From B to G a 17th major, or a major 3d, twice compounded, in the ratio of 1. 5.

From B to E a 19th, or a 5th, twice compounded, in the ratio of 1. 6.

If we compare their mutual relations we have  
2. 3.

## TO THE SECOND PART. 103

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|-----------------------------|----------------------|
| 2. 3. a fifth,              | 3. 4. a fourth,      |
| 2. 4. an octave,            | 3. 5. a sixth major, |
| 2. 5. a tenth, or 3d major, | 3. 6. an octave.     |
| 2. 6. a compound 5th,       | 4. 5. a major third, |
|                             | 4. 6. a fifth,       |
|                             | 5. 6. a minor third, |

And if the fourth string had been doubled, we should have had the minor sixth in the ratio of 5. 8, which would have completed the whole number of consonant intervals; but as there are only three letters in the whole combination, I have limited the series to six numbers, in order to preserve the fundamental bass E in the grave, with its harmonics, a minor 3d and a 5th, in the ratio of 6. 5. 4.

By these harmonics, which are formed from the different *Lengths* of strings in an arithmetic series, we have the true original principle of the minor mode, which has, by descending from unity, a *minor* 3d, next the fundamental or gravest sound at E 6.

I shall now examine the *Vibrations* by the same six numbers, at Plate X. fig. L.

These,

These, like the former, proceed from unity, but in a contrary direction; for as the number of vibrations in a given time is expressed by the same series as the lengths of strings 1, 2, 3, 4, 5, 6, the slower vibrations are the grave, and the quicker are the acute.

Thus the string that vibrates once is an octave graver than the string which vibrates twice in the same time 1. 2; and for the same reason two vibrations will be a fifth graver than three in the same time 2. 3.

If we proceed with the vibrations, we have all the simple and compound intervals expressed by the same ratios from grave to acute, as they were before by the different lengths of strings from acute to grave; also the *major* 3d next to the fundamental, F, A, C, or 4, 5, 6; whereas the *Lengths* of strings placed the *minor* 3d next the gravest or fundamental sound, 6, 5, 4.

From these opposite properties in the same musical strings we discover the two species of thirds,  
and

## TO THE SECOND PART. 105

and in them the essential principles for the formation of the minor and major scales, which in another place will be more fully explained.

I shall now make some observations on the production of consonant intervals as they are derived from a single vibration; by which the true character of a fundamental bass and its harmonic powers will be found the great and leading principle of a theory of harmonics.

In examining the several intervals which proceed from unity, we find that notwithstanding their great variety their effects are all directed to one point, in order to mark and distinguish one original, or primary, sound, superior to the others.

To illustrate this I shall take the whole series, as set down in the harmonic tables, Plate XI. &c. which are formed in consequence of the vibrations of strings, and are the foundation of the second diagram, as the different lengths are of the first.

P

Here

Here we may observe how a given sound at unity in generating its harmonics strengthens itself, and increases its powers by its octaves; thus 1, 2, octave (after which its fifth 2, 3, exists in its way to its next octave 1, 2, 3, 4, which at the same time generates a fourth 3, 4, after the original sound is reinforced by two octaves) proceeds to the imperfect concord the third major 4, 5, or (which is the same) its seventeenth major 1, 5, in its way to its nineteenth 1, 6, or octave to its fifth 3, 6, and finally finishes the whole creation of harmonics with its third octave, 1, 2, 3, 4, 5, 6, 8.

There are two other imperfect concords, viz. the minor third 5, 6, and the minor sixth 5, 8; but as they are formed by the mutual relations, and not by the primary, they cannot be fundamental basses in a major scale.

In the operations above described it must be owned, that nature seems to proceed with the greatest caution in not permitting the fifth and third to appear until the bass advances to its different octaves; thus acquiring fresh power by their addition, it preserves

## TO THE SECOND PART. 107

preserves the first impressiion of a generator or fundamental bass, which otherwise might be weakened and overpowered by hearing the fifth and third too soon.

It appears also that the fifth, the next in degree of perfection as a concord to the octave, is doubled in the series, as 2, 3, and 4, 6; the fourth also (by the mutual relations, not fundamentally) is doubled as another perfect concord 3, 4, and 6, 8. But these fourths are only so many octaves to unity, as 1, 2, 4, 8, and therefore are a great addition to its power.

It appears also, that as neither of the thirds or sixes is doubled, one may conclude this is a sufficient reason for their being distinguished as imperfect concords; and being only once heard in the whole combination, they are prevented from becoming predominant, or from diminishing the natural powers of the fundamental bass.

From these observations the character and perfection of a fundamental bass must be admitted as the original principle of harmonic combination;

P 2

since

since it contains within its octaves all the consonant intervals, which, when united and sung together, yield one of the most pleasing and perfect combinations that can be produced by harmony. But this character does not absolutely depend on its being accompanied with the whole creation of its harmonics, since they are supposed to exist in as full and ample a manner in F, A, C, or 4, 5, 6, as if the whole series in the harmonic tables (which are comprehended in three letters to each root) had been applied : and into whatever position these harmonics may be transposed, as from F, A, C, to A, C, F, or C, F, A, or to any other form, yet F will be the original generator, though less powerful in proportion as the harmonics are further removed from their original place of gravity.

From what has been observed on this subject, it appears to a demonstration how every fundamental bass generates its own harmonics, either by the different *lengths*, or the *vibrations* of strings.

It now remains to shew the relation and connection that, one fundamental bass has with another, in

## TO THE SECOND PART. 109

In order to discover the true principles by which the motions of the different harmonics are regulated so as to constitute melody.

The harmonics of a given found, or fundamental bass, can have no motion, or change, but among themselves, unless the fundamental bass moves to another bass, whose relation to it is such as produces the most agreeable alteration possible among the harmonics.

What this relation is with respect to these basses comes now to be explained.

In the arithmetic series, which has been applied to the different lengths of strings, or their vibrations, we find only *these three primes*, 2, 3, 5; but as the ratio 2, 3, is a more perfect relation than 4, 5, or 5, 6, it must therefore have a preference in directing the motion of the fundamental bass.

As the harmonics have been generated from unity, so the motion of the fundamental basses must be directed from the same point.

Thus

Thus if we take a given sound as a principal generator expressed by unity, and compare it with 3, according to the different lengths of strings, we have 1, 3, or a compound fifth descending; and by a like comparison with their vibrations, we have also a compound fifth ascending from unity. See Plate XII. where C is made the principal generator, and has an immediate communication with its two products, F its fifth below, and G its fifth above.

These are therefore the first and primary motions of the principal generator, and their relations are the most perfect, as they form a geometric series in the ratio of 3; for as the vibrations give 1, 3, ascending, and the lengths of strings give 1, 3, descending, the ratios will be equally expressed by continuing the series, as 1, 3, 9; in which the mean, or second, term is the principal, as unity was before: now if each of these three terms be multiplied by the arithmetic series, or only a part of it, as 4, 5, 6, [Pl. XIII.] or 6, 5, 4, [Pl. XIV.] according to the species of third wanted next the fundamental, we shall have the harmonics of each,  
from

## TO THE SECOND PART. III

from which the systems of tetrachord and diapason are formed.

Hence it appears that a theory of harmonics depends on the *union* of a *geometric* with an *arithmetic* series. The first directs the most perfect motion of the fundamental bass, as from 3, 1, or 3, 9, and by the other we discover the harmonics of each.

These principles being applied to the lengths and vibrations of strings yield all the intervals proper to form the minor or major scales ; and if the geometric progression be continued in the ratio of 3, as in the harmonic tables, we shall have an increasing series of flat dieses by the different lengths of strings, [Pl. XIV.] and of sharps by their vibrations, [Pl. XIII.] together with that vast variety of intervals proper for every mode and gender, and for the mutations.

But before I proceed, it will be proper to take notice of some few operations of numbers that they may be more readily applied to musical intervals.

As

As every musical sound stands in some relation to a given one, or unity, it follows that each must be expressed by its particular number in the harmonic tables, in order to shew its relation to unity, or to any other number, or sound.

There are two sorts, or species, of motion in musical sounds mentioned by the Ancients.

The first the diastematic, or moving by skip, which seems natural to the gravity of the fundamental bass.

The other the systematic, which confined the sounds to the natural degrees of voice, peculiar to melody.

To preserve these species of motion distinct, two sorts of progression are required, which, as they are essential to this theory, are formed into harmonic tables.

The first is the geometric, which is a continued multiplication from unity in the ratio of 3 ;  
as

## TO THE SECOND PART. 113

as 1, 3, 9, 27, 81, &c. These are placed in the margin on the left of each table, and stand as so many Roots, from which the harmonics are formed by the application of the second progression, that is, the arithmetic; which is placed in every column next the root in this series 1, 2, 3, 4, 5, 6, whose common difference is 1. See Plates XIII and XIV. To this progression the 8th is added, in order to extend the whole to the third diapason, which comprehends, as has been observed before, the whole variety of consonant intervals.

The musical letters stand in the next column, as harmonics to the root; and their relation to each other, or to the root, is determined by the arithmetic series, which also expresses the ratio of each interval.

Each root of the geometric series is the multiplier of the arithmetic, and is the common difference of that series: thus the root 3 is the common difference in this series, 3, 6, 9, 12, 15, 18, the harmonics to that root, which are placed immediately after the letters. Also root 9 will multiply

Q

multiply the arithmetic series, and discover its harmonics, as follows: 9, 18, 27, 36, 45, 54. Hence we not only have their mutual relations, but those which arise from comparing them with the harmonics of any other root, however remote.

The harmonics of each root, as thus described, are in their lowest denomination, and the numbers parallel in the succeeding columns are their respective octaves, made so by multiplying the first, and every succeeding, product of the root by 2.

Thus by multiplying either of the primes 2, 3, 5, by 2, the product will be octave; and if continued, as in these tables, will be a series of geometric proportionals in the ratio of 2: but as a series of octaves, applied as basses or roots, can make no alteration in the harmonics, which will always be the same, the other two primes 3 and 5 will abundantly supply all that is necessary.

We have already seen the great variety which arises from a progression in the ratio of 3; it remains now to proceed to the other, which is also geometric,

## TO THE SECOND PART. 115

geometric, in the ratio of 5 ; thus, 5, 25, 125. This series is also formed into harmonic tables ; the first at Pl. XV, represents the different lengths of strings, and corresponds with the first diagram : the second agrees with the vibrations Pl. XVI, and corresponds with the second diagram.

In these tables every root is multiplied by the arithmetic series, 6, 5, 4, or 4, 5, 6, by which we have a minor third 6, 5, next the fundamental bass in the first, and a major third 4, 5, in the second diagram ; and by adding 24, a multiple of root 3, taken from the triple progression, Pl. XVI, we have two species of thirds in the harmonics of root 5, in each diagram ; and by proceeding in the same manner with the quintuple progression, we have the same two species of thirds to every succeeding root ; and the semitone minor 24, 25, is not only the interval formed by these two species, but is also discovered in the quintuple progression, between E 120 of root 5, and E# 125 of root 25, in the second diagram.

Q 2.

As

## 116 INTRODUCTION

As it is of the greatest consequence in a theory of harmonics to know the original of all intervals which proceed from unity, the better to discover their several relations, I shall mention one more : this is the enharmonic diesis, which is formed by comparing 128, a multiple of root 1, Pl. XIII, with 125, root 25, in the quintuple progression, Pl. XVII.

In these harmonic tables the quintuple progression proceeds from 5, to 25, the next page parallel with it, and after to root 125, in the next or following page parallel with the former ; and in the same manner the roots proceed from left to right, as 15, 75, 375, quintuple, or in the ratio of 5. But if they proceed from one root to the next in the same column, the progression is in the triple, as 5, 15, 45, in the ratio of 3.

It must be observed, that the arithmetic series, which multiplies the root, expresses at the same time the ratio of the products : thus, if we multiply 15 by 6, 5, 4, we have 90, 75, 60, which are in the ratio of 6, 5, 4.

Having

## TO THE SECOND PART. 117

Having explained the principles on which these harmonic tables are constructed, it remains to shew the use of them in a theory of harmonics, by the following rules.

### *Rules relating to the RATIO.*

The harmonic tables contain a series, or progression, of roots from unity in the ratios of 3 and 5, and the harmonics to each are formed by the arithmetic series before described.

*Rule I.* Every root and its harmonics are multiplied by 2, by which we have a series of simple and compounded octaves continued in the ratio of 2.

II. Hence it follows, that every compounded number in the ratio of 2 may be reduced to its lowest denomination by a continued division by 2 ; and the compounds of the roots of 3 or 5 may be reduced in like manner by a division by 3 or 5.

III. The difference between one number, or term, and another, is the ratio, or difference, between

tween one found and another expressed by those numbers.

IV. If one term be compared with another in large numbers, they may be reduced to a simple ratio, by being placed as fractions and divided by a common divisor; thus  $\frac{32}{30} = \frac{16}{15}$  the common divisor will be one of the primes 2, 3, 5. In some cases the less term is subtracted from the greater, and the product will be the divisor; thus  $\frac{75}{50} = \frac{2}{3}$  a fifth, or  $\frac{48}{45} = \frac{16}{15}$  semitone major.

V. When two or more ratios are added together, they must be placed as fractions, and the numerator of the first, and the denominator of the second, must be divided by a common divisor, till brought to their lowest denomination. Thus if we add a tone major and minor together, the sum will

be a major third:  $1 \frac{2}{8} + \frac{10}{9} = \frac{5}{4}$ ; also  $1 \frac{5}{4} + \frac{6}{5} + \frac{8}{6} = \frac{2}{1}$ .

Thus the major and minor thirds being added to a fourth, the extreme sounds are octave to each other. Also a comma 81. 80, added to a  
tone

## TO THE SECOND PART. 119

tone minor 10. 9, is a tone major,  $1 \frac{9}{80} + \frac{1}{9} = \frac{9}{8}$ .

VI. It happens sometimes that we have no common divisor, in which case the two numerators, and the two denominators, must be multiplied into each other, and if the product has no common divisor, we are sure it is in its lowest denomination.

Thus a major third, and a tone major, added together,  $1 \frac{5}{4} + \frac{9}{8} = \frac{45}{32}$ , form a greater 4th, or tritonus.

VII. If a minor third be added to a deficient third, we have a common divisor only to one numerator and one denominator,  $1 \frac{6}{5} + \frac{32}{27}$ . Here 6 and 27 are divided by 3, which gives a new numerator and denominator; but as the division can go no farther, the two denominators must be multiplied for a new denominator, and the two numerators, in like manner, for a new numerator, and the product will be a femidiapente, or imperfect 5th. Thus,  $1 \frac{6}{5} + \frac{32}{27} = \frac{64}{45}$ .

VIII.

VIII. If, after the division is finished, there remain two quotients above, and but one below, or the contrary, the two quotients must be multiplied by each other, and placed as a fraction with the one : thus, if a tone major be added to a semitone major,  $1 \frac{9}{8} + \frac{16}{15} = \frac{6}{5}$ , we have a minor third for the product.

IX. In adding intervals together, the numerator of one and the denominator of the other, that are alike, are to be cancelled ; or, which is the same thing, we must write unity over and under each, and then proceed with the division as before.

Thus  $1 \frac{9}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$  ; then  $1 \frac{9}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$  ;

then  $1 \frac{9}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$  ; then  $1 \frac{9}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$  ;

then  $1 \frac{9}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$  ; then  $1 \frac{9}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8} = \frac{3}{2}$  ;

a fifth, or diapente.

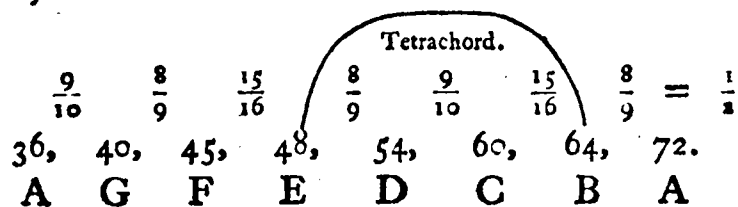
This

## TO THE SECOND PART. 121

This last operation includes the whole, which, had it not been to shew the progress of the division, would at first have been presented in this form only.

X. If we would know the intervals which compose the system of diapason, we must extract them from the harmonics of any three next terms or roots; as 1, 3, 9, or 3, 9, 27. The octave of the middle or second term, or principal fundamental bass, is to be placed first, and the succeeding sounds to follow in the order of a system; after which we must divide each with the next, by a common divisor, or by their difference, and we shall have the ratio of each interval, from acute to grave; whose sum, when added together, is equal to the diapason.

Example: from 1, 3, 9, of the first diagram, Rule IV.

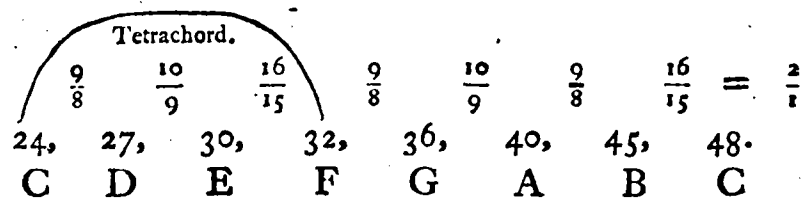


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We have here the ratio of  $36 \div 40 = \frac{9}{10}$  a tone minor, and  $40 \div 45 = \frac{8}{9}$  a tone major; and of the others, placed in fractions above, in this example.

XI. It must be observed, that in a descending series from acute to grave the fraction is formed by placing the less number above, but in an ascending series the less number of the fraction is below; as in the following example, taken from the three first terms of the second diagram, 1, 3, 9.



In these two examples of diapason the great advantage of the ratio is discovered; for though each is composed of three tones major, two minor, and two semitones major, yet their position in each is very different; and as the harmonious constitution of the tetrachord, which is the product of the three next terms, 1, 3, 9, or 3, 9, 27, &c. is the original of these and all other scales, we must look there for their essential differences.

Thus,

## TO THE SECOND PART. 123'

Thus, in the first tetrachord from C to D is a tone minor, which in the second is a tone major: also from D to E is a tone major in the first, and a tone minor in the second. Besides this, as the intervals in each diapason are the products of the series 1, 3, 9, which is in the ratio of 3, their motion or succession must be conformable to that series, which will not permit us to move from 40 to 45, because they are the products of 1, 9, and not 1, 3, or 3, 9, which regulate the motion of all the others.

From these observations we discover that the *geometrical* progression of the fundamental bass is the *governing principle*, and that the system of tetrachord, which made so great a part in the Grecian theories, had, from its simplicity, a preference to the diapason, which, when applied to the fundamental bass, or three terms, is incapable of proceeding as a system from one extreme to the other, unless the system is discontinued by an incomposite interval of a third, as in the following example, which is the same descending or ascending.

R 2

C D

|     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| C   | D   | E   | F   | G   | A   | C   | B   | C.  |
| 24, | 27, | 30, | 32, | 36, | 40, | 48, | 45, | 48. |
| 3,  | 9,  | 3,  | 1,  | 3,  | 1,  | 3,  | 9,  | 3,  |
| C   | G   | C   | F   | C   | F   | C   | G   | C.  |

In this progression it appears how great a share the second term 3, or principal generator, has in respect to the first and third terms 1 and 9.

XII. To know the relation of a root and either of its harmonics, they must be brought as near together as possible, by dividing the harmonic by 2. Thus, to find the ratio of 27. 9; as I cannot divide 27 by 2, I must multiply 9 by 2, which gives 18, and then (by Rule IV.)  $\frac{27}{18} = \frac{3}{2}$ , also of 49. 9; 9 must be multiplied till brought the nearest to 45, as  $\frac{45}{36} = \frac{5}{4}$ , a major third.

But in order to save the trouble of multiplying or dividing the terms to be compared, we may extract them from the harmonic tables.

XIII. Thus

## TO THE SECOND PART. 125

XIII. Thus if we compare the root 3 and 729, in the columns parallel with 3, we look for one of its multiples that comes the nearest to 729, and find it to be 768; then placing them as fractions, and proceeding by Rule IV.  $\frac{768}{729} = \frac{256}{243}$ , we have a limma, or the difference between C in a natural scale, and B in a scale with four sharps. Also if 81, a fifth to root 27, be compared to 5, a major third of root 1, we proceed as before, except in the division, and find the ratio  $\frac{81}{80}$  a comma, which is the difference of A in C scale, and A in G scale; and if we add the comma to the limma, by Rule V, we have a semitone major: thus  $\frac{256}{243} + \frac{81}{80} = \frac{16}{15}$ .

In the same manner we may proceed to discover by these tables all other intervals, however remote: as for instance, the *Pythagorean* comma 524288, and 531441, which is a comparison of a multiple of the first root, with the fifth of the twelfth power, root 177147.

It must be particularly observed, that every scale is perfected by the harmonics of the three  
next

next terms, and that the product of no other can enter into its composition: thus, 1, 3, 9, or 3, 9, 27, &c.

XIV. If we would know the difference between the first and last note of a series, we must observe that the ratio is formed so as to distinguish the ascending or descending intervals, by placing the greater or less number of the fraction uppermost; agreeably to Rule XI; after which we may proceed to add them together, according to Rule IX.

C G A E F C G

$$\text{Thus, } 1 \frac{3}{4} + \frac{10}{9} + \frac{3}{4} + \frac{16}{15} + \frac{3}{4} + \frac{3}{2} = \frac{3}{4}.$$

In this example the last note is a fourth lower than the first.

These rules being sufficient for the *addition* of intervals, I proceed to explain how a less interval may be *subtracted* from a greater, in order to discover the difference.

XV. In comparing one interval with another they must be placed as fractions, and the sign of multiplication

## TO THE SECOND PART. 127

multiplication put between them, to shew that the numerator of the first is to be multiplied by the denominator of the second ; and the denominator of the first by the numerator of the second. Thus the difference of a tone major when compared with a tone minor is a comma :  $\frac{9}{8} \times \frac{10}{9} = \frac{81}{80}$ , a comma. between an octave and fifth;  $\frac{2}{1} \times \frac{3}{2} = \frac{4}{3}$ , a fourth ; between a fifth and fourth,  $\frac{3}{2} \times \frac{4}{3} = \frac{9}{8}$ , a tone major ; between a fifth and sixth major,  $\frac{3}{2} \times \frac{5}{3} = \frac{10}{9}$ , a tone minor ; between a sixth minor and a fifth,  $\frac{8}{5} \times \frac{3}{2} = \frac{16}{15}$ , a femitone major.

If the products are not in their lowest denomination, after this cross multiplication of the ratios is finished, they must be divided by a common divisor. Thus, if from a fifth we subtract a major third, the remainder is a minor third :  $\frac{3}{2} \times \frac{5}{4} = \frac{15}{8} = \frac{150}{144} = \frac{25}{24}$ , a minor third.

If we subtract a femitone major from a tone minor, we have a femitone minor :  $\frac{10}{9} \times \frac{16}{15} = \frac{160}{135} = \frac{32}{27}$ . The common divisor here is 6, the difference between

tween 150 and 144, (see Rule IV.) and the product is a semitone minor.

If from a minor third a deficient third is taken, the residue is a comma :  $\frac{6}{5} \times \frac{32}{27} = \frac{162}{160} = \frac{81}{80}$ , a comma.

XVII. When the multiplication of the ratios is finished, and the products have no common divisor, we may be sure they are in their lowest denomination: as for example, the difference between the tritone 45. 32, and the semi-diapente 64. 45,  $(\frac{45}{32} \times \frac{64}{45})$  is  $= \frac{2048}{2025}$ , their lowest denomination.

XVIII. If the ratio, or difference, is not well understood, we must look for the numbers in the harmonic tables in either of the diagrams: as for example, 2048 is a multiple of root 1 B, in the first, or of F, root 1, in the second diagram; and 2025 is Cb, a minor third to the fundamental bass, root 405, in the quintuple of the first, or is E#, a major third to the fundamental bass at root 405, in the quintuple of the second, diagram. If, after this, a more satisfactory knowledge of this ratio is wanted, we may add it to some other small quantity

## TO THE SECOND PART. 129

tity as at Rule V. For example,  $\frac{2048}{2025} + \frac{81}{80} = \frac{128}{125}$ ,  
an enharmonic diesis.

Also a semitone minor added,  $\frac{2048}{2025} + \frac{25}{24} = \frac{256}{243}$ ,  
a limma. Or a greater semitone minor added,  
 $\frac{2048}{2025} + \frac{135}{128} = \frac{16}{15}$ , a semitone major.

To account for these two species of minor semitones, as well as the two species of major, we must subtract either a semitone major, or a semitone minor, from a tone major, and the residue will be in the first case a greater semitone minor, and in the second a greater semitone major.

For example,  $\frac{9}{8} \times \frac{16}{15} = \frac{135}{128}$  exceeds  $\frac{25}{24}$  by a comma;  
and  $\frac{9}{8} \times \frac{25}{24} = \frac{27}{25}$  exceeds  $\frac{16}{15}$  by a comma.

From these divisions of the tone major, the impossibility of an *equal* division of intervals is very evident; for if we divide the tone minor, it will be also into unequal parts. Example,  $\frac{10}{9} \times \frac{16}{15} = \frac{25}{24}$ , a semitone minor. And the same inequality will be found in dividing all other intervals discovered in a theory of harmonics.

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There are other methods by which the addition and subtraction of intervals might have been explained; but as these appeared to be the least difficult, I have therefore given them a preference, and doubt not but they will be found sufficient when applied to the harmonic tables; in which, every interval, however minute, may be discovered, as also its root, or fundamental bass, together with its scale and relation to all others, whether with minor or major thirds; for I must observe, that there are as many positions of these two species of scales, as there are flat or sharp dièses, exclusive of the two natural scales.

These are the principal *Rules* in respect to the Ratio, by which the following theory will be understood without much difficulty.

THE

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THE  
THEORY  
OF  
HARMONICS.

PART THE SECOND.

SECTION I.

**A**MONG the works of the Grecians, published by M. Meibomius, we have an Introduction to Music by Guadentius, the Philosopher, a favourer of the Ratio. In the 17th page of his Introduction, he has laid down *two Diagrams*, which, being in their degrees composed of tones major and limmas, have given occasion to the most formidable objection to the Grecian Theory.

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These

These two diagrams I come now to examine. They are to be considered as *one general and universal system*, in which all particular systems are contained.

A particular system may be limited to some part of an universal one, but is always a regular collection, or composition, of many things; by which a chain of principles is linked together, and their several parts follow, and depend on each other.

In explaining these diagrams there are two things to be kept constantly in view: the one, the different *lengths* of strings, which is the foundation of the first diagram; and the other, their different *vibrations*, which is the foundation of the second.

These diagrams, as they now stand, are composed of tones major and limmas, which are placed before the large numbers in the first, and after them in the second, diagram. See Pl. xviii. The fifteen names are common to each; but the numbers placed before the names represent the lengths of strings in the first diagram, and those placed after represent the vibrations in the second.

But in order to discover the original series of which these numbers are composed, they must be divided by 2, to bring them to their lowest denomination.

| $\frac{9}{8}$ | $\frac{9}{8}$ | $\frac{256}{243}$ | $\frac{9}{8}$ | $\frac{256}{243}$ | $\frac{9}{8}$ |
|---------------|---------------|-------------------|---------------|-------------------|---------------|
| 2592          | 2304          | 2048              | 1944          | 1728              | 1536          |
| 1296          | 1152          | 1024              | 972           | 864               | 768           |
| 648           | 576           | 512               | 486           | 432               | 384           |
| 324           | 288           | 256               | 243           | 216               | 192           |
| 162           | 144           | 128               |               | 108               | 96            |
| 81            | 72            | 64                |               | 54                | 48            |
|               | 36            | 32                |               | 27                | 24            |
|               | 18            | 16                |               |                   | 12            |
|               | 9             | 8                 |               |                   | 6             |
|               |               | 4                 |               |                   | 3             |
|               |               | 2                 |               |                   |               |
|               |               | 1                 |               |                   |               |

From these reduced numbers a geometrical progression is formed in the ratio of 3; as 1, 3, 9, 27, 81, &c. in the harmonic tables belonging to each diagram. Pl. XIII and XIV.

I shall now place this series in the following order, beginning with unity, and annex to each number one of the seven musical letters; the choice of which is determined by the greatest number of fifths ascending and descending without being obliged to make use of either flats or sharps.

| First Diagram.  | Lengths of strings descending from unity.   |   |    |    |     |     |
|-----------------|---------------------------------------------|---|----|----|-----|-----|
| B               | E                                           | A | D  | G  | C   | F   |
| I               | 3                                           | 9 | 27 | 81 | 243 | 729 |
| F               | C                                           | G | D  | A  | E   | B   |
| Second Diagram. | Vibrations of strings ascending from unity. |   |    |    |     |     |

From

From this description we may observe, that the different lengths of strings are inverse, or reciprocal, as their different vibrations; that by these properties in musical strings, the series in the first diagram must descend in the ratio of 3, by a necessity equal with that which obliges the second to ascend from unity in the same ratio.

We also discover, that instead of a scale of very untunable degrees, such as are tones major and limmas, we have a continued series of triples, in geometric proportion, each of which being multiplied by an arithmetic series 1, 2, 3, 4, &c. generates its own harmonics and all other consonant intervals, and must therefore be considered as the root, or principal, by which those harmonics exist, as may be seen in the harmonic tables, Pl. xiv, &c.

This progression from each to the next is in a compound ratio of a fifth, viz. a twelfth; by which means we have the octave to 1, in our way to the fifth, 1, 2, 3. This octave appears necessary to every root; not only to approach the fifth in its true ratio 2. 3, but also to unite the two great principles on which this theory so much depends, which are the *geometric* with the *arithmetic* progression.

These octaves necessarily multiply the first series of seven roots into fourteen, when ranged in a scale, as in the two diagrams, Pl. xviii; but as the Grecians deemed this an imperfect system, they added another sound, viz. *Proslambanomenos*, in the ratio of 1. 4, by which the whole was comprehended in two octaves, when formed into degrees, which they

they reckoned the most perfect system, as containing all the less. See Pl. XIX; in which the two diagrams are inverted, and the musical letters applied to each sound.

That a series, in the ratio of 3, should be formed into a scale of degrees, consisting of tones major and limmas, seems necessary to preserve the numbers according to their magnitude, as they represent the different lengths of strings and their vibrations; to describe both of which properly required the inversion of the scales to be placed in opposition to each other, in order to shew that the same number which represents an acute sound in the first, was the same which represented a grave sound in the second. But as they could not in this form be applied to either of the genders, it cannot be doubted but that some other form was necessarily implied, and generally known to be the only rational foundation of a theory of harmonics.

Nevertheless it must be owned, that for want of a more particular explanation of this doctrine, the writers of succeeding ages have been mistaken, and have formed only one scale of degrees out of the two diagrams; and some writers on this subject have explained grave sounds as high, and acute as low.

The whole theory thus disordered, and every principle of science extinguished, gave occasion to the great objection to the Grecians' having any knowledge of harmony; namely, that the imperfect concords,  $\frac{5}{4}$ ,  $\frac{6}{5}$ ,  $\frac{5}{3}$ , and  $\frac{8}{5}$ , which are so essential to harmony, are not to be found in their true ratio.

This

This objection must remain unanswerable, if no other application than that of a scale of untunable degrees can be discovered. But the contrary of this has already appeared, by reducing the large numbers to their lowest denomination, by which we have found their real application in a geometric progression in the ratio of 3.

If we examine the two diagrams, (Pl. xviii.) the Proslambanomenos 2304 in the first diagram, is the lowest or gravest sound; but the numbers of the Proslambanomenos of the second diagram, as the gravest sound, are 648. Also the highest, or Nete hyperbolæon, of the first is 576, whereas the highest of the second is 2592. In the same manner (Pl. xix.) the numbers in the first, to which A or B are annexed, are the same as those of G or F in the second, as D is the only note to which the same numbers are applied, and is of the root 27 in each, by being the same distance from unity in each.

The inversion of the scales, agreeably to the two great principles, makes no alteration among the intervals, which are the same above and below, as appears by the ratios, and by the black notes, which distinguish the limma, as the open notes do the tones major.

These large numbers in the diagrams have already been discovered to be a geometric progression ascending and descending, in the ratio of 3, as 1, 3, 9, &c. and have each been distinguished as a root, when multiplied by the arithmetic series, 1, 2, 3, 4, 5, &c. by which operation we discover all possible consonance belonging to that root, which cannot be applied to any

any other. As these harmonics are all in their true ratio to the root, as well as among themselves, their relation and agreement with the harmonics of any other root, may very easily be discovered by the rules already laid down for the addition or subtraction of ratios.

It appears in the harmonic tables, Pl. XIV, &c. that the product of each root is comprized in three letters: to discover therefore which of these three letters is the fundamental bass, we must bring them the nearest that is possible to each other, and the gravest or lowest of the three will be the fundamental bass.

Thus in the first diagram of the harmonic tables, 6, 5, 4, are the nearest to each other, and their relations stand thus  $\frac{5}{6} + \frac{4}{5} = \frac{2}{3}$ , where 6 is the grave sound, and consequently the fundamental bass.

But the contrary of this is true in the tables of the second diagram, where 4, 5, 6, place 4 as the fundamental bass, as  $\frac{5}{4} + \frac{6}{5} = \frac{3}{2}$ .

Hence we discover that no root in the first diagram can be a fundamental bass, as they all are in the second; for instance, in the first, unity, or B, is the root, yet E 6 is the fundamental bass; but in the second, unity, or F, is the root, and also the fundamental bass, whether it stands as 1, 2, or 4; for these octaves make no difference with respect to its power as a fundamental bass.

T

It

It must be observed, that these differences are occasioned by the two first principles of the lengths of strings, and their vibrations; the first places unity in the acute, and descends to the grave, as the numbers increase; and the other places unity in the grave, and in like manner as the numbers increase ascends to the acute.—To express which, it was necessary to insert the ratios, and place the largest number of the first contrary to what it is in the second example.

This combination 6, 5, 4, or 4, 5, 6, forms the trias harmonica, which being sung together, produces the most perfect union of consonance by their mutual agreement.

But that the roots, the fundamental basses, and their harmonics may be well understood, by those not perfectly acquainted with a theory formed on these principles, Pl. xx and xxi are designed to illustrate them.

Plate xx, comprehends the seven roots of the first diagram, in their lowest denomination, from unity to 729, which are placed in the first column, 1, 3, 9, &c. descending from acute to grave in the ratio of 3.

The three letters in the next column express the trias harmonica, of which the gravest is the fundamental bass, before explained.

In the third column are the numbers, or products of the roots when multiplied by 6, 5, 4, by which a comparison of the harmonics of one root is easily made with those of any other,  
by

by the rules before explained for the addition, or subtraction, of intervals.

In the following columns are the multiples of the root by 2, in large numbers, which correspond with those in the first diagram, at Pl. xviii.

Pl. xxi, containing the second diagram, has the same number of columns as the first, and for the same reasons; except instead of descending from acute to grave, it ascends from grave to acute, and the root in this is the fundamental bass, contrary to what it was in the other; and as the progression advances from 27 to 729, the series of sharps is discovered in their proper order; F #, C #, G #, D #: as the flats, B b, E b, A b, D b, are in the first diagram, from 27 to 729. We have also the harmonics for the minor mode or scale in the first diagram, and for the major in the second.

From what has been explained, it appears, that the principles on which this theory is founded are divided into two parts, and that each is of equal importance to the whole; that the ratio of concord, or discord, in the first is the same in the second diagram, but in a contrary direction from unity; that the *minor* scale is not formed *out* of the *major*, but each is *original*, and depends on distinct principles; each has its roots, its fundamental basses, and harmonics, and whatever else is necessary to complete the most perfect theory, by supplying every demand that *Melody* or *Harmony* can make.

S E C T I O N II.  
O F T H E T E T R A C H O R D.

The Tetrachord, in the Diatonic gender, is a system the most natural, and agreeable to the voice of man. Its composition consists of four sounds, or three intervals, of which the semitone is the first, and is succeeded by the tone minor, and finishes with the tone major, in the first diagram.

These three intervals, being the product of any three next terms, in a geometric progression in the ratio of 3, are fixed and unalterable in the diagram to which they belong; and cannot therefore be considered as a simple melody only, but as a composition of several, whose union with, and dependence on the three terms, or fundamental basses, makes the whole truly harmonical, as appears by the two tetrachords formed in Pl. xxii, fig. 1 and 2, extracted from the two diagrams.

In the first, we have all the sounds proper for the minor scale, and in the second the major is completed.

The lowest space of each tetrachord has the roots, 1, 3, 9, with the fundamental basses in the space above, after which, the three next spaces contain the three melodies, with figures annexed to each, to shew their relation to each other, and to the fundamental bass. The upper of the three spaces is the tetrachord, B, C, D, E, moving from left to right, with the ratio of each interval placed above, as determined by the roots, or fundamental basses; and the whole makes a most agreeable succession

succession of harmonic sounds, either ascending or descending, and the different positions of tones, major and minor, are distinguished according to the scale; thus, from C to D, which is a tone minor in the first, is a tone major in the second; also from D to E, which is a tone major in the first, is a tone minor in the second tetrachord.

It follows next to examine the several parts of this harmonic composition of tetrachord, to see if any thing characteristic can be discovered in the motion of these harmonics, as they appear to have an absolute dependence on the change of the fundamental bass.—I shall make some observations on fig. 1, which, as there is very little difference between this and fig. 2, may be applied equally to both.

When the fundamental bass moves from 1 to 3, the upper melody rises a semitone major  $\frac{16}{15}$ ; the second melody a tone minor; and the third is a ligature, or continues in the same tension.

The next movement of the fundamental bass is from 3 to 9, which raises the upper part, or melody, a tone minor higher; the second is a ligature; and the third part, or melody, ascends a semitone major.

The third and last movement is from 9 to 3, which obliges the upper melody to ascend a tone major; the second is a ligature; and the third melody descends a semitone major.

Thus we discover three things in this composition of the tetrachord.

The

The first is, that when the bass moves from 1 to 3, or from 3 to 9, there are only two sounds that move among the harmonics, and the third sound is a ligature, by being common to both.

Of these two sounds that move, one is either a tone minor  $\frac{10}{9}$ , or major  $\frac{9}{8}$ ; and the other is constantly a semitone major  $\frac{16}{15}$ .

Now as the semitone major and the ligature are found to attend every change of the bass, they are without doubt a distinguishing part of the character of the diatonic gender, and will, as such, be constantly found among the harmonics of the fundamental basses in the ratio of 3.

In each tetrachord the semitone is marked with a horizontal line, and the ligature with a semi-circle or bind.

The next observation to be made is, that no tone major  $\frac{9}{8}$  is found among the melodies, till the fundamental bass moves from 9 to 3, and then that interval appears, and completes the formation of the tetrachord in the upper melody; for as there are but three species of intervals in the diatonic system, it seems necessary for the principal melody to be composed of them all, in order to establish a system, founded in nature, on the most certain and obvious principles.

The third observation is, that the tetrachord is found to be a particular melody, generated by the fundamental bass, and subject to harmonic powers, and must, therefore, be considered

sidered in a secondary relation with respect to first principles, directly contrary to the method observed by all Theorists, who have made it a first and original one, by mistaking the effects for the cause, without any regard to minor or major scales: by which means they have confined their observations to a simple melody, contracting a system founded on harmonic principles to three simple intervals, without considering that the scale or system of the diatonic octave was so blended and united with the tetrachord, that all its intervals were determined by the same three terms, which formed the harmonious constitution of the tetrachord.

To illustrate this truth, I have at fig. 3 and 4, Pl. XXII, selected from among the harmonics of 1, 3, 9, of the tetrachords, fig. 1 and 2, the sounds proper for a scale of diatonic degrees, which are ranged and disposed agreeably to the order of minor and major scales, together with the roots or fundamental basses, to which they respectively belong. But as they stand opposite to each other, we more readily discover the difference between intervals formed for melody in the diatonic, and those proper for a bass; or, as the Grecians have distinguished, between systematic and diastematic intervals.

To understand the composition of these scales, it must be observed, that the roots, 1, 3, 9, are placed in the first or lowest space; the fundamental basses are in the second, expressed by the multiples of the roots, the more easily to discover the interval formed between the system or scale, and the bass, which is set down as a ratio in the third space. The fourth space contains the numbers, which are to be compared with

with those of the fundamental basses; and the fifth space contains the letters which those numbers represent, together with the ratio of the interval, from any letter to the next.

This ratio is discovered by comparing one number with the next in the fourth space; thus,  $\frac{36}{40} = \frac{9}{10}$ , and the like. The ratio, or interval between the bass and the melody, is discovered by comparing the numbers in the second space, with those of the fourth; thus,  $\frac{35}{72} = \frac{1}{2}$  an octave.

I have been particular in explaining these scales, because by the same method the tetrachord and its harmonics, and the scale which is included in that system, may be formed from any three next terms in the harmonic tables, as 3, 9, 27, or 9, 27, 81: observing, however, that in order to bring the numbers nearest to each other, we may multiply or divide by 2 any number in the tetrachord for that purpose; thus, in the present case, in the tetrachord fig. 1, C 30 is an octave different from C in the scale fig. 3, because 60, 64, is nearer each other than 30, 64: also, at fig. 2, E 60, is not so near as 30 is to 32; the first being a compound, and the last a simple ratio.

I shall now examine the system of diapason or octave, as it has had a preference to the tetrachord by some modern theorists.

In the first scale, fig. 3, Pl. xxii, the fundamental bass moves from E 48, to D 54; or, which is the same, from 1 to 9. Also in the second scale, fig. 4, from F 32, to G 36; or,

or, as before, from 1 to 9, which in its nearest degree is a tone major,  $\frac{8}{9}$ .

Thus the original character of the fundamental bass, which had its motion from a geometric progression in the ratio of 3, is lost; and the diatonic is substituted in its place as a bass, contrary to every principle of harmonics.

From these observations we discover the impossibility of forming a theory of harmonics from the system of diapason; for it appears, that this series is not capable of continuing a succession from one extreme to the other, as there is no communication in either scale between 40 and 45; or, which is the same, between 1 and 9, and the utmost that can be made of the system of diapason has already been explained in page 122. (Rule XI.) In which an incomposite interval preserves the geometric progression of the bass, by the interposition of 48, between 40 and 45; after which, the remaining part of the series is perfectly regular, and conformable to the three terms in the ratio of 3.

Thus the perfection of the tetrachord, and the reputation of those, whose theories for many ages have been regulated by its harmonic powers, must be acknowledged; but as this is not the only instance in which it is found superior to the harmonic system of diapason, or to any other system, I shall proceed to the conjunction and disjunction of the tetrachords.

## U                      SECTION

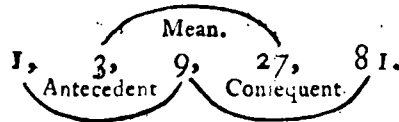
## SECTION III.

OF THE CONJUNCTION AND DISJUNCTION.  
OF TETRACHORDS.

This distinction of *Conjunction* and *Disjunction* will best be understood by considering, that as a tetrachord is the product of any three next terms in the geometric progression, it follows that any new term added must unavoidably make some alteration among the harmonics; in consequence of which, a new scale will be formed. For this reason, it is absolutely necessary to fix and determine these alterations by principles the most certain.

For this purpose I shall take three tetrachords, and place them as antecedent, mean, and consequent, in order to compare the harmonics of one tetrachord with those of another, to discover their difference.

Thus they will stand as at Pl. xxiii and xxiv, and the progression will be



I shall begin with the mean, 3, 9, 27, and comparé it with the consequent 9, 27, 81, to which it is conjoined: for it must be observed, that the mean has no other motion, than to the antecedent, or consequent.

As it is indifferent which diagram is taken for this comparison, the ratios being the same in both, I shall take those at Pl. xxiv of the second diagram, fig. 2.

In

In comparing C, root 3, with the major 3d, C #, root 81, a semitone minor of the greatest magnitude is found in the ratio of  $\frac{135}{128}$ , instead of  $\frac{25}{24}$ . Also the 3d to root 3, is a comma different from the 5th of root 81. (See the harmonic tables.) As the same irrational intervals exist in comparing the mean tetrachord with the antecedent, they need not be repeated; but we may proceed to the antecedent and consequent. In comparing a multiple 256 of root 1 F, with 243 E, which is the fifth to root 81, we have the limma of the ancients; also in comparing 32, a multiple of root 1, with root 27, we have a 3d deficient by a comma  $\frac{32}{27}$ .

Besides these defective intervals, others may be found, if these were not sufficient, to shew the impossibility of their existing in one and the same scale; and therefore they cannot affect the composition of the tetrachord, which has been found extensive in its harmonic powers, and perfect in all its parts. Instead, therefore, of objecting to these intervals, which are only to be found in comparing the composition of one perfect system with another, they are so many proofs of the perfection and regularity of this theory, in which no new term can be added to the first three, without giving immediate information of the change of the scale, by the alteration of one, or more ratios; for as long as the geometrical progression is observed in the roots, so long each scale will be perfect: and if we consider the various revolutions among the harmonics, which must unavoidably happen between 80 and 81; or, 128 and 135; or, between 243 and 256; it will be impossible to discover any interval among any three next terms, that is not the most perfect, as well as necessary for the mu-

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tations

tations of the scales; for as the succession of sounds is natural and agreeable, the variety which they produce by the mutations is one of the most animating parts of musical composition.

Thus the origin and necessity of these intervals are discovered, and the objections of modern theorists removed.

The *Disjunction* of the tetrachords has the same principles for its foundation, as the conjunction; viz. the motion of the fundamental bass, in a geometric progression; to preserve which, this distinction is unavoidable in a theory of harmonics, but does not appear of any consequence, when considered only as a melody, independent of the fundamental bass.

In Pl. xxiv, there are three tetrachords and two disjunctions, fig. 4.

These tetrachords are formed from the same roots as those above, fig. 2, but are disposed in a different order. The first tetrachord begins at 9, and ends at 27; the next begins at root 3: but as there is the same objection to 27, 3, as there was before to 1, 9; or 8, 9; tone major, the interposition of another root, 9, at the disjunction becomes absolutely necessary to preserve the progression in the ratio of 3, as 27, 9, 3, and not 27, 3.

The same observation is to be applied to the last note at 9, in the second tetrachord, and the first of the third, root 1; where, for the same reason, 3 must open the communication between 9 and 1, to preserve the progression 9, 3, 1, and prevent

vent a diatonic motion in the fundamental basses : but, besides this, there are two other reasons of equal importance ; the first is the mutations of the scales, which decrease the number of flat dieses, as at fig. 3, Pl. xxiii, and also the sharps, fig. 4, Pl. xxiv. For from two flat and two sharp dieses, which are necessary among the harmonics in the first tetrachord, fig. 3 and 4, we change to one in the second, and proceed to the natural scale at the third tetrachord ; but the contrary of this may be observed in conjoined tetrachords of each diagram ; where, from the natural scale, the mutation was made first to one, and then to two dieses. See fig. 1 and 2, Pl. xxiii and xxiv. The second reason is, that the melody descends in the second diagram, and ascends in the first, a tone major from the last note of one tetrachord, to the first note of the next tetrachord, by disjunction ; which the Greeks distinguished by the name of the diezeutic tone, or tone of disjunction : now as this is a tone major, and is preceded by a tone major in the former tetrachord, and as two tones major can never succeed to each other in the same scale, the disjunction became absolutely necessary, not only to form a new phrase and a new scale, but also to prevent the disagreeable effect of a very sudden and unexpected change of the dieses, in moving from root 27, to 3.

From the nature of this tone of disjunction we may acquit the Grecians of the charge, so often brought against them, of forming their diatonic gender with tones major and limmas only ; since the harmonic explanation of their tetrachords proves the contrary.

It

It must also be observed, that the ligature and semitone, two principal characters among the different melodies of every tetrachord, as has been already explained, are formed by the motion of the fundamental bass in the ratio of 3, and attend every such motion; and that the flat and sharp dieses begin in each diagram at root 27, and may be continued in an increasing series, in proportion to the number of conjoined tetrachords, from one to five, or more, dieses; the flats descending by fifths, and the sharps ascending by fifths, in this order.

$$\left\{ \begin{array}{l} B \flat, \quad E \flat, \quad A \flat, \quad D \flat, \quad G \flat, \quad \text{descending.} \\ 1, \quad 2, \quad 3, \quad 4, \quad 5, \\ F \sharp, \quad C \sharp, \quad G \sharp, \quad D \sharp, \quad A \sharp. \quad \text{ascending.} \end{array} \right\}$$

But contrary to this, the disjunction of the tetrachords decreases the number of flat or sharp dieses, from 5 to 4, and 3, &c. and thus the mutations may be made from any given or principal scale a fifth higher, or a fifth lower, by the increase or decrease of the flat, or sharp diesis; or, which is the same, by the conjunction or disjunction of the tetrachords.

From these observations on the conjunction and disjunction of the tetrachords, the distinction between them appears the most necessary to support a theory in all its parts, on the most satisfactory and rational principles. We have seen several melodies determined by the progression of the fundamental bass, from which different scales have been formed, and the number of dieses peculiar to each has been limited, without a possibility of the least alteration, unless a new term is introduced, which leads either to a conjunction, or a disjunction of the tetrachords,  
in

in which the agreement between the scales is found the most perfect; and the irrational intervals, which have already been explained, are removed to too great a distance to give the least offence. Thus in the present case, if E, D, is a tone minor in the antecedent tetrachord, fig. 2, Pl. xxiv, and a tone major in the consequent, yet, as we must pass through the mean or middle tetrachord, before such alteration can take place, no disagreeable consequence can be discovered, especially as the mutations are supported by the progression of the fundamental bass, in the ratio of 3, whose harmonics are always so perfect, as to admit of no alteration whatever, unless a new term be introduced; the consequence of which is, a new scale will immediately be formed in the place of the former, without any imperfection in the ratio of its harmonics.

## SECTION IV.

## EXPLANATION OF THE QUINTUPLE PROPORTION.

The three *primes* are 2, 3, and 5; but as a progression of the first, 2, 4, 8, &c. can yield no variety, the harmonics being the same in all; and as the triple progression 1, 3, 9, &c. has been fully explained, there remains only the quintuple to be considered, by which we shall discover a *new* motion in the fundamental bass of ascending, or descending a third major or minor, which by a judicious mixture with the triple proportion, will produce a variety of intervals not yet investigated, and add greatly to the powers of the mutations.

Among

Among the many discoveries made in this theory by the triple proportion of the fundamental bass, no semitone minor has yet been found in the ratio of 25, 24.

The usual method of discovering this interval has been by dividing the tone minor into two parts; or, which is the same thing, subtracting a semitone major from a tone minor,  $\frac{10}{9} + \frac{16}{15} = \frac{25}{24}$ , and the remainder is a semitone minor. (See Rule 15, p. 126.) If this interval were the only object in view, we might be content with either of these methods.

Some have asserted, that the ancients had not 5 among the number of their primes, yet at the same time acknowledge that the enharmonic diesis is an interval found in their theories, which is in the ratio of 128, 125. Now as 125 is a multiple of 5, it follows that 5 must be one of the primes, as well as 2 and 3. The arithmetic series, which has all along been applied to the roots in the triple progression, will in like manner be applied to the quintuple, and will demonstrate this truth.

The quintuple proportion is the only one now left to complete a theory of harmonics; for, besides the semitone minor, the fundamental bass will acquire a new motion of ascending and descending a third, and a very considerable addition will be made to each of the diagrams; for the first, which consists of scales with minor thirds, will be improved with those of major, and the second diagram, which consists of scales with  
major

major thirds, will in like manner have their respective minor scales.

The quintuple progression, like that of the triple, is of no consequence in harmonics unless united with the arithmetic series: for from the first to the third term in each is discord, as  $\overline{1, 9}$ , or  $\overline{5, 125}$ ; and therefore each as a root, must be multiplied by the arithmetic series 4, 5, 6, or 6, 5, 4, according to the diagram to which they are to be applied. Thus  $5 \times 6 = 30$ ,  $5 \times 5 = 25$ ,  $5 \times 4 = 20$ , and the products are the harmonics to that root, 5; also  $25 \times 6 = 150$ ,  $25 \times 5 = 125$ ,  $25 \times 4 = 100$ , are the products, or harmonics of root 25; and if the same multiplication is repeated at root 125, we shall have the harmonics to that root, or to any other in the quintuple; but it must be observed that 1, 5, or 5, 25, are double compounded ratios, being each a seventeenth major; yet as F, A, or A, C#, are major thirds without any distinction of compound or simple thirds, so these are brought the nearest into a simple ratio, 4, 5, or 20, 25, which is the same; in like manner as 1, 3, was before brought to 2, 3; but this will not be difficult, as the letters are annexed to each number in the harmonic tables of the quintuple, which immediately succeed those of the triples in each diagram, Pl. xv and xvi, which have been explained page 115. But as the roots in these tables move in the ratio of 3, as 5, 15, 45, there are two other Plates, xxv and xxvi, which are intended to discover the easy communication between the minor and major scales, with the same number of flat or sharp dieses in each diagram; by which the mutations from a minor to a major scale, or the contrary, will be readily understood.

X

Each

Each of these tables is divided into four sections. The three columns of the first section contain seven roots from unity to root 729, in the ratio of 3, together with the letters, and the numbers which express their relation as harmonics to each root, in the same manner as they stand in the other harmonic tables.

In the second diagram the 3d to the root in the first section is the root to the second throughout, after being brought to its lowest denomination. Thus, 20 is reduced to 5, the root of the second section, which, when multiplied by the arithmetic series 4, 5, 6, or 6, 5, 4, gives the harmonics of root 5, as 20, 25, 30. Also if 60 be taken as a root, its lowest form is 15, which, being multiplied as before, gives 60, 75, 90, for its harmonics.

It must be remembered, that in the first diagram of the quintuple, (Pl. xxv.) the upper letter is the root, and the lowest letter in each root is the fundamental through every section.

As there is no alteration in the thirds to each fundamental, the fifth to each root in the first section is applied as a third to that of the second section; as from 24, root 1, to 20, root 5; by which means we have two species of thirds to each root, or fundamental, throughout the three last sections. Thus, 24, from root 1, is applied to root 5, and 72 to root 60, or 15; in consequence of which, we have the two species of thirds to each fundamental, and the semitone minor in its true ratio 24, 25; and by pursuing the same method with the other five roots the same advantages will follow.

The

The third section is formed from the second, and the fourth from the third, in the same manner as the second was from the first; that is, the major third to the root of the second section is 25, which is the root of the third; and the third major to the root of the third section is in like manner carried forward, as a root to the fourth section 125, and the other operations follow the same as at first; and each of these roots is multiplied by 4, 5, 6, or 6, 5, 4.

Every three next terms in the same section, by being in the ratio of 3, form a tetrachord, and consequently a scale peculiar to those three terms only; and the mean or middle term is the principal sound, or tone, to which all the other sounds in the scale refer, as the generator of the whole; in the same manner as has before been observed in the triple progression. There is, however, a very great difference between these two proportions: for any three next terms in the triple may move in progression, as 1, 3, 9; but it is not so in the quintuple, 1, 5, 25, or 5, 25, 125.

Therefore, as the triple proportion has been found to be the original of the quintuple, and that the latter has not, like the former, a progression of three terms, it follows that the motion of the fundamental bass in the quintuple must return to that of the triple, immediately after the change of rising or falling a third.

Thus in the first diagram, Pl. xxv, the motion from the fundamental bass A 72, to F 90 and B♭ 270; or from A 72, to F 90 and C 30; is better than 72, 90, and D♭ 450; or from D♭ 216,

X 2

to

to F 90 and A $\flat$  150; because the fundamental bass, by moving first a third and then one or more fifths, in the triple proportion, leaves us in no kind of suspense as to the mutation of the scale; but a motion of two greater, or two less, thirds in succession throws all the obscurity that is possible on the mutation, and leaves us in a painful uncertainty, till it is succeeded by the triple proportion to determine the scale.

The same observations may be made on the second diagram, Pl. xxvi, where the motion of the fundamental bass from C 48, to E 60 and A 20, or from 48, to 60 and B 180, is very good, and leaves us in no doubt with respect to the scale; but it is not so from 48, to 60 and G $\sharp$  300, in which the motion of two major thirds in succession in the bass makes so great an alteration in the ratios of the intervals, as cannot be admitted, unless on some very extraordinary occasion, to support a particular sentiment, in which reason has little to-do in the government of the passions.

The motion from F 16 to C $\sharp$  100; or, which is the same, from F, root 1, to C $\sharp$  25, or from A, root 5, to E $\sharp$ , root 125, is equally exceptionable with the former; because in this case the fundamental bass moves an interval, consisting of two major thirds, in the ratio of  $\frac{25}{16}$ .

As it seems contrary to the quintuple proportion to move a fundamental bass a minor third, it is therefore necessary before we proceed to explain the nature of this motion.

The

The change of harmonics in the triple proportion, as from 1 to 3, or from 3 to 9, has been always attended with a ligature, or one found common to both roots; agreeably to the rule which Euclid (p. 21) has laid down to be observed in the mutations.

This will appear at Pl. xxv, in which the fundamental bass of root 9, which is D 216, may move a minor third to F 90, which, in the quintuple, is the product of the bass A 72 at root 3; for in this change we have one, or, if we please, two sounds that are common. But if A 72, at root 3, should move to B $\flat$  270, the product of root 9, we have no ligature or common sound attending the change; and therefore it appears that there is no communication between 72 and 270, without first moving to F 90, the product of 72.

Hence we have a positive law, which forbids a fundamental bass to move a semitone, in like manner as before to move a tone major 9, 8, in the triple progression.

The same rules must be observed in the second diagram, Pl. xxvi; where G 144 may move to E 60; but C 48 cannot move to B 180, for the reason before given, not having a common sound.

Thus it appears that the motion of a minor third in the fundamental bass has its original in the quintuple proportion, as well as the major third; and the laws which govern one, are equally applicable to the other.

These

These and the preceeding tables of the first and second diagram contain all the various species of intervals, both rational and irrational ; for by uniting the arithmetic with the triple proportion, and sometimes with the quintuple, the scale or tone is perfected, as well as the mutations proper for every mode.

We have also not only two scales with minor and major thirds, peculiar to one or more flat or sharp dieses, but the two natural scales with different species of thirds.

Each scale is the product of any three next terms ; and the second, or mean term, is the tone or principal sound of that scale. The relative scales, connected with the principal one, are regulated by the increase or decrease of one diesis more or less than what completed the first proposed scale ; whereby three minor and three major scales may be used in every mode. But these five auxiliary scales make no alteration in the name of the mode, which is notwithstanding distinguished by that of the principal tone of the scale first taken. Thus if we take any three next terms, as G 9, D 27, A 81, which may represent as many principal tones of as many different scales, the second or mean term 27 is that which governs the whole, and gives its name to the mode. And agreeably to what was observed before of the increase or decrease of the numbers of dieses, 27 has two, 81 has three, and 9 has only one diesis ; and as the formation of two scales of a different kind depends on the same number of dieses, we have six scales in the whole, which complete the mode and all its mutations

But

But as an example on this subject will better be understood, I have, at Pl. xxvii, extracted from the first diagram, Pl. xxv, three roots proper to form three scales, as at fig. 1, in which the principal tone of each is in the proportion of 9, 27, or 81, with a minor third to each; and if considered as a mode, the scale which governs it is the second or mean term 27, which, to be perfect, requires two dieses, with three at root 81, and one only at root 9.

Opposite to these, at fig. 2, are the roots of three more scales, in the same proportion of 9, 27, 81, notwithstanding they are expressed by large numbers.

Each of these has major thirds, and the same number of dieses with those which are parallel with them, to form their different scales. Thus G 27, a minor third, with two dieses, and B ♭ 270, with two dieses a major third, correspond in like manner with each other, as does D 9 with F 90, or C 81 with E ♭ 810.

The pitch of the principal tone of the mode may be at G 27, with a minor third, or at B ♭ 270, with a major third, and the other five scales are the auxiliaries; and the whole being regulated by the number of dieses, forms one mode only, according to the doctrine of the ancients.

From this explanation of the geometric proportion in the ratio of 3, it appears to be an *universal leading principle* in a theory of harmonics, and extends its powers through every essential part.

Thus

Thus the three terms of every scale or tetrachord, the three first tetrachords of the Grecian diagram conjoined, or, which is the same thing, the three principal scales in the same kind of every mode, are formed by these proportions. Besides which, the three hexachords of Guido, the moll, the natural, and the quadro, together with the three cliffs of the moderns, F, C, and G, the order also and succession of the flat and sharp dieses are governed and regulated by the same geometric proportion in the ratio of 3; and as the two Grecian diagrams have already been found throughout in the same ratio, there can be no doubt of the consummate knowledge which the Greeks had of Music as a science, and of Harmony; without which a progression in the ratio of 3 could be of little or no use.

## SECTION V.

### OF THE MUTATIONS.

The Mutations (before fully explained according to the directions given by Euclid) may be reduced to the five following rules:

I. A mutation, or change from one scale to another, may be a fifth higher or lower; that is, from 27 to 81, or the contrary; or from 27 to 9, and the contrary. See Pl. xxvii, fig. 1 and 3.

II. A third higher or lower, which will be from the minor to the major scale, or the contrary.

III. A

III. A dissonant mutation may be by rising or falling a tone 9, 8, which is done by moving from 9 to 81, or the contrary.

IV. A mutation is made by changing the genus of a scale; as from D with one flat to D with two sharps, or the contrary.

V. By a change of the mode; when the first movement is in one mode, and a second movement succeeds in another, which sometimes bears a relation to the first, and at other-times none at all.—Instances of this are to be met with in many Cantatas, Recitatives, &c.

## SECTION VI.

### OF DISCORDS.

The motion of the fundamental bass having been determined by the triple and quintuple proportion, and their several concords discovered by the arithmetic series, it remains to consider the origin of discords, their different species, and how they become an essential part of harmonics.

The ancient writers, in treating of discords, have passed rather too slightly over a subject of so great importance.

Aristoxenus, in his division of intervals, says, (p. 16) "The first division of intervals is that by which they differ from among themselves in magnitude."

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"The

“ The second, that by which the consonant differ from “ the dissonant ;” (p. 45) and yet he gives no other rule of dissonance, or discord, than the difference between diapente and diatessaron, which is a tone major 9, 8.

Euclid, in his division of intervals, (which I have before taken notice of) says, (p. 8) that “ The consonant intervals “ differ from the dissonant ” and that “ the latter are placed “ between the former ;” and then proceeds to tell us, that “ consonance is a mixture of sounds differing in acuteness and “ gravity, but that dissonance, on the contrary, refusing to mix, “ hurts the ear with harshness.”

From a writer of Euclid's eminence more might have been expected, especially as Aristoxenus had said so little on the subject.

Nicomachus comes nearer the point ; for he tells us (p. 11, &c.) that Pythagoras, in passing by a smith's shop, heard different sounds, from the striking of hammers on an anvil, which engaged his attention so much as to put him upon trying some experiments, to discover the exact relation those sounds had to each other : and after different trials of the weights, he proceeded to try some experiments with strings, and discovered the proportions of 6, 8, 9, 12 ; and by comparing either of the extreme with either of the mean proportions, as 6, 8, =  $\frac{3}{4}$ , or 6, 9, =  $\frac{2}{3}$ , or 12, 9, =  $\frac{4}{3}$ , and 12, 8, =  $\frac{3}{2}$ , he discovered the diatessaron and diapente in their true ratios, and also the diapason from the two extremes, 6, 12, =  $\frac{1}{2}$  : but with respect to the

the mean proportions 9, 8, he found no agreement in their relation, and therefore pronounced them discord; and by inserting the several degrees of sound between 6, 8, 9, 12, he discovered the diatonic gender, composed of two tetrachords disjoined, equal to the system of diapason.

What use Pythagoras made of this discovery we are not informed, for Nicomachus, who wrote this little manuel in a hurry to oblige a lady, excuses himself from being more particular by the shortness of the time, but promises to treat every part more at large in his Commentaries; however as no such book has come down to us, we must be satisfied with what is now before us.

This account of Pythagoras has been objected to by many writers; but as the numbers and their ratio correspond with the different lengths of strings and their vibrations, it is sufficiently satisfactory on the present occasion.

In examining these proportions 6, 8, 9, 12, we find,

1. That by reducing these numbers to their lowest denomination we discover the three roots or fundamental basses, 1, 3, 9. See the two first columns in Pl. xxviii and xxix.

2. That by applying the arithmetic series to each of these three terms, the system of the tetrachord and the scale have already been formed in each diagram. Pl. xxii.

3. That the character of a principal sound or generator has been discovered in the mean or middle term, by the application  
Y 2 of

of the two first original principles of the different lengths and their vibrations. See Pl. XII; in which, the fifth above and fifth below unity are in the ratio of 3, which answers to the series, 1, 3, 9.

4. The fifth, being thus generated by either of these principles, discovers its union, as a principal harmonic sound with every fundamental bass, whether it has a consonant or a dissonant combination applied to it.

5. It has before been observed that the two products of 3, a generator, 1, 9, or 9, 1, cannot *succeed* to each other as fundamental basses; but this does not prevent their being *united* in harmonic combination; especially if by this method the different species of discords are formed, agreeably to the observations made by Aristoxenus, Euclid, and Pythagoras.

The first says, that "a discord is the difference between diatessaron and diapente;" which corresponds with the 9, 8, of Pythagoras; and Euclid says, "that the discord is placed between the concords;" in like manner 9, 8, is placed by Pythagoras between the concords, 6, 8, 9, 12. But Euclid proceeds further, and says "the discords will not mix like the concords;" Pythagoras removes this objection, by *combining* the discord with other sounds that will mix; of which the fifth, as was observed before, must always be one, as appears in this Pythagorean combination, 6, 8, 9, 12.

That this and other dissonant combinations, discovered by the union of the first and third terms, 1, 9, or 8, 9, may be better understood,

understood, I have placed them in the two Pl. xxviii and xxix.

The first is agreeable to the different lengths of strings, the second to their vibrations; and as the species of discords are the same in both, so the observations, which I shall make on one, may be applied to the other, except the difference which is found between the minor and major third; which will be taken notice of in its proper place.

The first column in Pl. xxix has the three terms, or fundamental basses, 1, 3, 9.

The second has the *Pythagorean* numbers 6, 8, 9, 12.

The third has the harmonics to the three terms.

The fourth column has a discord, which is the suspension of the fourth formed on C, a fundamental bass, to which F is a fourth, and is a discord to G, a fifth to the root C.

In the fifth column is another discord, viz. the suspension of the ninth, formed on F, a fundamental bass, to which A is a third major, C a fifth, G a ninth and a discord to the root.

The sixth column is a third term compounded, having a minor seventh to G, a fundamental bass, to which B is a major third, D a fifth, and F the discord is a minor seventh, and forms the compounded harmonics of the third term.

The

The seventh column is a double compound, having a ninth major or minor to G, a fundamental bass, to which B is a major third, D a fifth, F a seventh, and A  $\sharp$ , or  $\flat$  a discord and a double compounded third term, as in the first diagram the seventh column is a double compounded fourth term. Thus, by the union of the first and third terms, 1, 9, or 8, 9, these three species of discords are formed, and supported by their different fundamental basses, and their fifths.

The additional harmonic powers, which these three terms, or fundamental basses, acquire by the application of the discords, contribute very much to distinguish their different characters, by which their succession is easily discovered; as for example;

The first term from its situation, is incapable of receiving any other discord than the suspension of the ninth.

The second term may have either of the suspensions, but not the minor seventh.

The third term has more extensive powers than the other two; for besides the suspension of the fourth, or the ninth, it can receive the minor seventh and major third, which is peculiar to that term only, as will be explained in page 168.

Hence we may observe, that when Pythagoras determined 8, 9, to be discord, he could not, as many have imagined, intend a mere succession of sounds; (for then the tetrachord and other perfect systems must have been a succession of discords;) but on the contrary, a combination with other sounds. For if we consider, that all communication is cut off between the

the first and third term, 1, 9, as fundamental basses, it seems as if their *union* was reserved to be exerted in a fresh creation of harmonics, and to give new and distinct characters to the three terms.

Having explained the discords in Pl. xxix, I shall now compare them with those in Pl. xxviii.

The first five columns are the same, except that minor thirds are next the fundamental bass; but the sixth column in each being compared discovers a difference of very great consequence to themotion of the fundamental bass.

Each contains a seventh and a discord; the first has been found to have a major third, but the other on the contrary has a minor third next the fundamental. These discords, when compared with their respective fundamental basses, are found in the same ratio; but when compared with the other harmonic parts are very different. Thus,

Pl. xxix, from E to D, is a minor 7th in the ratio of 16, 9.

Pl. xxviii, from G to F, is also a minor 7th in the ratio of 9, 16.

Pl. xxix, from G to D, is a 5th deficient by a comma = 40, 27.

Pl. xxviii, B to F, is a femidiapente or imperfect 5, = 45, 64.

Pl. xxix, from B to D, is a minor 3d deficient by a comma, = 32, 27.

Pl. xxviii, from D to F, is also a minor 3d deficient by a comma, = 27, 32.

In

In comparing the seventh with the basses and their harmonics, we discover no other difference than that which arises from the two species of thirds to the fundamental basses; for these thirds, when compared with the sevenths, give two species of fifths, which is confirmed by adding each species of third to its particular fifth, and the product is the ratio of the seventh;

$$\frac{5}{6} + \frac{27}{40} = \frac{9}{10}; \text{ also } \frac{5}{4} + \frac{64}{45} = \frac{16}{9}.$$

However imperfect these fifths, as well as the deficient thirds may appear at first sight, yet as they are occasioned by the addition of a new harmonic, to which the others are compared, they cannot be objected to, not only because the whole combination is extracted from the scale, which admits of no sound that is not a product of the three terms, but also because each interval in the whole combination, when compared with the fundamental basses, is in its true ratio.

Hence it appears that all harmonics, whether simple or compound, are not to be questioned as to their mutual relations, provided they are true to the fundamental bass, whose powers they are intended to encrease, by supporting its operations.

This will appear very evident in the distinct characters of these two sevenths, which I shall explain in their turn: but as that with a major 3d, Pl. xxix, has something very interesting in its composition, I shall begin with that first.

This seventh is composed of three species of thirds, from G to B, B to D, and D to F, in the following ratios;

$$\underbrace{\frac{5}{4}} + \underbrace{\frac{6}{5}} + \underbrace{\frac{32}{27}} = \frac{16}{9}.$$

The

The D, and the F, which make the deficient third by a comma, are not only original sounds in the scale, but are also found in the geometric progression, 1, 3, 9, 27, and therefore are fixed and unalterable, whether they appear as fundamental basses, or harmonics.

The active principle of the semitone major has already been observed in the formation of the tetrachord, Pl. VII, Fig. 2 and 4, which is constantly formed, when the bass moves in the ratio of 3; and as there are two semitones in every scale, one of which attends the bass, from 1 to 3, and from 3 to 9, or the contrary; so, in this compounded harmonic, now under consideration, these two semitones move in a contrary direction, when the bass moves from 9 to 3. Thus the major third to 9 ascends a semitone major to the octave 3, the second term; and the discord, or minor seventh, descends a semitone major into the third, to root 3, the second term.

From the motion of these two semitones, we may observe two things of equal importance:

1. That all original discords must be considered as minor sounds, and must descend to the next degree to be resolved.

2. That as the major third ascended, so in like manner all major sounds must ascend to the next degree, and cannot descend without changing the scale by introducing a new term.

Z

From

From this natural motion in the minor and major sounds, we are conducted with the greatest certainty to the second term, or principal generator; as if by its powerful attraction it drew, and resolved every sound to itself, in order to finish the phrase with the most perfect termination.

From the vast variety found in harmonics, we may have as many basses as there are notes or letters in the combination. Thus this minor seventh, by changing the position from G, B, D, F, to B, D, F, G, or D, F, G, B, or to F, G, B, and D, forms other harmonics; but the first only is fundamental; and the other discords, which proceed from these inversions, are formed by the different positions of the relative basses, and distinguished by particular figures placed over each, to shew their relation to the fundamental bass.

These different basses diversify the proportions; and by placing the same thing in a variety of forms, the first impression is changed to others no less characteristic and entertaining. Thus G, the fundamental, may rise a major third to B with a sixth and imperfect fifth,  $\frac{6}{5}b$ ; or to D, a fifth above, with a compound sixth,  $\bar{6}$ , or, to F, a seventh above with a major fourth, or tritonus, and a second,  $\frac{4+}{2}$ .

These transpositions of the original combination do not alter the nature of the discord at F, or the major sound at B, since in every position the F must descend to the next degree to be resolved, and the B must ascend a semitone at the instant

instant the bass moves from 9, the third term, to 3, the second.

Hence we discover the difference between an original discord and others which are accidental; for if we take away the minor seventh F, we remove at the same time all the accidental discords, viz. the semidiapente or imperfect fifth, the fourth or tritonus, the second, and the deficient third, 32, 27; and the simple harmonics only remain, as at first, in their most perfect form.

I must observe that as melody depends so much on a diatonic succession of sounds, which have been discovered to be the immediate products of the fundamental basses, it appears here, that by the assistance of this minor seventh, we have a system formed by a communication from the octave of the third term to its seventh or minor sound, which proceeds to the third of the second term, as thus, G, F, E.

This succession is not only very agreeable in itself, but gives a character to the seventh, which cannot be allowed to either of the suspensions, the ninth or the fourth; that is, this seventh may be used without being prepared, though, like the other two discords, it must be resolved by descending to the next degree.

I have thus explained the most remarkable parts of these combinations, by which the character of an original discord is distinguished from those that are accidental; together with the

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active

active principles of the two semitones, which direct the motions of the minor and major sounds, so as to determine the motions of the original, as well as the accidental, discords in their different positions; by which we are conducted to the second term, or principal generator, and its harmonics, which are the several points to which these motions are directed, as the final or perfect repose.

Thus the same laws, which instruct us in the *formation* of the mass of harmony, teach us also how to give it *life* and *motion*; and if we attend to the several impressions made on our ear, by this dissonant combination, transposed to different positions, we discover that our expectation of the succeeding second term almost anticipates the change, and conducts the ear to its effect.

Nothing can be more pleasing to a curious and intelligent mind, than to discover the mutual agreement between the laws of science, and those dictated by our own feelings; to be able to account for those charming and agreeable ideas raised in the mind by the powers of harmony, and to reconcile them by principles the most rational and natural.

I shall now examine the other seventh, E, G, B, D, in the sixth column, Pl. xxviii, in order to discover by its motion the term that succeeds to it.

The ratios of the several intervals, which form this combination, have already been mentioned, in comparing them with those of the third term, and the only difference appears to arise from the two species of thirds next to the fundamental bass.

The

The seventh, like the other, must descend to be resolved : but the minor third has no motion like the former, but must continue a ligature at the time of the resolution, when the fundamental bass descends a fifth, by which another seventh is formed. This indeed changes the position of the harmonics, but with respect to the fundamental bass, they stand in the same relation as before, and will continue the same so long as the bass descends a fifth, at the time of the resolution, which moves into a minor third.

Now in a progression like this, where nothing is conclusive, and where even the scale is not always known, some essential alteration in the harmonics to make them more decisive, by bringing us nearer to the second term, is absolutely necessary ; and as a major sound is the only one discovered proper to produce this alteration, the last seventh must be resolved into a semitone major ; by which means we have a major third to the succeeding bass a fifth lower, which gives us the third term.

These harmonics of the third term bring us home to the second term, by the next motion of the fundamental bass a fifth lower.

Thus by the motion of this seventh, we not only discover the term that succeeds to it, but also its character, for as it is antecedent to the third term in the same manner as the third is to the second, it must therefore be a fourth term.

From this additional term, we have two sorts of comparisons in the ratio of 3.

The

The first is, as the first term is to the second, so is the second to the third, which terminates in an imperfect repose, with the simple harmonics on the third term.

The other is, as the fourth term is to the third, so is the second, which finishes and completes the final or perfect repose.

The progression of the fourth term may be continued in one scale, or may move into any other agreeably to the nature of the mutations; but must finally be resolved into the third term, which is decisive with respect to the scale, by being constantly succeeded by the second term.

It appears therefore, that the lengthening of the phrase is the consequence of the use of the fourth term, and in proportion to their number will be the length of the phrase: for as no progression can be formed by either of the three terms in the scale, it is absolutely necessary to find that power placed some where, not only for the sake of the phrase, but also to vary the sentiment, by combinations less perfect, and less conclusive, than the others: all of which are found in the fourth term only, by being farther removed from the second term.

As this fourth term, like the former, is a compounded harmonic, it has also four positions. Thus E, G, B, D, a seventh, or G, B, D, E, a sixth and fifth  $\frac{6}{5}$ , or B, D, E, G, a fourth and third  $\frac{4}{3}$ , or D, E, G, and B, a second and fourth  $\frac{4}{2}$ .

In

In all these positions D is the discord, and must descend to the next degree to be resolved; and if it moves a tone, it forms another fourth term, but if it descends a semitone, it forms a third term on a bass or root, a fifth lower.

As the ratios of all compound harmonics are formed from the products of the three first terms differently combined, so those of the fourth term must vary, according to the part of the scale from whence they are taken; for as no other sounds can be admitted into their composition, the formation of the scale is of the greatest importance in a theory of harmonics.

This discord of the fourth term is sometimes used in a descending system, without being prepared, as was before observed of the discord on the third term; but like that it must be resolved.

From the many observations made on the third and fourth terms, it does not appear that their distinct characters are occasioned by the seventh, when considered simply as a discord, but from its being united with a major or minor third.

It is from this union that the same species of discord discovers a much greater and more interesting variety of revolutions in harmonics than either of the suspensions; and as the seventh is so immediately concerned in all these operations, that even the fundamental bass is limited to descend a fifth, and sometimes a third, at the time of its resolution, and as the motion of the harmonics are regulated by the same cause, it follows that the  
seventh

seventh, from its extraordinary powers, must be considered as the *first* species of discord.

From this first species we have discovered a new term, which, by the great influence of the second term in every scale, is placed as antecedent to the third term.

This disposition determines it to be a fourth term, dependent in its resolution on the third; in the same manner as the third is on the second term. It appears therefore that, by the different applications of this first species of discord, and by attending to the motion and character of the fundamental bass, the harmonic powers are greatly increased.

Hence we have the simple and compounded harmonics limited to the different basses to which they belong: and if the same faithful guide continues to govern and direct the future operations, the double compounded harmonics will be discovered, and finish the characters of the third and fourth terms: as appears in the seventh column of Pl. xxviii, and xxix.

In this theory the motions of the fundamental basses are limited to that of ascending or descending a fifth, and sometimes a third; but there is no instance of their moving a tone, or semitone: from whence the true character of the melody is preserved in its natural motion of tones and semitones, whilst that of the fundamental bass demands a more masculine modulation.

Therefore as long as no other harmonics are admitted, but such as depend on the motion, and are the products, of the fundamental

damental basses, so long will these two opposite characters be preserved, namely, the systematic motion for melodies, and the diastematic for the fundamental basses.

## S E C T I O N VII.

### *Of the double compounded THIRD and FOURTH TERMS.*

It has been explained before that the formation either of the simple or compounded harmonics is a composition of thirds, in an ascending direction from a given sound or fundamental bass. Thus as the simple harmonics are a combination of two species of thirds, so another third of either species being added to them forms the compound, or discord of the seventh; or, as before explained, the third or fourth term.

If therefore we advance one step further on the same principles, and add another third to the former, we shall discover the utmost limits of harmonic combination in the double compounded third or fourth term. This harmonic consists of a third of either species, according to the term to which it is applied; of a fifth, a seventh, and a ninth, either major or minor, agreeably to the scale, and the relation that the combination has to the second term.

As this combination, from the variety of its proportions, is one of the most dissonant, and at the same time the most proper for the darker shades of musical composition, it cannot be unworthy of a particular consideration; especially as no

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such

such combination has yet appeared in any theory that I have seen, occasioned by an opinion, that the perfection of the diapason is such, as to contain all other intervals within itself, and that whatever exceeds it, is only a repetition, at the distance of an octave, of what existed before, by which all simple intervals are made compound by the addition of their octaves.

This reason, however specious, when applied to a simple melody, or a single succession of sounds, cannot be admitted with respect to combination, which depends on other principles, drawn from the character and motion of the fundamental bass. But that I may not advance any thing new, in so essential a part of harmonics as this is, without supporting it by the most undeniable evidence, I shall for this purpose select from this theory the following rules.

1. That the whole powers of harmonics are conducted by *four terms*.

2. That the three first are primary fundamental basses with perfect fifths, and generate their own harmonics, from which the scale is formed, and all other combinations peculiar to it.

3. That the fourth term has nothing original in its composition, but is formed from the products of the three first terms, and as its communication with the second term is cut off by the third term, it therefore stands as a secondary fundamental bass, of an inferior character to the third term, to which it must descend a fifth.

4. That

4. That the motion of these four terms in one scale is limited to the triple and quintuple proportion; but in no instance admits of any other, either in the ratio of 9, 8, tone major, or 16, 15, semitone major.

5. That the third and fourth terms may have compounded, or double compounded, harmonics; from which the first and second terms are excluded.

6. That the first and third terms only have an immediate communication with the second,

How far these rules will agree with the doctrine of keeping all harmonics in their first and most simple positions within the limits of the diapason, remains now to be examined.

The extreme flat seventh, and imperfect fifth, or femidiapente, with a minor third, form the combination  $B\flat, D, F, A\flat$ , (Pl. xxix.) which now comes under consideration, in order to discover its character as a fundamental bass.

The only found in the whole scale, on which these harmonics can be formed, is the major found, which is a product of the third term, and an imperfect concord of a third major. Now if this major found can support the character of a fundamental bass, all harmonic combination will be confined to the limits of the diapason; but if on the contrary it is found to have no one requisite proper for this character, the double

A a 2

com-

compounded harmonic must be established as the true and original combination.

It is determined by the first rule, that all harmonic combination is formed on four terms; therefore, if this major sound could be supposed to be one of the first three primary basses, it must have a perfect fifth, according to the second rule; which we find it has not: and if it is a fourth term, it cannot move to the second term, agreeably to the third and sixth rule; and if incapable of this motion as a fourth term, its two minor discords cannot be resolved by any other motion. Hence we discover that a *major sound* cannot be a *fundamental bass*, because its motion of a semitone major, 16, 15, is absolutely contrary to the fourth rule, as well as the whole theory, which limits the motions of the fundamental bass, to the triple and quintuple proportion only.

We may conclude from the fifth rule, that this combination is a part of a double compound; and as it has no other motion in the same scale, but into the second term, it can be derived from no other than a third term double compounded, agreeably to the sixth rule.

Thus we discover that these harmonics, excluding the fundamental bass, exceed the limits of the diapason, by a semitone in a minor scale, and in a major scale by a tone.

From these observations the first idea of a discord is preserved, which had its original form in the union of the first and  
third

third terms; to which is now added the harmonic third of the first term, not by any arbitrary will of our own, but by a necessity, which the motion of the fundamental bass obliges us to comply with.

The extreme flat seventh and imperfect fifth being found as parts of a double compound, peculiar to the third term, it may not be improper to give one example of their formation. In Pl. xxv, root 5, C 30, E  $\sharp$ , G, are the simple harmonics of the third term, to which the first term B  $\flat$ , 270, and its minor third, D  $\flat$ , 225, are added, which are taken from the quintuple of root 9, the first term in the triple; and the whole forms a double compounded harmonic on the third term: when the bass moves to F 90, the second term, the two minor discords are resolved, and the major sound E rises a semitone, by which the harmonics of the second term are completed.

Also at Pl. xxvi, the quintuple of root 9, B 180, D  $\sharp$ , F  $\sharp$ , a third term, is by the addition of A, C, taken from the quintuple of root 1, the first term, formed into a double compounded third term. But as its motion to the second, at the quintuple of root 3, together with the resolution of its two discords, and the ascending of the major sound, are the same as above, they need not be repeated.

From the same example taken from each of the diagrams we discover how the combination of harmonic sounds and their several relations depend on the fundamental bass; that by its animating powers they receive life and motion; that the character of a third term double compounded is established, in  
preference

preference to a major sound, which has no other motion than a semitone; which, for this and other reasons before given, can never be a fundamental bass.

There are two instances in which this combination deserves our particular notice.

The first is with respect to the motion of the semitone major. The active principle of this interval was first discovered in the harmonics of 1 to 3, or from 3 to 9, and the contrary; in which one semitone was formed by the change of each of these harmonics; but when the third term was composed of its minor seventh, we had two semitones formed by its motion into the second term; the one from a minor sound descending, and the other from a major sound ascending, as has been observed before. But in the present case, of a double compounded third term into the second, we have three semitones formed; the flat ninth and minor seventh descend a semitone as minor sounds to be resolved, and the major sound ascends another semitone into the second term, or its octave, which is the same.

The second instance relates to the systems, which make so great a part of the ancient harmonica.

These systems are of different magnitudes; but the least must constitute a minor or major third: which agrees with Euclid's definition, that "the least system contains more intervals than one," (p. 1.)

In moving therefore from the second to the third term twice, with their simple harmonics, we have a system of diatessaron, or  
four

four founds in their next degrees in the melody, as E, D, C, B, beginning with the third of the second term, and ending with the third of the third term, second diagram.

|        |             |   |
|--------|-------------|---|
| System | E, D, C, B. | } |
| Roots  | C, G, C, G. | } |

If the third term is compounded with its minor seventh, we have a system of six degrees, from the eighth of the third term to the third of the same term descending, while the third term moves to the second term at the resolution of the seventh, and repeats the same motion, till the whole is finished.

|        |                   |   |
|--------|-------------------|---|
| System | G, F, E, D, C, B. | } |
| Roots  | G, G, C, G, C, G. | } |

But when the third term is double compounded, we have a system of seven degrees, while the third term repeats its motion to the second term, during which the system descends from the minor ninth of the third term to its third, which determines the largest harmonic system which can be formed on any two fundamental basses.

|        |                                        |   |
|--------|----------------------------------------|---|
| System | A $\flat$ , G, F, E $\flat$ , D, C, B. | } |
| Roots  | G, G, G, C, G, C, G.                   | } |

These two last remarks shew us the gradual increase of the harmonic powers, from the simple to the double compounded third term, by the modulation of the semitones, and the variety and extent of the systems.

The

The observations made in the minor ninth, added to the third term, will in all respects be the same if the ninth is major; except, that in the resolution of the discords of the last two semitones only will be modulated, instead of three, which are found in the former; and that the first is formed from a scale with a minor third, the latter from one with a major; but their relation to the second term is the same, because each has a major third and perfect fifth to the fundamental bass, without which, it can never stand as a third term.

Hence the relation that any combination has to the second term must determine the character of the fundamental bass; for if there is no immediate communication with the second term, without first moving to the third, we can be in no doubt of its being a fourth term.

Thus if a double compounded harmonic has a minor third, it forms a double compounded fourth term, and as such leads us by the natural motion of its harmonics to a third term, agreeably to what has been observed before; viz. that the impression made on the mind by a fourth term, whether singly or doubly compounded, is so undetermined, that we wait with a kind of impatience, till relieved by the succeeding third term, which discovers at once the scale, the phrase, the second term, and whatever else is necessary for our information.

The particular composition of this double compounded harmonic, or fourth term, is formed on the same principles with the third term; with this difference only, that if the seventh is minor, the third must be minor, and not major; but if the seventh

venth is major, (which can never happen to the third term,) the third must also be major: as to the ninth, it may be a major, or a minor, third above the seventh, according to the scale.

The two discords, the ninth and seventh, are frequently used on the fourth, or third term, without being prepared, but nevertheless must be resolved: by descending to their next degrees, the root or fundamental bass in double compounded harmonics does not always appear, unless in compositions of four, five, or six parts; but its place is easily discovered by the motion of the harmonics.

## SECTION VIII.

### THE SECOND SPECIES OF DISCORD.

The principles of the third and fourth terms being established, and supported by the motion of the fundamental bass, the ninth, or *second species of discord*, comes next to be discovered.

To understand its genius and character it will be proper to explain two laws, to which this discord, as a suspension, is subject.

1. It must be prepared.
2. It must be resolved.

By the preparing of a discord is to be understood, that the sound exists as a concord one quantity of time, after which, by the motion of the fundamental bass, the same sound becomes a discord on the next quantity of time; and finally must be re-

B b

solved

solved on the third quantity of time, by descending to the next degree, either a tone or a semitone.

These operations depend on certain distinctions, to be observed in the divisions of time.

There are various species of musical time, but the divisions of each are comprehended in two or three equal quantities in a bar, or their multiples.

These bars are distinguished by a division of their equal parts into accented and unaccented. If a bar is composed of four equal quantities, the first and third are accented; the second and fourth are unaccented. If there are three quantities in a bar, the first only is accented, and the other two are unaccented. The preparing of a discord therefore must be on the unaccented part.

The same sound continues, or is a ligature, and forms the discord on the following accented part, after which it must descend to be resolved on the succeeding unaccented part, either a tone or a semitone.

Hence it appears, that all discords, prepared and resolved, require three quantities of time; the first quantity prepares the ear for its existence on the second, and the third satisfies the ear by the sweetness of its resolution.

This discord, like the minor seventh, has its origin in the union of the first and third terms; as may be seen at Pl. xxviii, in

in the fifth column, where D is its fundamental bass, with its perfect fifth and minor third; and again, at Pl. xxix, in the fifth column, where F is the fundamental bass, with its perfect fifth and major third.

As this discord now stands, we may observe that the three terms are employed in its formation, and that the second term is placed a fifth from each of the extremes, to which a major or minor third to the fundamental bass is added, which completes its harmonics.

The construction of these two ninths is in all respects the same, except the minor third to the first, and the major to the second; which not only makes this discord proper for the two scales, but also for the different parts of the scales.

By this we discover, that though its original is on a primary fundamental bass, yet it may exist on a secondary fundamental bass, and may therefore be applied not only to either of the three terms, but also to a fourth term, if properly prepared and resolved.

But as the character and genius of every discord can only be discovered by its motion and relation to the second term, it will be proper to compare this ninth with the minor seventh, to enable us to judge of its being a new species of discord, from its different preparations and resolutions, as well as from its motion and relation to the second term.

The ninth may be prepared by being a fifth, but not by being a third, to the fundamental bass.

B b 2

The

The seventh is prepared by being a fifth or third to the fundamental bass.

The fundamental bass continues two quantities of time to receive the resolution of the ninth into the eighth.

The fundamental bass falls a fifth, or a third, to receive the resolution of the seventh into the third, or eighth.

The ninth may be applied to the first term, and after its resolution the second term may succeed.

The seventh cannot exist on a first term, without changing its character to a fourth term, and being obliged to move to other fourth terms, before it can arrive at the second term.

The ninth may be applied to the second term without changing its character, or the scale.

The seventh cannot be applied to a second term without changing both.

The ninth must always be prepared, unless united with the seventh, as a double compounded harmonic.

The seventh is frequently used in descending systems without being prepared.

These differences between the two discords must be owned to be very great, more especially as the motion of the fundamental bass, its character, and relation to the second term, are so particularly determined.

This is the great object to which all harmonics should be directed; and unless some distinction marks the phrase, the sentiment, and the distance of the fundamental bass, which leads

us

us to the principal generator, or second term, our pleasure must abate in proportion to the difficulties we have to encounter.

As all inversions teach us to diversify an original harmony, without deviating from the natural genius and character of the fundamental bass, I would recommend a constant attention to the fundamental bass, its motion and harmonics, particularly those which precede any species of discord: the same attention must be paid to those which succeed, and are the resolution of the discords; for although these two species of discord, the seventh and the ninth, should each be prepared by a fundamental bass descending a fifth, yet, by an attention to the phrase, to the sounds that move, and those which remain as ligatures, the species of discord will be easily determined. For if the third to the fundamental, which prepares the discord, is the ligature, we are sure the seventh is the discord; but if the fifth of that fundamental bass should be the ligature, the second species of discord, or ninth, must undoubtedly be applied.

Hence the discord is very early known by its preparation: as to its resolution, which is of such consequence to the phrase and the succeeding harmonics, we can be under no difficulty, if we remember that the natural motion of all original discords is to *descend* a tone or semitone, and observe the situation of the ninth, which by being placed one degree above the eighth of the fundamental bass, must, by a kind of necessity, move to it; but the contrary of this is true with respect to the seventh, which is placed a note under the octave of the fundamental bass, to which it cannot ascend, but must wait for another fundamental bass to be resolved into a third.

I must

I must also observe, that although the ninth is sometimes found on a secondary fundamental bass, yet it has a more natural connection with the three primary, and for that reason has a near relation to the second term. But it is not so with this seventh, which, as it constitutes the principal character of the fourth term, can never be otherwise applied, nor have any immediate relation to the second term.

There is one observation more I must make with respect to the ninth, which is, that there is no instance of its being applied, where we may not with propriety apply the simple harmonies, if it were not to vary the sentiment, by employing a double quantity of time. This enables us to suspend the ninth, by which means the succeeding sound arrives with a sweeter and more graceful effect.

I hope therefore it will not be thought improper to distinguish this discord as the *second species*, and to call it the suspension of the ninth. From the many distinctions made by comparing it with the seventh, if properly attended to, it must be impossible to mistake it, either with respect to its character, or the similitude of the figuring, as applied to the relative basses; for its preparation and resolution on fundamental basses are so peculiar to itself, as to prevent any possibility of its being taken for any other discord.

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## SECTION IX.

## THE THIRD SPECIES OF DISCORD.

The Diatessaron, or perfect fourth, intended now for the *third* species of discord, has before been demonstrated as a perfect concord in the ratio of 3, 4.

How this duplicity of character can belong to the same interval is the subject of our present enquiry.

It must not be forgot, that the products of the first, second, and third terms, fixed and determined the place of every sound, which formed the scale, as well as all possible combination of consonant sounds; and that neither of these three terms alone could extend its powers to the formation of discord.

Their *Union* therefore in part, or in the whole, became absolutely necessary for the attainment of this great end; and at the same time demonstrates the perfection of a theory, which obliges us to draw every thing from its *original principles*.

This has in part been already explained in the two first species of discords, where the first term was added as a compound harmonic to the third term, and formed the minor seventh, or *first* species of discord; and by applying the third term, as harmonic to the first, the suspension of the ninth, or *second* species of discord was formed.

From

From the same original principles this *third* species of discord has its existence ; for by the union of the first and third terms we have the suspension of the fourth ; see Pl. xxviii and xxix ; which, by being united with the fifth to the fundamental bass, and subject to the laws before prescribed to the suspension of the ninth, must therefore be prepared and resolved.

The formation of this discord depends on the motion of the fundamental bass, with its simple harmonics rising a fifth on an accented part of the bar. This motion not only prepares the discord, but determines its combination to be an eighth, fifth, and fourth.

The fourth is prepared by the eighth of the preceding fundamental bass, and is substituted as a discord in the present combination, in lieu of the third, on an accented part of a bar, for one quantity of time. This suspension of the fourth must descend to the next degree, on the next unaccented part of the bar, to restore the third to its natural place, and complete the simple harmonics,

Thus a suspension appears to be only a power (subject to certain conditions) of preventing, or keeping back, the motion of one sound for a time ; at the expiration of which it is put into its natural state, and then the harmonics are perfected.

This power, as applied to harmonics, contributes greatly to vary the sentiment, by employing three quantities of time  
with

with the suspension, when two only would have been sufficient without it.

This consideration alone makes discords necessary, not only to distinguish the motion of the fundamental bass, but also to mark the time, and enforce the expression of the harmonics,

This suspension, or third species of discord, is formed in each diagram; (see Pl. xxviii and xxix, column 4.) and may therefore be resolved into a minor or major third, by descending a tone, or a semitone.

These two different resolutions will enable us to apply this discord to a fourth term, as well as to a third and second; but, from the nature of the scale, it can never be applied to a first term; for as that is the only fundamental bass below the second term, and has no perfect fifth below, it cannot, consistently with the scale and its character of a *first* term, admit any other discord but the second species, or ninth.

The second term is also limited to the second and third species of discords only; viz. the ninth and fourth. But the third and fourth terms may receive either of the three species of discords; and the first species may be either singly or doubly compounded, agreeably to the distinct character of the third and fourth terms before described.

As these limitations of the discords distinguish the different terms to which they are applied; so, by a comparison of their  
C c preparations

preparations and resolutions, their several characters will be sufficiently known to determine the relation of any fundamental bass to the second term, or principal sound.

The 7<sup>th</sup> }  
 The 9<sup>th</sup> } is prepared by a { 3<sup>d</sup>, and the fundamental bass descends 5.  
 The 4<sup>th</sup> } { 5<sup>th</sup>, and the fundamental bass descends 5.  
 { 8<sup>th</sup>, and the fundamental bass ascends 5.

The 7<sup>th</sup> }  
 The 9<sup>th</sup> } is resolved into a { 3<sup>d</sup>, and the fundamental bass descends 5.  
 The 4<sup>th</sup> } { 8<sup>th</sup>, and the fundamental bass continues.  
 { 3<sup>d</sup>, and the fundamental bass continues.

It has already been found, p. 178, that a fundamental bass preceding the third term a fifth higher, and descending a fifth to it, must be a secondary fundamental bass, or fourth term; so, if several fundamental basses precede the third term in this order of fifths, they must all be deemed so many fourth terms; for as they have nothing in their composition the least conclusive, either in relation to the phrase, or principal generator, till the third term appears, and discovers their remote relation to the second term, it follows, that the phrase must necessarily be lengthened, according to the number of fourth terms preceding the third and second term, which determines the phrase and the scale.

This third species of discord having been explained, together with its preparation by an eighth, and the motion of the fundamental bass ascending a fifth, I shall next describe it when prepared by a seventh. This preparation may be made by the seventh of a third or fourth term, by the fundamental bass

bass descending a fifth, and may be resolved into a third, either major or minor, the bass continuing on ; or may be resolved into another seventh if the root descends a fifth, and will have the best effect if on a third term, in order to proceed to the second term, on which it may exist as a fourth ; or it may be united with the whole harmonics of the third term, which will make a ninth a major seventh and fourth, and be resolved into the simple harmony of the second term on the next quantity of time.

The different positions of the harmonics of this third species of discord are many, but with respect to the figuring there can be only three ; viz.  $\frac{5}{4}$ ,  $\frac{5}{2}$ ,  $\frac{7}{4}$  ; and, as was before observed, by a proper attention to the motion of the fundamental bass that precedes this discord, and discovers its preparation by the fundamental bass rising five, there will be found so great a difference from a discord prepared by one falling five, as must remove every appearance of difficulty.

From what has been above explained concerning the three species of discord, we find that they have received their original existence from the *union* of the *first* and *third* terms ; that by the motion of the fundamental bass, which determines their preparations and resolutions, we discover their true character, in what particulars they differ from each other, and their relation to the second term ; and that they are original, because they comprehend all other dissonances whatever by their inversions, or the different positions of the harmonic basses, without altering the rules before prescribed for the motion of the fundamental bass.

C c 2

Before

Before I draw to a conclusion, I would take notice of one very great mistake made by modern theorists, which, if they had determined the fact by the ratio, could not have escaped them. We meet with it in Kircher's third book, page 145, and in Zarlino, page 363, in the third book of his *Institutioni*; the same has been adopted by all succeeding writers. What I mean is their description of the enharmonic gender.

They have made the first interval in the tetrachord of the enharmonic gender a semitone minor, as B, B $\times$ , C, and E; instead of the enharmonic diesis, which is B, C $\flat$ , C $\sharp$ , E. The deception of marking B with a single cross, instead of a double one, is equally exceptionable. By this mistake they have placed the enharmonic diesis the second interval in the tetrachord, when it ought to have been the first. But to prove this beyond a doubt, the tetrachord in question is in the first diagram, in which there is no B $\sharp$ . See the harmonic tables, Pl. xiv, where B 128 in the triple, when compared with C $\flat$  in the quintuple progression, root 25, is the enharmonic diesis in its true ratio  $\frac{128}{125}$ ; to which if a semitone minor is added,  $\frac{25}{24}$ , they will make a semitone major, and the residue of the tetrachord will be a ditone, or third major,  $\frac{5}{4}$ , an imperfect concord, not peculiar to the enharmonic, as the same interval is found in the diatonic gender, in which the third minor is the incomposite interval, or residue of the chromatic gender.

The

## The CONCLUSION.

HAVING considered at large in the *First* Part of this Work the Principles of the Grecian Doctrine of the Harmonica, as maintained by the *Aristoxenians*, and in this *Second* Part as supported by the *Pythagoreans*, in the Doctrine of the Ratio, it may be useful to make a few summary reflections on these two very extraordinary theories. — The former, maintained by Aristoxenus, Euclid, and Bacchius Senior, cannot be examined with too great attention, as it abounds with whatever is curious and interesting to the subject. The order and disposition of the several systems, particularly the tetrachord, from which the scales are formed, together with the number of flat and sharp dieses, by which the minor and major scales are completed; above all the seven species of the diapason, in which the tetrachords are comprehended, and regulated by the conjunction and disjunction; are among the many instances of the perfection of this theory.

The mese, or tone of disjunction, is of the greatest consequence; for it not only prevents two eights, two fifths, or two major thirds in succession, but stops us from moving into another scale, unless by the intervention of an additional bass as a temperament, which marks and discovers the alteration necessary for a new scale. — The genders supply the imperfect consonances; viz. the major third in the enharmonic, and the minor in the chromatic gender; each of which being taken from the diapason, there remain the two species of sixths; in the same manner as the diapente, added to the diatessaron, completes the magnitude of diapason. By the same method the diatonic moll forms the interval of a superfluous second, which,  
together

together with the extreme flat seventh, completes the diapason.

The great advantage of discovering these intervals contributes to the perfecting of this theory ; for notwithstanding the seven species of diapason contained them all, yet their formation would have been attended with great difficulty, and their magnitude must sometimes have been exceptionable ; as from B to D in the first diagram, and from F to D in the second, are each a minor third deficient by a comma : yet it must be observed, that although the Aristoxenians did not admit of the ratio in their theory, their division of intervals was strictly conformable to it. Thus the hemitone minor and major, which form a part of the chromatic gender, when added together are the sum of the tone minor, and not the tone major. As to the spifs, or smaller intervals, they are so necessary to the mutations, that nothing could be perfect without them ; and though they may be taken out of the different species of diapason, yet the genders regulate and determine the magnitude of all the less intervals, with much greater ease and certainty.

If again we examine the rules laid down for the mutations, they are simple and easy to be understood ; for they are comprehended in the perfect consonant mutation, to ascend or descend a fifth ; the imperfect mutation, to ascend or descend a third ; and the dissonant mutation, to ascend or descend a tone. These mutations are regulated by adding one diesis, or by taking one away from the first proposed number ; by which means the mode is formed and supplied with all its varieties.

In

In the First part of this Work I did not treat of the Discords ; not that I was without materials ; but as a Second Part, supported by the ratio, has succeeded to the former, in that I have explained the three species of original discords, agreeably to the numbers applied by Pythagoras to discover the true ratio of concord and discord ; viz. 6, 8, 9, 12. I was desirous of doing honour to his memory, as the first we know of, who discovered the magnitude of musical intervals by the ratio ; which very properly succeeds the two diagrams of Gaudentius, having the same principles applied to each.

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But here I must beg leave to pause one moment, to pay a tribute of gratitude most justly due to the merit of that great and excellent theorist and professor of Music, the learned Dr. *Pepusch* ; from whom, in an early part of life, I received the first principles of the application of numbers to musical intervals. As I loved and honoured him while living, so I greatly lamented his death, which happened at a time when I stood most in need of his assistance. His church music, preserved in the Ancient Academy, is his best eulogy as a Composer.

While I pay this tribute due to the memory of a departed friend, I am happy to embrace the opportunity of acknowledging the very friendly assistance I have received in this work from the Rev. *J. Trebeck*, and the Rev. *G. S. Townley*.

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I shall

I shall now make some observations on the *Second* Part, or the doctrine of the Ratio, maintained by Gaudentius, and by Pythagoras.

The great advantage of discovering the magnitude of all musical intervals by the Ratio cannot but be agreeable to men of science and lovers of truth. The Ratio has brought out of obscurity the two diagrams of Gaudentius and removed several objections made by writers of the first reputation.

These diagrams consisting of tones major and limmas occasioned the first and principal objection to the Grecian harmonica, as seeming destitute of the imperfect concords, without which, it is impossible to have harmony, or even melody; which last the Grecians are acknowledged to have had in the greatest perfection. With respect to the imperfect concords, it is very extraordinary that no writer, whom I have seen, ever attempted to discover them in the genders: for as to the seven species of the diapason, from whence the genders are formed, as well as all other musical intervals, it was not possible to extract them from thence, unless the position and harmonious constitution of those diapasons, together with their *Inversion*, had been first discovered, and perfectly understood by the moderns.

With respect to the objection above-mentioned to the diagrams of Gaudentius, it was so true, that nothing could remove it but the application of numbers, or the harmonic principles: these were no sooner applied, than the clouds of darkness disappeared, and the *great*, the *perfect* and *immutable* systems of the diagrams, instead of being (as was imagined) a scale of  
sounds

sounds intended for the voice, are found to be roots, principal tones, or keys of several scales, formed into systems, in the ratio of tones major and limmas. However extraordinary these diagrams may appear, yet (as before observed) the same exist in our own theory; for if the principal tones of different scales be placed in the same order of degrees, they will be each to the next in the ratio of tone major or limma. Hence all the objections made to the Grecian harmonica on account of these Diagrams, must vanish, or must in an equal degree affect our own: which would be a task too difficult for any one to prove.

It is very extraordinary that authors of the first reputation should condemn the Grecians for having no imperfect concords, when they are so easily discovered in the three genders, even according to their own definitions. But authors are sometimes tempted to take upon trust, whatever they meet with in the works of men of character, rather than give themselves the trouble to discover the truth.

Five has been rejected as one of the primes, though the enharmonic diesis (which they all agree is a part of that gender) cannot be formed without it. For as 125 is a multiple of 5, in the quintuple proportion, it must therefore be one of the primes. The quintuple proportion is sometimes useful in the mutations; but as it is incapable of a progression, it must always be succeeded by the triple proportion. Though in neither of these proportions can the mutations of the enharmonic diesis, or of the limma, be found: the former is not applied in combination, nor mentioned by Euclid, when he treats of the mutations; which is a confirmation that it was only necessary to determine the exact

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magnitude

magnitude of the ditone, or major third, in the enharmonic gender; as the chromatic was with respect to the minor third in the chromatic gender. The limma cannot be applied in combination, nor in the mutations, and therefore can be found only in the diagrams, being absolutely necessary for their formation, as systems of principal sounds.

I would here observe, that in the course of this work it has been shewn, that the fundamental bass cannot without a temperament ascend a tone or semitone, nor descend a tone or semitone; yet exceptions to these general laws are found in modern compositions in distinct phrases, in the Cadenza fugita, and in extra-harmonic sounds, which balk the cadence, and interrupt the regular progression of the fundamental bass.

These, and many other remarks which have been made in the course of this work, seemed to me necessary to remove every objection that could materially affect any essential part of the *Grecian Harmonica*: in which we discover the laws of science so intimately united and blended with those of nature, that the least deviation from the latter must in an equal degree be an injury to the simplicity of the former. That simplicity excludes the necessity of a profound knowledge in Geometry, in order to discover the original of the minor and major scales, of concords, discords, genders, modes, and other parts of the ancient Harmonica, which have long been perplexed and obscured by spheres, squares, cylinders, cones, and other geometrical figures, that have contributed little or nothing to the improvement of the practical Musician.

The

The *Aristoxenians* discovered the Agreement of concord and discord, and determined the magnitude of musical intervals by the laws of their own natural Feelings.

The *Pythagoreans* applied themselves to discover the *Ground* of those laws ; that by some leading and universal principle they might preserve the voice of nature in its most pure and original state. Hence they found that all melodies originated in, and were regulated by, the fundamental bass. Hence their diagrams were confined to a geometric proportion in the ratio of 3 : and the harmonious construction of the tetrachord, the conjunction and disjunction of tetrachords, the minor and major scales, the flat and sharp dieses, were all regulated and determined by this first unerring principle, a progression in the ratio of 3.

Here I cannot but take notice how generally this universal principle has regulated all musical compositions, even to this day ; and in many instances, where we cannot suppose it to be understood. Guido has placed his three hexachords (as was before observed) in the same ratio : and if we examine the three cliffs of the moderns F, C, and G, and the tuning of many stringed instruments, as in particular the Violin, Tenor, and Violoncello, their relation to each other is the same. Moreover the several species of discord (formed out of three terms by the union of the first and third terms in *combination*, whose motion could not be permitted in *succession*) their preparations and resolutions, the mutations with the same species of thirds, are all governed by the same ratio. Could all this be by Chance ? rather, is there not an absolute necessity, that All shall comply

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with

with principles founded in Nature? to whose laws we must be obedient, whether they are thoroughly understood or not.

Thus this wonderful theory of the Grecians, established on the most solid foundation, proceeds from a given sound, or principal generator, and advances with its creative powers, to generate its two products, the first and third terms, from whence arise all consonance and dissonance, scales and systems, major and minor, an encreasing series of flats and sharps, the tone major and minor, the different species of semitones, and all other intervals adapted to the mutations; the three genders, diatonic, chromatic, and enharmonic, the triple and quintuple progression of the fundamental bass, and whatever else is necessary to adorn, enrich and perfect the Science of Harmony.

It is almost impossible to enumerate all the advantages with respect to musical information and a profound knowledge of harmony, contained in the *Grecian Harmonica*. The simplicity and agreement of its several parts, established on laws that coincide with natural feelings and remain unalterable, are considerations sufficient to recommend it to all those who either wish to excel as Composers, or are desirous of attaining a *true knowledge of the*  
SCIENCE of MUSIC.

F I N I S.

A  
GENERAL INDEX  
TO THE  
THEORY  
OF  
HARMONICS.



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G g

The

|                                                        |       |                      |
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N.B. By a mistake in *numbering* the Plates, No. XI. was omitted; in consequence of which, Plate X. is immediately followed by Plate XII.

| Page. | Line. |                          |                                |
|-------|-------|--------------------------|--------------------------------|
| 24,   | 1,    | for <i>of Harmony</i> ,  | read <i>of the Harmonica</i> . |
| 27,   | 3,    | for <i>dieses</i> ,      | read <i>diesis</i> .           |
| 31,   | 22,   | for <i>dieses</i> ,      | read <i>diesis</i> .           |
| 50,   | 20,   | for <i>Pate</i> ,        | read <i>Plate</i> .            |
| 72,   | 6,    | for <i>different</i> ,   | read <i>sufficient</i> .       |
| 100,  | 14,   | for <i>next</i> ,        | read <i>less</i> .             |
| 117,  | 6,    | for <i>'prog sion</i> ,  | read <i>progression</i> .      |
| 131,  | 3,    | for <i>Guadentius</i> ,  | read <i>Gaudentius</i> .       |
| 167,  | 10,   | for <i>them otion</i> ,  | read <i>the motion</i> .       |
| 169,  | 7,    | for Pl. VII.             | read Pl. XXII.                 |
| 173,  | 1,    | for <i>the seventh</i> , | read <i>this seventh</i> .     |



# GRECIAN SCALE EXTRACTED FROM EUCLID.

## MAJOR SYSTEM.

## MINOR SYSTEM.

The diagram illustrates the Grecian Scale, divided into Major and Minor systems. The Major system is shown at the top, and the Minor system is shown at the bottom. The scale is represented by two staves, with notes and intervals labeled in Greek. The Major system is divided into four tetrachords, each consisting of four notes. The Minor system is divided into four tetrachords, each consisting of four notes. The intervals between notes are labeled with Greek terms, and the tetrachords are labeled with Greek terms. The diagram also shows the relationship between the Major and Minor systems, with notes and intervals labeled in Greek.

*Major System Notes (from highest to lowest):*  
 Απειρη, Βαρυπυνη, Μεσοπυνη, Οξυπυνη, Βαρυπυνη, Μεσοπυνη, Οξυπυνη, Βαρυπυνη, Μεσοπυνη, Οξυπυνη, Απειρη.

*Minor System Notes (from highest to lowest):*  
 Προδωμανομενος, Υψηλότε, Υψηλότε, Λιχανος, Υψηλότε, Μεσον, Υψηλότε, Μεσον, Λιχανος, Μεσον, Διωνος, Μεσε, Τριτε, Συνημμενον, Παρανετε, Συνημμενον, Διωνος, Νετε, Συνημμενον, Παραμεσε, Τριτε, Διεzeugμενον, Παρανετε, Διεzeugμενον, Διωνος, Νετε, Διεzeugμενον, Τριτε, Υπερβολων, Παρανετε, Υπερβολων, Διωνος, Νετε, Υπερβολων.

*Tetrachords in Major System:*  
 Tetrachords conjoined (between Βαρυπυνη and Οξυπυνη)  
 Tetrachords disjoined (between Απειρη and Βαρυπυνη, between Οξυπυνη and Βαρυπυνη, between Μεσοπυνη and Οξυπυνη, between Απειρη and Βαρυπυνη)

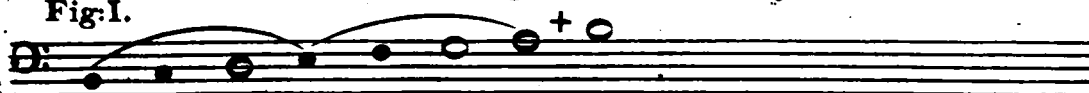
*Tetrachords in Minor System:*  
 Tetra. Υψηλότε (between Προδωμανομενος and Υψηλότε)  
 Tetra. Μεσον (between Υψηλότε and Λιχανος)  
 Tetra. Συνημμενον (between Λιχανος and Υψηλότε)  
 Tetra. Διεzeug (between Υψηλότε and Μεσον)  
 Tetra. Υπερβολ (between Υψηλότε and Λιχανος)

# The seven Species of Diapafon.

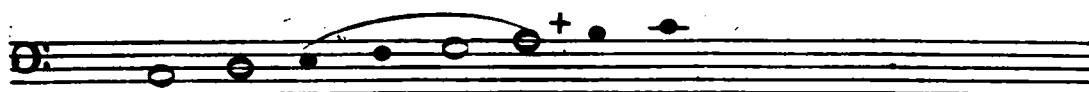
First Diagram.

Fig. I.

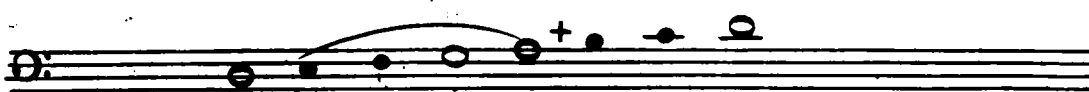
1 Mixolydian.



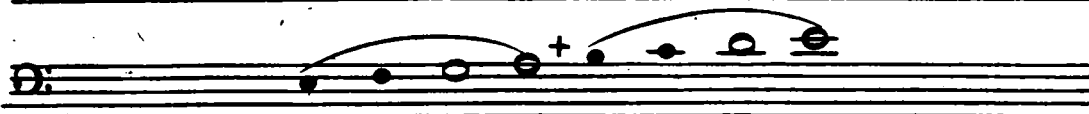
2 Lydian.



3 Phrygian.



4 Dorian.



5 Hypolydian.



6 Hypophrygian.



7 Hypodorian.



Fig. II.

1



H.

2



N.

3



L.

4



J.

5



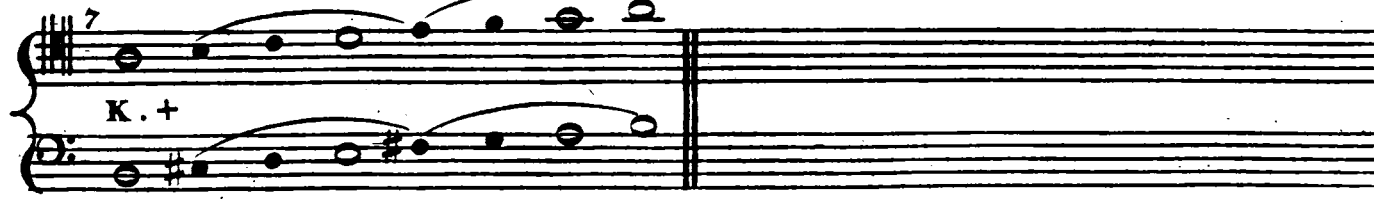
P.

6



M.

7



K. +

# The Seven Species of Diapafon.

Plate III

## FIRST DIAGRAM.

The diagram shows seven species of Diapafon, each represented by two staves (treble and bass clef). The notation includes various musical symbols such as notes, rests, accidentals, and dynamic markings. The first row is labeled '1st Species' and '1st'. The second row is labeled '2d Species' and '2d'. The third row is labeled '3d Species' and '3d'. The fourth row is labeled '4th Species' and '4th'. The fifth row is labeled '5th Species' and '5th'. The sixth row is labeled '6th Species' and '6th'. The seventh row is labeled '7th Species' and '7th'.

# EUCLID'S DIVISIONS of the TETRACHORD.

Plate IV.

## FIRST DIAGRAM.

DIAPASON.

| TETRACHORD.                                                                                      |  | TETRACHORD. |                        |
|--------------------------------------------------------------------------------------------------|--|-------------|------------------------|
| K.                                                                                               |  |             |                        |
| L.                                                                                               |  |             |                        |
| N.                                                                                               |  | =30         | ENHARMONIC.            |
| P.                                                                                               |  | =30         | SESQUIALTER CHROMATIC. |
| R.                                                                                               |  | =30         | TONLÆUM CHROMATIC.     |
| S.                                                                                               |  | =30         | DIATONIC MOLL.         |
| T.                                                                                               |  | =30         | DIATONIC SYNTONE.      |
| W.                                                                                               |  | =30         | CHROMATIC MOLL.        |
|                                                                                                  |  |             |                        |
| enharmonic diesis    semitone minor    semitone major    7 <sup>th</sup> major                   |  |             |                        |
|                                                                                                  |  |             |                        |
| tone    7 <sup>th</sup> minor    superfluous 2    deficient 7 <sup>th</sup>                      |  |             |                        |
|                                                                                                  |  |             |                        |
| deficient 3 <sup>d</sup> superfluous 6 <sup>th</sup> minor 3 <sup>rd</sup> major 6 <sup>th</sup> |  |             |                        |
|                                                                                                  |  |             |                        |
| major 3 <sup>d</sup> minor 6 <sup>th</sup> deficient 4 <sup>th</sup> superfluous 5 <sup>th</sup> |  |             |                        |
|                                                                                                  |  |             |                        |
| perfect 4 <sup>th</sup> perfect 5 <sup>th</sup> tritone or greater 4 <sup>th</sup> semidiapente. |  |             |                        |

## Plate V.

enharmonic diefis . semitone minor. semitone major. 7<sup>th</sup> major.

tone. 7<sup>th</sup> minor. superfluous 2<sup>nd</sup> deficient 7<sup>th</sup>

deficient 3<sup>rd</sup> superfluous 6<sup>th</sup> minor 3<sup>rd</sup> major 6<sup>th</sup>

major 3<sup>rd</sup> minor 6<sup>th</sup> deficient 4<sup>th</sup> superfluous 5<sup>th</sup>

perfect 4<sup>th</sup> perfect 5<sup>th</sup> semidiapente. tritone or greater 4<sup>th</sup>

## SECOND

**DIAGRAM.**

**Fig 2**

### Second Diagram

**Fig. 3**

# I

**Fig: 4**

## VII

# The Seven Species of Diapafon.

Plate VII

T.

SECOND DIAGRAM. U.

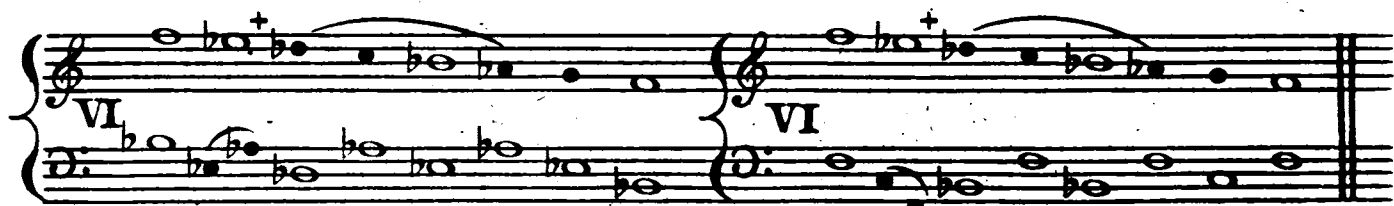
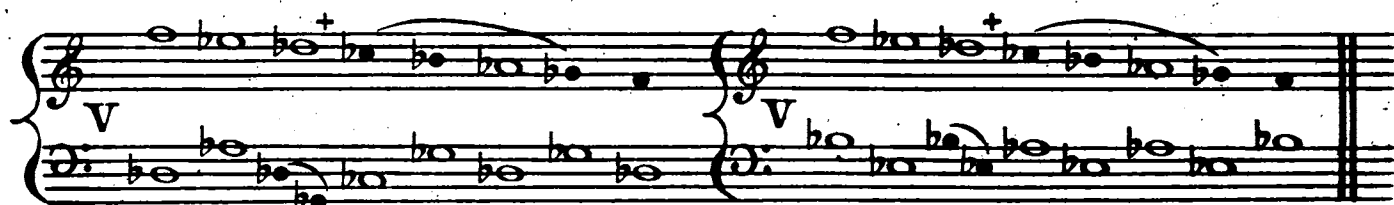
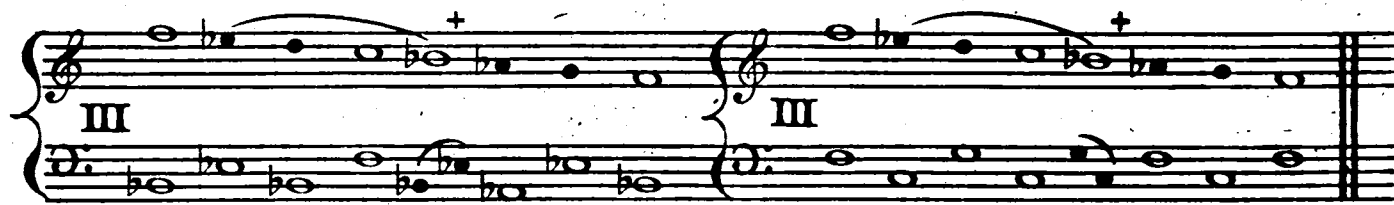
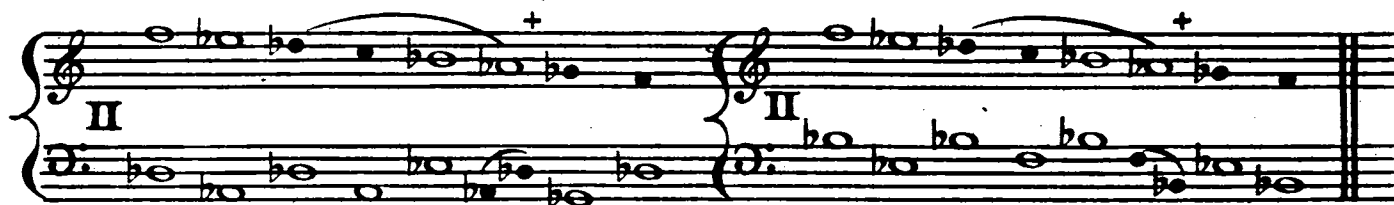


Plate VIII.

Scales with Major and Minor thirds, extracted from the Harmonics of the Three Terms.

7 Sharps

6 Sharps V<sup>th</sup> Species

5 Sharps II<sup>nd</sup> Species

4 Sharps VI<sup>th</sup> Species

3 Sharps III<sup>rd</sup> Species

2 Sharps VII<sup>th</sup> Species

1 Sharp IV<sup>th</sup> Species

Natural First Species of Diapason descending and ascending.

1<sup>st</sup> 2<sup>d</sup> 3<sup>d</sup> Term.

Scales with Major and Minor thirds extracted from the Harmonies of the Three Terms.

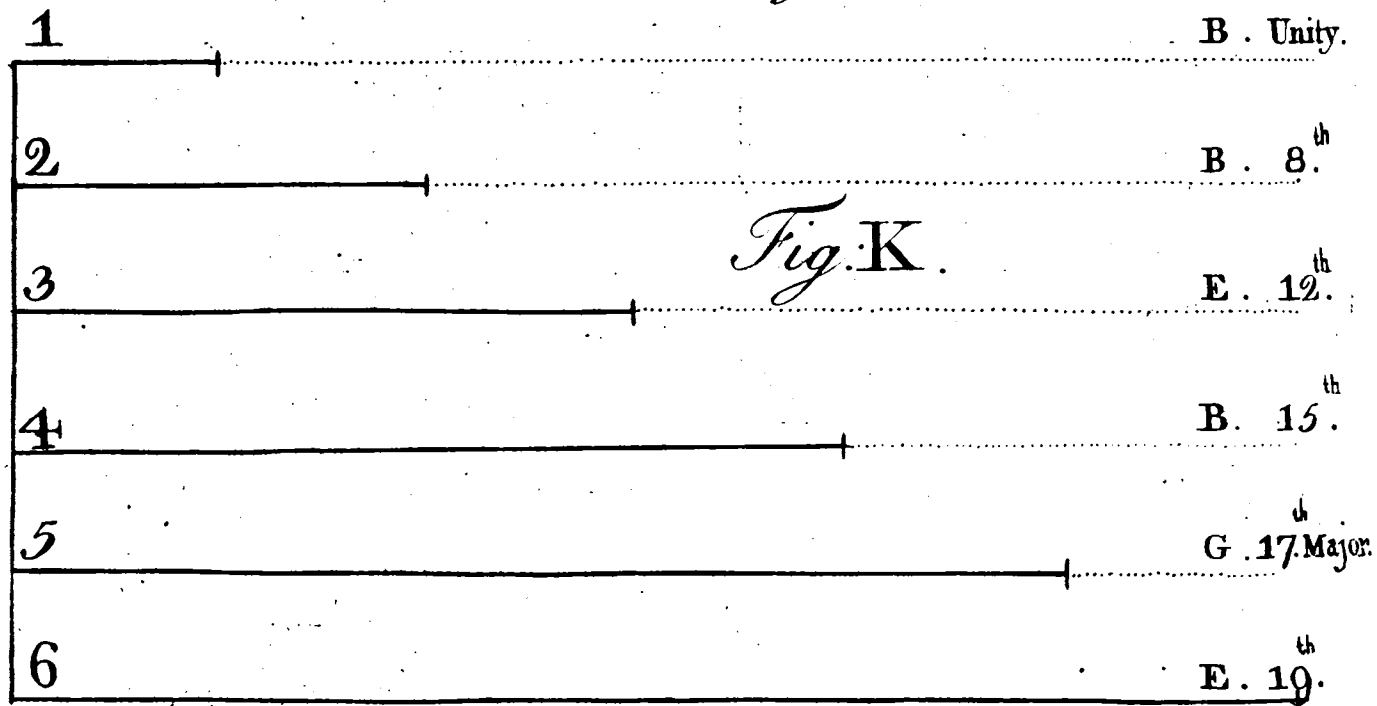
The image displays seven musical systems, each containing two staves (treble and bass clef) and a bracketed label. Each system represents a scale with its major and minor thirds extracted. The scales are as follows:

- System 1:** Labeled "1 Flat IV<sup>th</sup> Species". The scale is F major (F, G, A, Bb, C, D, E, F). The major thirds are F-A, G-Bb, A-C, Bb-D, C-E, D-F. The minor thirds are F-G, G-A, A-Bb, Bb-C, C-D, D-E, E-F.
- System 2:** Labeled "2 Flats VII<sup>th</sup> Species". The scale is Bb major (Bb, C, D, Eb, F, G, A, Bb). The major thirds are Bb-D, C-F, D-G, Eb-A, F-A, G-Bb. The minor thirds are Bb-C, C-D, D-Eb, Eb-F, F-G, G-A, A-Bb.
- System 3:** Labeled "3 Flats III<sup>th</sup> Species". The scale is Eb major (Eb, F, G, Ab, Bb, C, D, Eb). The major thirds are Eb-G, F-A, G-Bb, Ab-B, Bb-C, C-D. The minor thirds are Eb-F, F-G, G-Ab, Ab-Bb, Bb-C, C-D, D-Eb.
- System 4:** Labeled "4 Flats VI<sup>th</sup> Species". The scale is Ab major (Ab, Bb, C, Db, Eb, F, G, Ab). The major thirds are Ab-C, Bb-D, C-Eb, Db-F, Eb-G, F-Ab. The minor thirds are Ab-Bb, Bb-C, C-Db, Db-Eb, Eb-F, F-G, G-Ab.
- System 5:** Labeled "5 Flats II<sup>th</sup> Species". The scale is Gb major (Gb, Ab, Bb, Cb, Db, Eb, F, Gb). The major thirds are Gb-Bb, Ab-Cb, Bb-Db, Cb-Eb, Db-F, Eb-Gb. The minor thirds are Gb-Ab, Ab-Bb, Bb-Cb, Cb-Db, Db-Eb, Eb-F, F-Gb.
- System 6:** Labeled "6 Flats V<sup>th</sup> Species". The scale is Cb major (Cb, Db, Eb, Fb, Gb, Ab, Bb, Cb). The major thirds are Cb-Eb, Db-Fb, Eb-Gb, Fb-Ab, Gb-Bb, Ab-Cb. The minor thirds are Cb-Db, Db-Eb, Eb-Fb, Fb-Gb, Gb-Ab, Ab-Bb, Bb-Cb.
- System 7:** Labeled "7 Flats". The scale is Db major (Db, Eb, F, Gb, Ab, Bb, C, Db). The major thirds are Db-F, Eb-Gb, F-A, Gb-Bb, Ab-C, Bb-Db. The minor thirds are Db-Eb, Eb-F, F-Gb, Gb-Ab, Ab-Bb, Bb-C, C-Db.

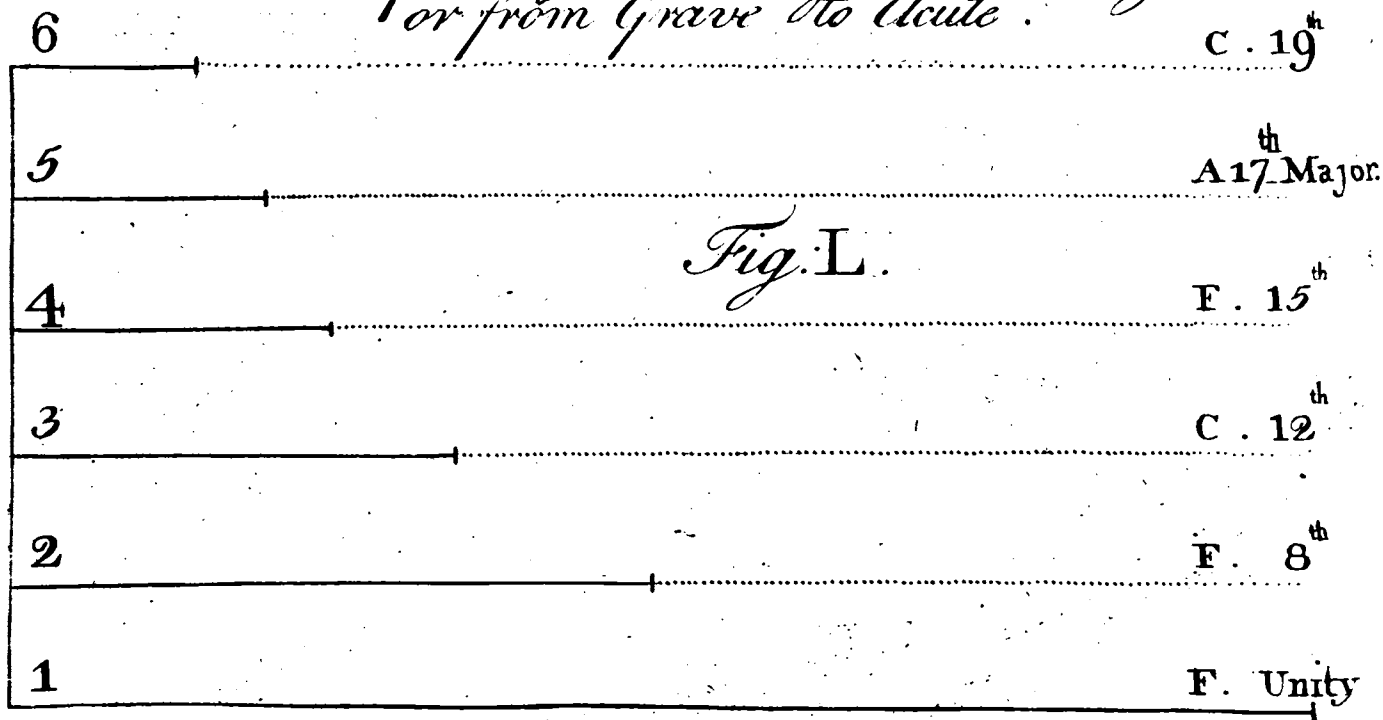
In each system, the first staff contains the scale, and the second staff contains the extracted major and minor thirds. The notation uses whole notes and rests, with accidentals (flats) indicating the key signature.

Plate X.

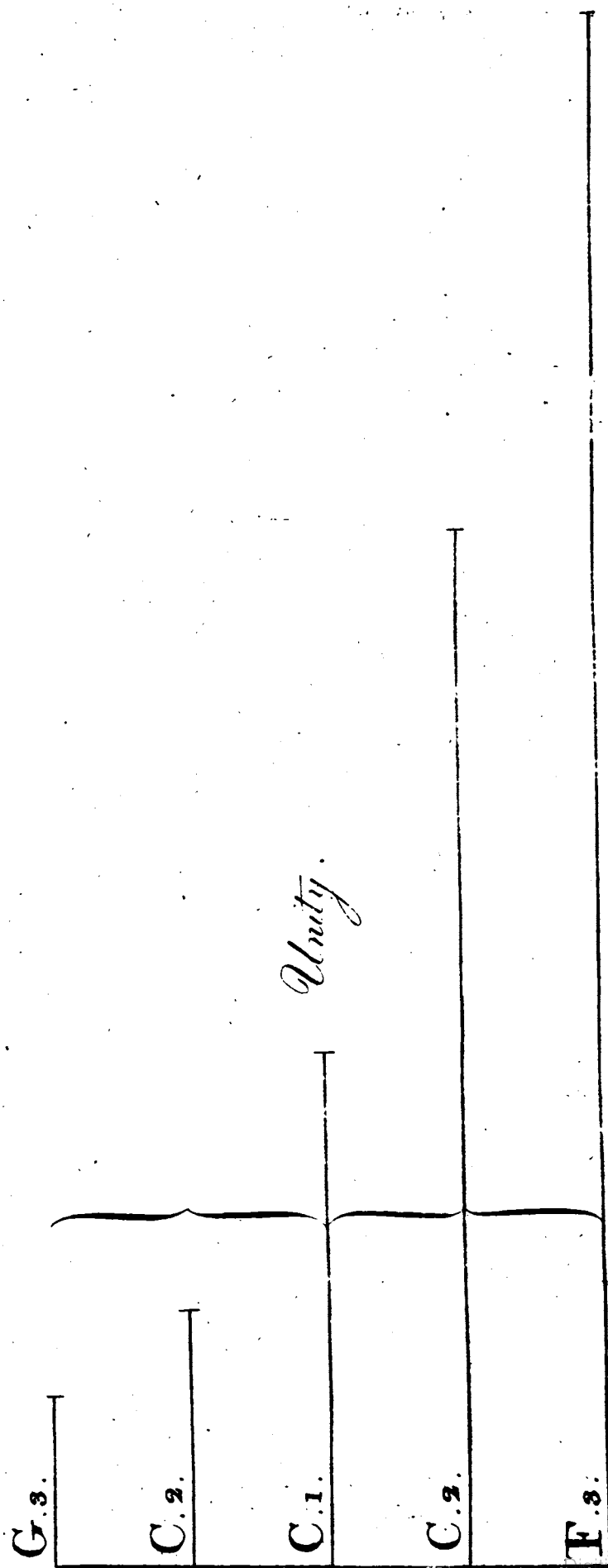
*Lengths of Strings descending from Unity,  
or from Acute to Grave.*



*Vibrations of Strings ascending from Unity,  
or from Grave to Acute.*



*Vibrations from Unity.*



*Lengths of Strings from Unity.*

HARMONIC TABLE.  
SECOND DIAGRAM.

|                          |   |    |     |     |     |      |      |      |       |       |       |        |
|--------------------------|---|----|-----|-----|-----|------|------|------|-------|-------|-------|--------|
| ROOT 27 & ITS HARMONICS. | 8 | D  | 216 | 432 | 864 | 1728 | 3456 | 6912 | 13824 | 27648 | 55296 | 110592 |
|                          | 6 | A  | 162 | 324 | 648 | 1296 | 2592 | 5184 | 10368 | 20736 | 41472 | 82944  |
|                          | 5 | F# | 135 | 270 | 540 | 1080 | 2160 | 4320 | 8640  | 17280 | 34560 | 69120  |
|                          | 4 | D  | 108 | 216 | 432 | 864  | 1728 | 3456 | 6912  | 13824 | 27648 | 55296  |
|                          | 3 | A  | 81  | 162 | 324 | 648  | 1296 | 2592 | 5184  | 10368 | 20736 | 41472  |
|                          | 2 | D  | 54  | 108 | 216 | 432  | 864  | 1728 | 3456  | 6912  | 13824 | 27648  |
|                          | 1 | D  | 27  | 54  | 108 | 216  | 432  | 864  | 1728  | 3456  | 6912  | 13824  |
| ROOT 9 & ITS HARMONICS.  | 8 | G  | 72  | 144 | 288 | 576  | 1152 | 2304 | 4608  | 9216  | 18432 | 36864  |
|                          | 6 | D  | 54  | 108 | 216 | 432  | 864  | 1728 | 3456  | 6912  | 13824 | 27648  |
|                          | 5 | B  | 45  | 90  | 180 | 360  | 720  | 1440 | 2880  | 5760  | 11520 | 23040  |
|                          | 4 | G  | 36  | 72  | 144 | 288  | 576  | 1152 | 2304  | 4608  | 9216  | 18432  |
|                          | 3 | D  | 27  | 54  | 108 | 216  | 432  | 864  | 1728  | 3456  | 6912  | 13824  |
|                          | 2 | G  | 18  | 36  | 72  | 144  | 288  | 576  | 1152  | 2304  | 4608  | 9216   |
|                          | 1 | G  | 9   | 18  | 36  | 72   | 144  | 288  | 576   | 1152  | 2304  | 4608   |
| ROOT 3 & ITS HARMONICS.  | 8 | C  | 24  | 48  | 96  | 192  | 384  | 768  | 1536  | 3072  | 6144  | 12288  |
|                          | 6 | G  | 18  | 36  | 72  | 144  | 288  | 576  | 1152  | 2304  | 4608  | 9216   |
|                          | 5 | E  | 15  | 30  | 60  | 120  | 240  | 480  | 960   | 1920  | 3840  | 7680   |
|                          | 4 | C  | 12  | 24  | 48  | 96   | 192  | 384  | 768   | 1536  | 3072  | 6144   |
|                          | 3 | G  | 9   | 18  | 36  | 72   | 144  | 288  | 576   | 1152  | 2304  | 4608   |
|                          | 2 | C  | 6   | 12  | 24  | 48   | 96   | 192  | 384   | 768   | 1536  | 3072   |
|                          | 1 | C  | 3   | 6   | 12  | 24   | 48   | 96   | 192   | 384   | 768   | 1536   |
| ROOT 1 & ITS HARMONICS.  | 8 | F  | 8   | 16  | 32  | 64   | 128  | 256  | 512   | 1024  | 2048  | 4096   |
|                          | 6 | C  | 6   | 12  | 24  | 48   | 96   | 192  | 384   | 768   | 1536  | 3072   |
|                          | 5 | A  | 5   | 10  | 20  | 40   | 80   | 160  | 320   | 640   | 1280  | 2560   |
|                          | 4 | F  | 4   | 8   | 16  | 32   | 64   | 128  | 256   | 512   | 1024  | 2048   |
|                          | 3 | C  | 3   | 6   | 12  | 24   | 48   | 96   | 192   | 384   | 768   | 1536   |
|                          | 2 | F  | 2   | 4   | 8   | 16   | 32   | 64   | 128   | 256   | 512   | 1024   |
|                          | 1 | F  | 1   | 2   | 4   | 8    | 16   | 32   | 64    | 128   | 256   | 512    |

# SECOND DIAGRAM. HARMONIC TABLE

continued

|                                    |   |    |       |       |       |        |        |        |         |
|------------------------------------|---|----|-------|-------|-------|--------|--------|--------|---------|
| Root 2187 $\frac{1}{3}$ HARMONICS. | 8 | F# | 17496 | 34992 | 69984 | 139968 | 279936 | 559872 | 1119744 |
|                                    | 6 | C# | 13122 | 26244 | 52488 | 104976 | 209952 | 419904 | 839808  |
|                                    | 5 | A# | 10935 | 21870 | 43740 | 87480  | 174960 | 349920 | 699840  |
|                                    | 4 | F# | 8748  | 17496 | 34992 | 69984  | 139968 | 279936 | 559872  |
|                                    | 3 | C# | 6561  | 13122 | 26244 | 52488  | 104976 | 209952 | 419904  |
|                                    | 2 | F# | 4374  | 8748  | 17496 | 34992  | 69984  | 139968 | 279936  |
|                                    | 1 | F# | 2187  | 4374  | 8748  | 17496  | 34992  | 69984  | 139968  |
| Root 729 $\frac{1}{3}$ HARMONICS.  | 8 | B  | 5832  | 11664 | 23328 | 46656  | 93312  | 186624 | 373248  |
|                                    | 6 | F# | 4374  | 8748  | 17496 | 34992  | 69984  | 139968 | 279936  |
|                                    | 5 | D# | 3645  | 7290  | 14580 | 29160  | 58320  | 116640 | 233280  |
|                                    | 4 | B  | 2916  | 5832  | 11664 | 23328  | 46656  | 93312  | 186624  |
|                                    | 3 | F# | 2187  | 4374  | 8748  | 17496  | 34992  | 69984  | 139968  |
|                                    | 2 | B  | 1458  | 2916  | 5832  | 11664  | 23328  | 46656  | 93312   |
|                                    | 1 | B  | 729   | 1458  | 2916  | 5832   | 11664  | 23328  | 46656   |
| Root 243 $\frac{1}{3}$ HARMONICS.  | 8 | E  | 1944  | 3888  | 7776  | 15552  | 31104  | 62208  | 124416  |
|                                    | 6 | B  | 1458  | 2916  | 5832  | 11664  | 23328  | 46656  | 93312   |
|                                    | 5 | G# | 1215  | 2430  | 4860  | 9720   | 19440  | 38880  | 77760   |
|                                    | 4 | E  | 972   | 1944  | 3888  | 7776   | 15552  | 31104  | 62208   |
|                                    | 3 | B  | 729   | 1458  | 2916  | 5832   | 11664  | 23328  | 46656   |
|                                    | 2 | E  | 486   | 972   | 1944  | 3888   | 7776   | 15552  | 31104   |
|                                    | 1 | E  | 243   | 486   | 972   | 1944   | 3888   | 7776   | 15552   |
| Root 81 $\frac{1}{3}$ HARMONICS.   | 8 | A  | 648   | 1296  | 2592  | 5184   | 10368  | 20736  | 41472   |
|                                    | 6 | E  | 486   | 972   | 1944  | 3888   | 7776   | 15552  | 31104   |
|                                    | 5 | C# | 405   | 810   | 1620  | 3240   | 6480   | 12960  | 25920   |
|                                    | 4 | A  | 324   | 648   | 1296  | 2592   | 5184   | 10368  | 20736   |
|                                    | 3 | E  | 243   | 486   | 972   | 1944   | 3888   | 7776   | 15552   |
|                                    | 2 | A  | 162   | 324   | 648   | 1296   | 2592   | 5184   | 10368   |
|                                    | 1 | A  | 81    | 162   | 324   | 648    | 1296   | 2592   | 5184    |

continued

# Second Diagram Harmonic Table

continued.

|              |   |     |         |
|--------------|---|-----|---------|
| Root 177147. | 8 | A*  | 1417176 |
|              | 6 | E*  | 1062882 |
|              | 5 | C** | 885535  |
|              | 4 | A*  | 708588  |
|              | 3 | E*  | 531441  |
|              | 2 | A*  | 354294  |
|              | 1 | A*  | 177147  |
| Root 59049.  | 8 | D*  | 452392  |
|              | 6 | A*  | 354294  |
|              | 5 | F** | 295245  |
|              | 4 | D*  | 236196  |
|              | 3 | A*  | 177147  |
|              | 2 | D*  | 118098  |
|              | 1 | D*  | 59049   |
| Root 19683.  | 8 | G*  | 157464  |
|              | 6 | D*  | 118098  |
|              | 5 | B*  | 98415   |
|              | 4 | G*  | 78732   |
|              | 3 | D*  | 59049   |
|              | 2 | G*  | 39366   |
|              | 1 | G*  | 19683   |
| Root 6561.   | 8 | C*  | 52488   |
|              | 6 | G*  | 39366   |
|              | 5 | E*  | 32805   |
|              | 4 | C*  | 26244   |
|              | 3 | G*  | 19683   |
|              | 2 | C*  | 13122   |
|              | 1 | C*  | 6561    |

# HARMONIC TABLE. FIRST DIAGRAM.

|                      |   |           |     |     |     |      |      |      |       |       |       |        |
|----------------------|---|-----------|-----|-----|-----|------|------|------|-------|-------|-------|--------|
| HARMONICS of Root 1. | 1 | B         | 1   | 2   | 4   | 8    | 16   | 32   | 64    | 128   | 256   | 512    |
|                      | 2 | B         | 2   | 4   | 8   | 16   | 32   | 64   | 128   | 256   | 512   | 1024   |
|                      | 3 | E         | 3   | 6   | 12  | 24   | 48   | 96   | 192   | 384   | 768   | 1536   |
|                      | 4 | B         | 4   | 8   | 16  | 32   | 64   | 128  | 256   | 512   | 1024  | 2048   |
|                      | 5 | G         | 5   | 10  | 20  | 40   | 80   | 160  | 320   | 640   | 1280  | 2560   |
|                      | 6 | E         | 6   | 12  | 24  | 48   | 96   | 192  | 384   | 768   | 1536  | 3072   |
|                      | 8 | B         | 8   | 16  | 32  | 64   | 128  | 256  | 512   | 1024  | 2048  | 4096   |
|                      |   |           |     |     |     |      |      |      |       |       |       |        |
| Root 3.              | 1 | E         | 3   | 6   | 12  | 24   | 48   | 96   | 192   | 384   | 768   | 1536   |
|                      | 2 | E         | 6   | 12  | 24  | 48   | 96   | 192  | 384   | 768   | 1536  | 3072   |
|                      | 3 | A         | 9   | 18  | 36  | 72   | 144  | 288  | 576   | 1152  | 2304  | 4608   |
|                      | 4 | E         | 12  | 24  | 48  | 96   | 192  | 384  | 768   | 1536  | 3072  | 6144   |
|                      | 5 | C         | 15  | 30  | 60  | 120  | 240  | 480  | 960   | 1920  | 3840  | 7680   |
|                      | 6 | A         | 18  | 36  | 72  | 144  | 288  | 576  | 1152  | 2304  | 4608  | 9216   |
|                      | 8 | E         | 24  | 48  | 96  | 192  | 384  | 768  | 1536  | 3072  | 6144  | 12288  |
|                      |   |           |     |     |     |      |      |      |       |       |       |        |
| Root 9.              | 1 | A         | 9   | 18  | 36  | 72   | 144  | 288  | 576   | 1152  | 2304  | 4608   |
|                      | 2 | A         | 18  | 36  | 72  | 144  | 288  | 576  | 1152  | 2304  | 4608  | 9216   |
|                      | 3 | D         | 27  | 54  | 108 | 216  | 432  | 864  | 1728  | 3456  | 6912  | 13824  |
|                      | 4 | A         | 36  | 72  | 144 | 288  | 576  | 1152 | 2304  | 4608  | 9216  | 18432  |
|                      | 5 | F         | 45  | 90  | 180 | 360  | 720  | 1440 | 2880  | 5760  | 11520 | 23040  |
|                      | 6 | D         | 54  | 108 | 216 | 432  | 864  | 1728 | 3456  | 6912  | 13824 | 27648  |
|                      | 8 | A         | 72  | 144 | 288 | 576  | 1152 | 2304 | 4608  | 9216  | 18432 | 36864  |
|                      |   |           |     |     |     |      |      |      |       |       |       |        |
| Root 27.             | 1 | D         | 27  | 54  | 108 | 216  | 432  | 864  | 1728  | 3456  | 6912  | 13824  |
|                      | 2 | D         | 54  | 108 | 216 | 432  | 864  | 1728 | 3456  | 6912  | 13824 | 27648  |
|                      | 3 | G         | 81  | 162 | 324 | 648  | 1296 | 2592 | 5184  | 10368 | 20736 | 41472  |
|                      | 4 | D         | 108 | 216 | 432 | 864  | 1728 | 3456 | 6912  | 13824 | 27648 | 55296  |
|                      | 5 | B $\flat$ | 135 | 270 | 540 | 1080 | 2160 | 4320 | 8640  | 17280 | 34560 | 69120  |
|                      | 6 | G         | 162 | 324 | 648 | 1296 | 2592 | 5184 | 10368 | 20736 | 41472 | 82944  |
|                      | 8 | D         | 216 | 432 | 864 | 1728 | 3456 | 6912 | 13824 | 27648 | 55296 | 110592 |
|                      |   |           |     |     |     |      |      |      |       |       |       |        |

# HARMONIC TABLE. FIRST DIAGRAM.

continued

|                      |   |                |       |       |       |        |        |        |         |
|----------------------|---|----------------|-------|-------|-------|--------|--------|--------|---------|
| HARMONICS of Root 81 | 1 | G              | 81    | 162   | 324   | 648    | 1296   | 2592   | 5184    |
|                      | 2 | G              | 162   | 324   | 648   | 1296   | 2592   | 5184   | 10368   |
|                      | 3 | C              | 243   | 486   | 972   | 1944   | 3888   | 7776   | 15552   |
|                      | 4 | G              | 324   | 648   | 1296  | 2592   | 5184   | 10368  | 20736   |
|                      | 5 | E <sub>b</sub> | 405   | 810   | 1620  | 3240   | 6480   | 12960  | 25920   |
|                      | 6 | C              | 486   | 972   | 1944  | 3888   | 7776   | 15552  | 31104   |
|                      | 8 | G              | 648   | 1296  | 2592  | 5184   | 10368  | 20736  | 41472   |
| Root. 243.           | 1 | G              | 243   | 486   | 972   | 1944   | 3888   | 7776   | 15552   |
|                      | 2 | G              | 486   | 972   | 1944  | 3888   | 7776   | 15552  | 31104   |
|                      | 3 | F              | 729   | 1458  | 2916  | 5832   | 11664  | 23328  | 46656   |
|                      | 4 | C              | 972   | 1944  | 3888  | 7776   | 15552  | 31104  | 62208   |
|                      | 5 | A <sub>b</sub> | 1215  | 2430  | 4860  | 9720   | 19440  | 38880  | 77760   |
|                      | 6 | F              | 1458  | 2916  | 5832  | 11664  | 23328  | 46656  | 93312   |
|                      | 8 | C              | 1944  | 3888  | 7776  | 15552  | 31104  | 62208  | 124416  |
| Root. 729.           | 1 | F              | 729   | 1458  | 2916  | 5832   | 11664  | 23328  | 46656   |
|                      | 2 | F              | 1458  | 2916  | 5832  | 11664  | 23328  | 46656  | 93312   |
|                      | 3 | B <sub>b</sub> | 2187  | 4374  | 8748  | 17496  | 34992  | 69984  | 139968  |
|                      | 4 | F              | 2916  | 5832  | 11664 | 23328  | 46656  | 93312  | 186624  |
|                      | 5 | D <sub>b</sub> | 3645  | 7290  | 14580 | 29160  | 58320  | 116640 | 233280  |
|                      | 6 | B <sub>b</sub> | 4374  | 8748  | 17496 | 34992  | 69984  | 139968 | 279936  |
|                      | 8 | F              | 5832  | 11664 | 23328 | 46656  | 93312  | 186624 | 373248  |
| Root 2187.           | 1 | B <sub>b</sub> | 2187  | 4374  | 8748  | 17496  | 34992  | 69984  | 139968  |
|                      | 2 | B <sub>b</sub> | 4374  | 8748  | 17496 | 34992  | 69984  | 139968 | 279936  |
|                      | 3 | E <sub>b</sub> | 6561  | 13122 | 26244 | 52488  | 104976 | 209952 | 419904  |
|                      | 4 | B <sub>b</sub> | 8748  | 17496 | 34992 | 69984  | 139968 | 279936 | 559872  |
|                      | 5 | G <sub>b</sub> | 10935 | 21870 | 43740 | 87480  | 174960 | 349920 | 699860  |
|                      | 6 | E <sub>b</sub> | 13122 | 26244 | 52488 | 104976 | 209952 | 419904 | 839808  |
|                      | 8 | B <sub>b</sub> | 17496 | 34992 | 69984 | 139968 | 279936 | 559872 | 1119744 |

# First Diagram. Harmonic Table continued

|                            |   |     |         |
|----------------------------|---|-----|---------|
| HARMONICS OF<br>ROOT 6561. | 1 | E♭  | 6561    |
|                            | 2 | E♭  | 13122   |
|                            | 3 | A♭  | 19683   |
|                            | 4 | E♭  | 26244   |
|                            | 5 | C♭  | 32805   |
|                            | 6 | A♭  | 39366   |
|                            | 8 | E♭  | 52488   |
|                            |   |     |         |
| ROOT 19683.                | 1 | A♭  | 19683   |
|                            | 2 | A♭  | 39366   |
|                            | 3 | D♭  | 59049   |
|                            | 4 | A♭  | 78732   |
|                            | 5 | F♭  | 78415   |
|                            | 6 | D♭  | 118098  |
|                            | 8 | A♭  | 157464  |
|                            |   |     |         |
| ROOT 59049.                | 1 | D♭  | 59049   |
|                            | 2 | D♭  | 118098  |
|                            | 3 | G♭  | 177147  |
|                            | 4 | D♭  | 236196  |
|                            | 5 | B♭♭ | 295245  |
|                            | 6 | G♭  | 354294  |
|                            | 8 | D♭  | 452392  |
|                            |   |     |         |
| ROOT 177147.               | 1 | G♭  | 177147  |
|                            | 2 | G♭  | 354294  |
|                            | 3 | C♭  | 531441  |
|                            | 4 | G♭  | 708588  |
|                            | 5 | E♭♭ | 885535  |
|                            | 6 | C♭  | 1062882 |
|                            | 8 | G♭  | 1417176 |
|                            |   |     |         |

**FIRST DIAGRAM.  
QUINTUPLE.**

Plate XV.

|            |   |            |       |       |       |        |        |        |        |         |
|------------|---|------------|-------|-------|-------|--------|--------|--------|--------|---------|
| Root 5.    | 4 | G          | 20    | 40    | 80    | 160    | 320    | 640    | 1280   | 2360    |
|            |   | E $\sharp$ | 24    | 48    | 96    | 192    | 384    | 768    | 1536   | 3072    |
|            | 5 | E $\flat$  | 25    | 50    | 100   | 200    | 400    | 800    | 1600   | 3200    |
|            | 6 | C          | 30    | 60    | 120   | 240    | 480    | 960    | 1920   | 3840    |
| Root 15.   | 4 | C          | 60    | 120   | 240   | 480    | 960    | 1920   | 3840   | 7680    |
|            |   | A $\sharp$ | 72    | 144   | 288   | 576    | 1152   | 2304   | 4608   | 9216    |
|            | 5 | A $\flat$  | 75    | 150   | 300   | 600    | 1200   | 2400   | 4800   | 9600    |
|            | 6 | F          | 90    | 180   | 360   | 720    | 1440   | 2880   | 5760   | 11520   |
| Root 45.   | 4 | F          | 180   | 360   | 720   | 1440   | 2880   | 5760   | 11520  | 23040   |
|            |   | D $\sharp$ | 216   | 432   | 864   | 1728   | 3456   | 6912   | 13824  | 27648   |
|            | 5 | D $\flat$  | 225   | 450   | 900   | 1800   | 3600   | 7200   | 14400  | 28800   |
|            | 6 | B $\flat$  | 270   | 540   | 1080  | 2160   | 4320   | 8640   | 17280  | 34560   |
| Root 135.  | 4 | B $\flat$  | 540   | 1080  | 2160  | 4320   | 8640   | 17280  | 34560  | 69120   |
|            |   | G $\sharp$ | 648   | 1296  | 2592  | 5184   | 10368  | 20736  | 41472  | 82944   |
|            | 5 | G $\flat$  | 675   | 1350  | 2700  | 5400   | 10800  | 21600  | 43200  | 86400   |
|            | 6 | E $\flat$  | 810   | 1620  | 3240  | 6480   | 12960  | 25920  | 51840  | 103680  |
| Root 405.  | 4 | E $\flat$  | 1620  | 3240  | 6480  | 12960  | 25920  | 51840  | 103680 | 207360  |
|            |   | C $\sharp$ | 1944  | 3888  | 7776  | 15552  | 31104  | 62208  | 124416 | 248832  |
|            | 5 | C $\flat$  | 2025  | 4050  | 8100  | 16200  | 32400  | 64800  | 129600 | 259200  |
|            | 6 | A $\flat$  | 2430  | 4860  | 9720  | 19440  | 38880  | 77760  | 155520 | 311040  |
| Root 1215. | 4 | A $\flat$  | 4860  | 9720  | 19440 | 38880  | 77760  | 155520 | 311040 | 622080  |
|            |   | F $\sharp$ | 5832  | 11664 | 23328 | 46656  | 93312  | 186624 | 373248 | 746496  |
|            | 5 | F $\flat$  | 6075  | 12150 | 24300 | 48600  | 97200  | 194400 | 388800 | 777600  |
|            | 6 | D $\flat$  | 14610 | 29220 | 58440 | 116880 | 233760 | 467520 | 935040 | 1870080 |

# FIRST DIAGRAM. QUINTUPLE.

|            |   |     |       |       |        |        |        |         |         |
|------------|---|-----|-------|-------|--------|--------|--------|---------|---------|
| Root 25.   | 4 | E♭  | 100   | 200   | 400    | 800    | 1600   | 3200    | 6400    |
|            |   | C♯  | 120   | 240   | 480    | 960    | 1920   | 3840    | 7680    |
|            | 5 | C♭  | 125   | 250   | 500    | 1000   | 2000   | 4000    | 8000    |
|            | 6 | A♭  | 150   | 300   | 600    | 1200   | 2400   | 4800    | 9600    |
| Root 75.   | 4 | A♭  | 300   | 600   | 1200   | 2400   | 4800   | 9600    | 19200   |
|            |   | F♯  | 360   | 720   | 1440   | 2880   | 5760   | 11520   | 23040   |
|            | 5 | F♭  | 375   | 750   | 1500   | 3000   | 6000   | 12000   | 24000   |
|            | 6 | D♭  | 450   | 900   | 1800   | 3600   | 7200   | 14400   | 28800   |
| Root 225.  | 4 | D♭  | 900   | 1800  | 3600   | 7200   | 14400  | 28800   | 57600   |
|            |   | B♭  | 1080  | 2160  | 4320   | 8640   | 17280  | 34560   | 69120   |
|            | 5 | B♭♭ | 1125  | 2250  | 4500   | 9000   | 18000  | 36000   | 72000   |
|            | 6 | G♭  | 1350  | 2700  | 5400   | 10800  | 21600  | 43200   | 86400   |
| Root 675.  | 4 | G♭  | 2700  | 5400  | 10800  | 21600  | 43200  | 86400   | 172800  |
|            |   | E♭  | 3240  | 6480  | 12960  | 25920  | 51840  | 103680  | 207360  |
|            | 5 | E♭♭ | 3375  | 6750  | 13500  | 27000  | 54000  | 108000  | 216000  |
|            | 6 | C♭  | 4050  | 8100  | 16200  | 32400  | 64800  | 129600  | 259200  |
| Root 2025. | 4 | C♭  | 8100  | 16200 | 32400  | 64800  | 129600 | 259200  | 518400  |
|            |   | A♭  | 9720  | 18440 | 36880  | 73760  | 147520 | 295040  | 590080  |
|            | 5 | A♭♭ | 10125 | 20250 | 40500  | 81000  | 162000 | 324000  | 648000  |
|            | 6 | F♭  | 12150 | 24300 | 48600  | 97200  | 194400 | 388800  | 777600  |
| Root 6075. | 4 | F♭  | 24300 | 48600 | 97200  | 194400 | 388800 | 777600  | 1555200 |
|            |   | D♭  | 29160 | 58320 | 116640 | 233280 | 466560 | 933120  | 1866240 |
|            | 5 | D♭♭ | 30375 | 60740 | 121480 | 242960 | 485920 | 971840  | 1943680 |
|            | 6 | B♭♭ | 36450 | 72900 | 145800 | 291600 | 583200 | 1166400 | 2332800 |

# FIRST DIAGRAM. QUINTUPLE.

|            |   |     |       |        |        |        |        |         |
|------------|---|-----|-------|--------|--------|--------|--------|---------|
| Root 37    | 4 | Cb  | 500   | 1000   | 2000   | 4000   | 8000   | 16000   |
|            |   | Ab  | 600   | 1200   | 2400   | 4800   | 9600   | 19200   |
|            | 5 | Abb | 625   | 1250   | 2500   | 5000   | 10000  | 20000   |
|            | 6 | Fb  | 750   | 1500   | 3000   | 6000   | 12000  | 24000   |
| Root 1125  | 4 | Fb  | 1500  | 3000   | 6000   | 12000  | 24000  | 48000   |
|            |   | Db  | 1800  | 3600   | 7200   | 14400  | 28800  | 57600   |
|            | 5 | Dbb | 1875  | 3750   | 7500   | 15000  | 30000  | 60000   |
|            | 6 | Bbb | 2250  | 4500   | 9000   | 18000  | 36000  | 72000   |
| Root 3375  | 4 | Bbb | 4500  | 9000   | 18000  | 36000  | 72000  | 144000  |
|            |   | Gb  | 5400  | 10800  | 21600  | 43200  | 86400  | 172800  |
|            | 5 | Gbb | 5625  | 11250  | 22500  | 45000  | 90000  | 180000  |
|            | 6 | Ebb | 6750  | 13500  | 27000  | 54000  | 108000 | 216000  |
| Root 10125 | 4 | Ebb | 13500 | 27000  | 54000  | 108000 | 216000 | 432000  |
|            |   | Cb  | 16200 | 32400  | 64800  | 129600 | 259200 | 518400  |
|            | 5 | Cbb | 16875 | 33750  | 67500  | 135000 | 270000 | 540000  |
|            | 6 | Abb | 20250 | 40500  | 81000  | 162000 | 324000 | 648000  |
| Root 10125 | 4 | Abb | 40500 | 81000  | 162000 | 324000 | 648000 | 1296000 |
|            |   | Fb  | 48600 | 97200  | 194400 | 388800 | 777600 | 1555200 |
|            | 5 | Fbb | 50625 | 101250 | 202500 | 405000 | 810000 | 1620000 |
|            | 6 | Dbb | 60750 | 121500 | 243000 | 486000 | 972000 | 1944000 |

# SECOND DIAGRAM. QUINTUPLE.

| QUINTUPLE. |   |                |      |       |       |       |        |        |        |        |  |
|------------|---|----------------|------|-------|-------|-------|--------|--------|--------|--------|--|
| Root 1215. | 6 | D*             | 7290 | 14580 | 29160 | 58320 | 116640 | 233280 | 466560 | 933120 |  |
|            | 5 | B*             | 6075 | 12150 | 24300 | 48600 | 97200  | 194400 | 388800 | 777600 |  |
|            |   | B <sub>h</sub> | 5832 | 11664 | 23328 | 46656 | 93312  | 186624 | 373248 | 746496 |  |
|            | 4 | G*             | 4860 | 9720  | 19440 | 38880 | 77760  | 155420 | 310840 | 621680 |  |
| Root 405.  | 6 | G*             | 2430 | 4860  | 9720  | 19440 | 38880  | 77760  | 155520 | 311040 |  |
|            | 5 | E#             | 2025 | 4050  | 8100  | 16200 | 32400  | 64800  | 129600 | 259200 |  |
|            |   | E <sub>h</sub> | 1944 | 3888  | 7776  | 15552 | 31104  | 62208  | 124416 | 248832 |  |
|            | 4 | C*             | 1620 | 3240  | 6480  | 12960 | 25920  | 51840  | 103680 | 207360 |  |
| Root 135.  | 6 | C*             | 810  | 1260  | 3240  | 6480  | 12960  | 25920  | 51840  | 103680 |  |
|            | 5 | A*             | 675  | 1350  | 2700  | 5400  | 10800  | 21600  | 43200  | 86400  |  |
|            |   | A <sub>h</sub> | 648  | 1296  | 2592  | 5184  | 10368  | 20736  | 41472  | 82944  |  |
|            | 4 | F*             | 540  | 1080  | 2160  | 4320  | 8640   | 17280  | 34560  | 69120  |  |
| Root 45.   | 6 | F*             | 270  | 540   | 1080  | 2160  | 4320   | 8640   | 17280  | 34560  |  |
|            | 5 | D*             | 225  | 450   | 900   | 1800  | 3600   | 7200   | 14400  | 28800  |  |
|            |   | D <sub>h</sub> | 216  | 432   | 864   | 1728  | 3456   | 6912   | 13824  | 27648  |  |
|            | 4 | B              | 180  | 360   | 720   | 1440  | 2880   | 5760   | 11520  | 23040  |  |
| Root 15.   | 6 | B              | 90   | 180   | 360   | 720   | 1440   | 2880   | 5760   | 11520  |  |
|            | 5 | G*             | 75   | 150   | 300   | 600   | 1200   | 2400   | 4800   | 9600   |  |
|            |   | G <sub>h</sub> | 72   | 144   | 288   | 576   | 1152   | 2304   | 4608   | 9216   |  |
|            | 4 | E              | 60   | 120   | 240   | 480   | 960    | 1920   | 3840   | 7680   |  |
| Root 5.    | 6 | E              | 30   | 60    | 120   | 240   | 480    | 960    | 1920   | 3840   |  |
|            | 5 | C*             | 25   | 50    | 100   | 200   | 400    | 800    | 1600   | 3200   |  |
|            |   | C <sub>h</sub> | 24   | 48    | 96    | 192   | 384    | 768    | 1536   | 3072   |  |
|            | 4 | A              | 20   | 40    | 80    | 160   | 320    | 640    | 1280   | 2560   |  |

# SECOND DIAGRAM. QUINTUPLE.

Plate XVII

|            |   |     |       |       |        |        |        |         |         |
|------------|---|-----|-------|-------|--------|--------|--------|---------|---------|
| Root 6075. | 6 | F## | 36450 | 72900 | 145800 | 291600 | 583200 | 1166400 | 2332800 |
|            | 5 | D## | 30375 | 60750 | 121500 | 24300  | 486000 | 972000  | 1944000 |
|            |   | D#  | 29160 | 58320 | 116640 | 233280 | 466560 | 933120  | 1866240 |
|            | 4 | B#  | 24300 | 48600 | 97200  | 194400 | 388800 | 777600  | 1555200 |
| Root 2025. | 6 | B#  | 12150 | 24300 | 48600  | 97200  | 194400 | 388800  | 777600  |
|            | 5 | G## | 10125 | 20250 | 40500  | 81000  | 162000 | 324000  | 648000  |
|            |   | G#  | 9720  | 19440 | 38880  | 77760  | 155520 | 311040  | 622080  |
|            | 4 | E#  | 8100  | 16200 | 32400  | 64800  | 129600 | 259200  | 518400  |
| Root 675.  | 6 | E#  | 4050  | 8100  | 16200  | 32400  | 64800  | 129600  | 259200  |
|            | 5 | C## | 3375  | 6750  | 13500  | 27000  | 54000  | 108000  | 216000  |
|            |   | C#  | 3240  | 6480  | 12960  | 25920  | 51840  | 103680  | 207360  |
|            | 4 | A#  | 2700  | 5400  | 10800  | 21600  | 43200  | 86400   | 172800  |
| Root 225.  | 6 | A#  | 1350  | 2700  | 5400   | 10800  | 21600  | 43200   | 86400   |
|            | 5 | F## | 1125  | 2250  | 4500   | 9000   | 18000  | 36000   | 72000   |
|            |   | F#  | 1080  | 2160  | 4320   | 8640   | 17280  | 34560   | 69120   |
|            | 4 | D#  | 900   | 1800  | 3600   | 7200   | 14400  | 28800   | 57600   |
| Root 75.   | 6 | D#  | 450   | 900   | 1800   | 3600   | 7200   | 14400   | 28800   |
|            | 5 | B#  | 375   | 750   | 1500   | 3000   | 6000   | 12000   | 24000   |
|            |   | Bb  | 360   | 720   | 1440   | 2880   | 5760   | 11520   | 23040   |
|            | 4 | G#  | 300   | 600   | 1200   | 2400   | 4800   | 9600    | 19200   |
| Root 25.   | 6 | G#  | 150   | 300   | 600    | 1200   | 2400   | 4800    | 9600    |
|            | 5 | E#  | 125   | 250   | 500    | 1000   | 2000   | 4000    | 8000    |
|            |   | Eb  | 120   | 240   | 480    | 960    | 1920   | 3840    | 7680    |
|            | 4 | C#  | 100   | 200   | 400    | 800    | 1600   | 3200    | 6400    |

# SECOND DIAGRAM. QUINTUPLE.

|             |   |     |       |        |        |        |        |         |
|-------------|---|-----|-------|--------|--------|--------|--------|---------|
| Root 10125. | 6 | D## | 60750 | 121500 | 243000 | 486000 | 972000 | 1944000 |
|             | 5 | B## | 50625 | 101250 | 202500 | 405000 | 810000 | 1620000 |
|             |   | B#  | 48600 | 97200  | 194400 | 388800 | 777600 | 1555200 |
|             | 4 | G## | 40500 | 81000  | 162000 | 324000 | 648000 | 1296000 |
| Root 3375.  | 6 | G## | 20250 | 40500  | 81000  | 162000 | 324000 | 648000  |
|             | 5 | E## | 16875 | 33750  | 67500  | 135000 | 270000 | 540000  |
|             |   | E#  | 16200 | 32400  | 64800  | 129600 | 259200 | 518400  |
|             | 4 | C## | 13500 | 27000  | 54000  | 108000 | 216000 | 432000  |
| Root 1125.  | 6 | C## | 6750  | 13500  | 27000  | 54000  | 108000 | 216000  |
|             | 5 | A## | 5625  | 11250  | 22500  | 45000  | 90000  | 180000  |
|             |   | A#  | 5400  | 10800  | 21600  | 43200  | 86400  | 172800  |
|             | 4 | F## | 4500  | 9000   | 18000  | 36000  | 72000  | 144000  |
| Root 375.   | 6 | F## | 2250  | 4500   | 9000   | 18000  | 36000  | 72000   |
|             | 5 | D## | 1875  | 3750   | 7500   | 15000  | 30000  | 60000   |
|             |   | D#  | 1800  | 3600   | 7200   | 14400  | 28800  | 57600   |
|             | 4 | B#  | 1500  | 3000   | 6000   | 12000  | 24000  | 48000   |
| Root 125.   | 6 | B#  | 750   | 1500   | 3000   | 6000   | 12000  | 24000   |
|             | 5 | G## | 625   | 1250   | 2500   | 5000   | 10000  | 20000   |
|             |   | G#  | 600   | 1200   | 2400   | 4800   | 9600   | 19200   |
|             | 4 | E#  | 500   | 1000   | 2000   | 4000   | 8000   | 16000   |

Plate XVIII. **DIAGRAMS OF GAUDENTIUS.**

**FIRST DIAGRAM.**

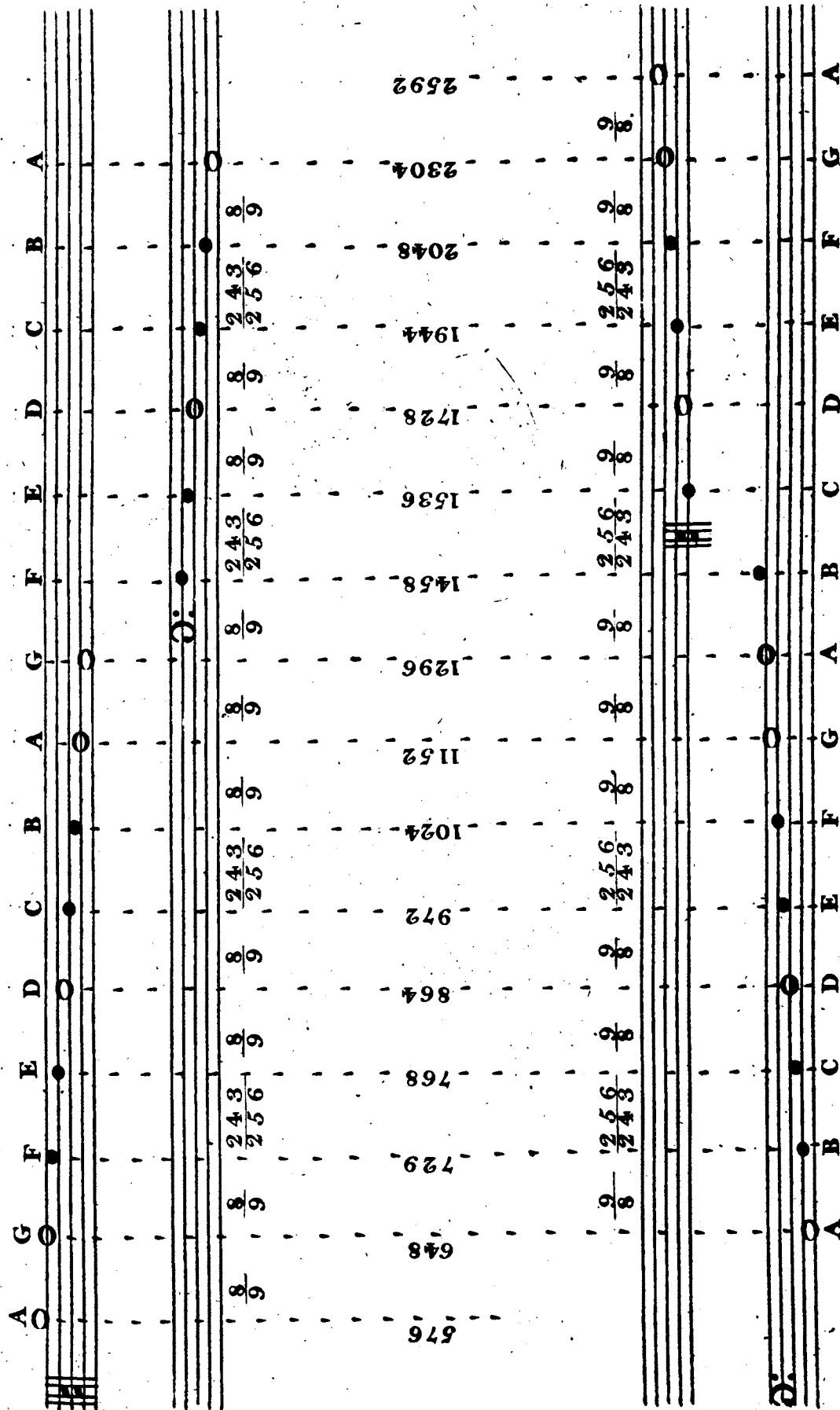
**SECOND DIAGRAM.**

|     |     |      |                              |      |
|-----|-----|------|------------------------------|------|
|     |     | 576  | <i>Nete hyperbolaon</i>      | 2592 |
| 9   | 8   |      |                              |      |
|     |     | 648  | <i>Hyperbolaon diatonos</i>  | 2304 |
| 9   | 8   |      |                              |      |
| 256 | 243 | 729  | <i>Trite hyperbolaon</i>     | 2048 |
|     |     | 768  | <i>Nete diezeugmenon</i>     | 1944 |
| 9   | 8   |      |                              |      |
|     |     | 864  | <i>Diezeugmenon diatonos</i> | 1728 |
| 9   | 8   |      |                              |      |
| 256 | 243 | 972  | <i>Trite diezeugmenon</i>    | 1536 |
|     |     | 1024 | <i>Paramesos</i>             | 1458 |
| 9   | 8   |      |                              |      |
|     |     | 1152 | <i>Mese</i>                  | 1296 |
| 9   | 8   |      |                              |      |
|     |     | 1296 | <i>Meson diatonos</i>        | 1152 |
| 9   | 8   |      |                              |      |
| 256 | 243 | 1458 | <i>Parhypate meson</i>       | 1024 |
|     |     | 1536 | <i>Hypate meson</i>          | 972  |
| 9   | 8   |      |                              |      |
|     |     | 1728 | <i>Hypaton diatonos</i>      | 864  |
| 9   | 8   |      |                              |      |
| 256 | 243 | 1944 | <i>Parhypate hypaton</i>     | 768  |
|     |     | 2048 | <i>Hypate hypaton</i>        | 729  |
| 9   | 8   |      |                              |      |
|     |     | 2304 | <i>Proslambanomenos</i>      | 648  |

|     |     |
|-----|-----|
| 9   | 8   |
| 9   | 8   |
| 256 | 243 |
| 9   | 8   |
| 9   | 8   |
| 256 | 243 |
| 9   | 8   |
| 9   | 8   |
| 256 | 243 |
| 9   | 8   |
| 9   | 8   |
| 256 | 243 |
| 9   | 8   |

# *Inversion of the Scales.*

First Diagram.



Second Diagram.  
*Inversion of the Scales.*

**FIRST DIAGRAM.**

*Roots, & their Octaves.*

| Roots | Harmonics |      |     |      |      |      |      |      |
|-------|-----------|------|-----|------|------|------|------|------|
| 1     | B         | 4    |     |      |      |      | 1024 | 2048 |
|       | G         | 5    |     |      |      |      |      |      |
|       | E         | 6    |     |      |      |      |      |      |
| 3     | E         | 12   |     |      |      | 768  | 1536 |      |
|       | C         | 15   |     |      |      |      |      |      |
|       | A         | 18   |     |      |      |      |      |      |
| 9     | A         | 36   |     |      |      | 1152 | 2304 |      |
|       | F         | 45   |     |      |      |      |      |      |
|       | D         | 54   |     |      |      |      |      |      |
| 27    | D         | 108  |     |      | 864  | 1728 |      |      |
|       | B $\flat$ | 135  |     |      |      |      |      |      |
|       | G         | 162  |     |      |      |      |      |      |
| 81    | G         | 324  |     | 648  | 1296 | 2592 |      |      |
|       | E $\flat$ | 405  |     |      |      |      |      |      |
|       | C         | 486  |     |      |      |      |      |      |
| 243   | C         | 972  |     | 972  | 1944 |      |      |      |
|       | A $\flat$ | 1215 |     |      |      |      |      |      |
|       | F         | 1458 |     |      |      |      |      |      |
| 729   | F         | 2916 | 729 | 1458 |      |      |      |      |
|       | D $\flat$ | 3645 |     |      |      |      |      |      |
|       | B $\flat$ | 4374 |     |      |      |      |      |      |

# SECOND DIAGRAM. Roots and their Octaves.

PlateXXI

| Roots | Harmonics |     |      |      |      |      |      |  |
|-------|-----------|-----|------|------|------|------|------|--|
|       | F# 4374   |     |      |      |      |      |      |  |
|       | D# 3645   |     |      |      |      |      |      |  |
| 729   | B 2916    | 729 | 1458 |      |      |      |      |  |
|       | B 1458    |     |      |      |      |      |      |  |
|       | G# 1215   |     |      |      |      |      |      |  |
| 243   | E 972     |     | 972  | 1944 |      |      |      |  |
|       | E 486     |     |      |      |      |      |      |  |
|       | C# 405    |     |      |      |      |      |      |  |
| 81    | A 324     |     | 648  | 1296 | 2592 |      |      |  |
|       | A 162     |     |      |      |      |      |      |  |
|       | F# 135    |     |      |      |      |      |      |  |
| 27    | D 108     |     |      | 864  | 1728 |      |      |  |
|       | D 54      |     |      |      |      |      |      |  |
|       | B 45      |     |      |      |      |      |      |  |
| 9     | G 36      |     |      |      | 1152 | 2304 |      |  |
|       | G 18      |     |      |      |      |      |      |  |
|       | E 15      |     |      |      |      |      |      |  |
| 3     | C 12      |     |      |      | 768  | 1536 |      |  |
|       | C 6       |     |      |      |      |      |      |  |
|       | A 5       |     |      |      |      |      |      |  |
| 1     | F 4       |     |      |      |      | 1024 | 2048 |  |

*Tetrachord first Diagram. Tetrachord second Diagram*  
*fig:1* *fig:2*

|       |                 |                |               |     |
|-------|-----------------|----------------|---------------|-----|
|       | $\frac{16}{15}$ | $\frac{10}{9}$ | $\frac{9}{8}$ |     |
|       | B — C           | D              | E             |     |
|       | 3 2             | 3 0            | 2 7           | 2 4 |
|       | G               | A — A          | A             |     |
|       | 4 0             | 3 6            | 3 6           | 3 6 |
|       | E — E           | F              | E             |     |
|       | 4 8             | 4 8            | 4 5           | 4 8 |
| F.B.  | E               | A              | D             | A   |
|       | 9 6             | 7 2            | 5 4           | 7 2 |
| Roots | 1               | 3              | 9             | 3   |

|       |                 |                |               |     |
|-------|-----------------|----------------|---------------|-----|
|       | $\frac{15}{16}$ | $\frac{9}{10}$ | $\frac{8}{9}$ |     |
|       | F — E           | D              | C             |     |
|       | 6 4             | 6 0            | 5 4           | 4 8 |
|       | C — C           | B              | C             |     |
|       | 4 8             | 4 8            | 4 5           | 4 8 |
|       | A               | G — G          | G             |     |
|       | 4 0             | 3 6            | 3 6           | 3 6 |
| F.B.  | F               | C              | G             | C   |
|       | 3 2             | 2 4            | 1 8           | 2 4 |
| Roots | 1               | 3              | 9             | 3   |

*The Scale, or Diatonic Octave in the first Diagram.*  
*fig:3* *Tetrachord.*

|       |               |                  |                 |                   |                 |                  |                   |                 |
|-------|---------------|------------------|-----------------|-------------------|-----------------|------------------|-------------------|-----------------|
|       | A             | $\frac{9}{10}$ G | $\frac{8}{9}$ F | $\frac{15}{16}$ E | $\frac{8}{9}$ D | $\frac{9}{10}$ C | $\frac{15}{16}$ B | $\frac{8}{9}$ A |
|       | 3 6           | 4 0              | 4 5             | 4 8               | 5 4             | 6 0              | 6 4               | 7 2             |
|       | $\frac{2}{1}$ | $\frac{6}{5}$    | $\frac{6}{5}$   | $\frac{3}{2}$     | $\frac{2}{1}$   | $\frac{6}{5}$    | $\frac{3}{2}$     | $\frac{2}{1}$   |
| F.B.  | 7 2 . A       | 4 8 . E          | 5 4 . D         | 7 2 . A           | 1 0 8 . D       | 7 2 . A          | 9 6 . E           | 1 4 4 . A       |
| Roots | 3             | 1                | 9               | 3                 | 9               | 3                | 1                 | 3               |

*The Scale, or Diatonic Octave, in the Second Diagram.*  
*fig:4* *Tetrachord.*

|       |               |                  |                   |                    |                  |                   |                  |                    |
|-------|---------------|------------------|-------------------|--------------------|------------------|-------------------|------------------|--------------------|
|       | C.            | $\frac{9}{8}$ D. | $\frac{10}{9}$ E. | $\frac{16}{15}$ F. | $\frac{9}{8}$ G. | $\frac{10}{9}$ A. | $\frac{9}{8}$ B. | $\frac{16}{15}$ C. |
|       | 2 4           | 2 7              | 3 0               | 3 2                | 3 6              | 4 0               | 4 5              | 4 8                |
|       | $\frac{2}{1}$ | $\frac{3}{2}$    | $\frac{5}{4}$     | $\frac{2}{1}$      | $\frac{3}{2}$    | $\frac{5}{4}$     | $\frac{5}{4}$    | $\frac{2}{1}$      |
| F.B.  | 1 2 . C.      | 1 8 . G          | 2 4 . C           | 1 6 . F            | 2 4 . C          | 3 2 . F           | 3 6 . G          | 2 4 . C            |
| Roots | 3             | 9                | 3                 | 1                  | 3                | 1                 | 9                | 3                  |

# FIRST DIAGRAM. TETRACHORDS CONJOINED.

**FIG. 1.**

|                              | ANTECEDENT.     |                |               | MEAN.           |                |               | CONSEQUENT.     |                |               |               |
|------------------------------|-----------------|----------------|---------------|-----------------|----------------|---------------|-----------------|----------------|---------------|---------------|
|                              | $\frac{16}{15}$ | $\frac{10}{9}$ | $\frac{9}{8}$ | $\frac{16}{15}$ | $\frac{10}{9}$ | $\frac{9}{8}$ | $\frac{16}{15}$ | $\frac{10}{9}$ | $\frac{9}{8}$ |               |
|                              | B               | C              | D             | E               | F              | G             | A               | B $\flat$      | C             | D             |
|                              | 512             | 480            | 432           | 384             | 360            | 324           | 288             | 270            | 243           | 216           |
|                              | $\frac{2}{3}$   | $\frac{5}{6}$  | $\frac{1}{2}$ | $\frac{2}{3}$   | $\frac{5}{6}$  | $\frac{1}{2}$ | $\frac{2}{3}$   | $\frac{5}{6}$  | $\frac{1}{2}$ | $\frac{2}{3}$ |
|                              | 768             | 576            | 864           | 576             | 432            | 648           | 432             | 324            | 486           | 324           |
| FUNDAMENTAL BASSES<br>Roots. | E<br>1          | A<br>3         | D<br>9        | A<br>3          | D<br>9         | G<br>27       | D<br>9          | G<br>27        | C<br>81       | G<br>27       |

# FIRST DIAGRAM. TETRACHORDS DISJOINED.

**FIG. 3.**

|           | $\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$ |               |               | $\frac{1}{1}$ | $\frac{9}{8}$ | $\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$ |               |               | $\frac{1}{1}$ | $\frac{9}{8}$ | $\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$ |               |               |               |
|-----------|----------------------------------------------|---------------|---------------|---------------|---------------|----------------------------------------------|---------------|---------------|---------------|---------------|----------------------------------------------|---------------|---------------|---------------|
|           | A                                            | B $\flat$     | C             | D             | D             | E                                            | F             | G             | A             | A             | B                                            | C             | D             | E             |
|           | 288                                          | 270           | 243           | 216           | 216           | 192                                          | 180           | 162           | 144           | 144           | 128                                          | 120           | 108           | 96            |
|           | $\frac{2}{3}$                                | $\frac{5}{6}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$                                | $\frac{5}{6}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$                                | $\frac{5}{6}$ | $\frac{1}{2}$ | $\frac{2}{3}$ |
|           | 432                                          | 324           | 486           | 324           | 432           | 288                                          | 216           | 324           | 216           | 288           | 192                                          | 144           | 216           | 144           |
| FB.<br>R. | D<br>9                                       | G<br>27       | C<br>81       | G<br>27       | D<br>9        | A<br>3                                       | D<br>9        | G<br>27       | D<br>9        | A<br>3        | E<br>1                                       | A<br>3        | D<br>9        | A<br>3        |

# SECOND DIAGRAM.

Plate XXIV.

## TETRACHORDS CONJOINED.

FIG. 2.

| ANTECEDENT.        |                |               |               | MEAN.           |                |               |               | CONSEQUENT.     |                |               |  |
|--------------------|----------------|---------------|---------------|-----------------|----------------|---------------|---------------|-----------------|----------------|---------------|--|
| $\frac{15}{16}$    | $\frac{9}{10}$ | $\frac{8}{9}$ |               | $\frac{15}{16}$ | $\frac{9}{10}$ | $\frac{8}{9}$ |               | $\frac{15}{16}$ | $\frac{9}{10}$ | $\frac{8}{9}$ |  |
| F                  | E              | D             | C             | B               | A              | G             | F#            | E               | D              |               |  |
| 512                | 480            | 432           | 384           | 360             | 324            | 288           | 270           | 243             | 216            |               |  |
| $\frac{2}{1}$      | $\frac{5}{4}$  | $\frac{3}{2}$ | $\frac{2}{1}$ | $\frac{5}{4}$   | $\frac{3}{2}$  | $\frac{2}{1}$ | $\frac{5}{4}$ | $\frac{3}{2}$   | $\frac{2}{1}$  |               |  |
| 256                | 384            | 288           | 192           | 288             | 216            | 144           | 216           | 162             | 108            |               |  |
| FUNDAMENTAL BASSES | F              | C             | G             | C               | G              | D             | G             | D               | A              | D             |  |
| ROOTS              | 1              | 3             | 9             | 3               | 9              | 27            | 9             | 27              | 81             | 27            |  |

# SECOND DIAGRAM.

## TETRACHORDS DISJOINED.

FIG. 4.

| $\frac{15}{16}$ $\frac{9}{10}$ $\frac{8}{9}$ |               |               |               | $\frac{1}{1}$ $\frac{8}{9}$ | $\frac{15}{16}$ $\frac{9}{10}$ $\frac{8}{9}$ |               |               |               | $\frac{1}{1}$ $\frac{8}{9}$ | $\frac{15}{16}$ $\frac{9}{10}$ $\frac{8}{9}$ |               |               |               |
|----------------------------------------------|---------------|---------------|---------------|-----------------------------|----------------------------------------------|---------------|---------------|---------------|-----------------------------|----------------------------------------------|---------------|---------------|---------------|
| G                                            | F#            | E             | D             | D                           | C                                            | B             | A             | G             | G                           | F                                            | E             | D             | C             |
| 288                                          | 270           | 243           | 216           | 216                         | 192                                          | 180           | 162           | 144           | 144                         | 128                                          | 120           | 108           | 96            |
| $\frac{2}{1}$                                | $\frac{5}{4}$ | $\frac{3}{2}$ | $\frac{2}{1}$ | $\frac{3}{2}$               | $\frac{2}{1}$                                | $\frac{5}{4}$ | $\frac{3}{2}$ | $\frac{2}{1}$ | $\frac{3}{2}$               | $\frac{2}{1}$                                | $\frac{5}{4}$ | $\frac{3}{2}$ | $\frac{2}{1}$ |
| 144                                          | 216           | 162           | 108           | 144                         | 96                                           | 144           | 108           | 72            | 96                          | 64                                           | 96            | 72            | 48            |
| F.B.                                         | G             | D             | A             | D                           | G                                            | C             | G             | D             | G                           | C                                            | F             | C             | G             |
| R.                                           | 9             | 27            | 81            | 27                          | 9                                            | 3             | 9             | 27            | 9                           | 3                                            | 1             | 3             | 9             |

FIRST DIAGRAM. QUINTUPLE.

Communication between the  
minor and major Scales.

| 1 <sup>st</sup> Sect.<br>Root 1. |    |       | 2 <sup>d</sup> Sect.<br>Root 5. |  |  | 3 <sup>d</sup> Sect.<br>Root 25. |  |  | 4 <sup>th</sup> Sect.<br>Root 125. |  |  |
|----------------------------------|----|-------|---------------------------------|--|--|----------------------------------|--|--|------------------------------------|--|--|
| Root 1.                          | B  | 16    | G. 20                           |  |  | Eb. 100                          |  |  |                                    |  |  |
|                                  | G  | 20    |                                 |  |  | C. 120. Cb. 125                  |  |  | Cb. 500.                           |  |  |
|                                  | E  | 24    | E. 24. Eb. 25                   |  |  | Ab. 150                          |  |  | Ab 600. Abb. 625                   |  |  |
| Root 3.                          | E  | 48    | C. 30                           |  |  | Ab. 300                          |  |  | Fb. 750                            |  |  |
|                                  | C  | 60    | C. 60                           |  |  | F. 360. Fb. 375                  |  |  | Fb 1500                            |  |  |
|                                  | A  | 72    | A. 72. Ab. 75                   |  |  | Db. 450                          |  |  | Db 1800. Dbb 1875                  |  |  |
| Root 9.                          | A  | 144   | F. 90                           |  |  | Db. 900                          |  |  | Bbb 2250                           |  |  |
|                                  | F  | 180   | E. 180                          |  |  | Bb. 1080. Bbb. 1125              |  |  | Bbb. 4500                          |  |  |
|                                  | D  | 216   | D. 216. Db. 225                 |  |  | Gb. 1350                         |  |  | Gb. 5400. Gbb. 5625                |  |  |
| Root 27.                         | D  | 432   | Bb. 270                         |  |  | Gb. 2700                         |  |  | Eb. 6750                           |  |  |
|                                  | Bb | 540   | Bb. 540                         |  |  | Eb. 3240. Ebb. 3375              |  |  | Ebb. 13500                         |  |  |
|                                  | G  | 648   | G. 648. Gb. 675                 |  |  | Cb. 4050                         |  |  | Cb. 16200. Cbb. 16875              |  |  |
| Root 81.                         | G  | 1296  | Eb. 810                         |  |  | Cb. 8100                         |  |  | Abb. 20250                         |  |  |
|                                  | Eb | 1620  | Eb. 1620                        |  |  | Ab. 9720. Abb. 10125             |  |  | Abb. 40500                         |  |  |
|                                  | C  | 1944  | C. 1944. Cb. 2025               |  |  | Fb. 12150                        |  |  | Fb. 48600. Fbb. 50625              |  |  |
| Root 243.                        | C  | 3888  | Ab. 2430                        |  |  | Fb. 24300                        |  |  | Dbb. 60750                         |  |  |
|                                  | Ab | 4860  | Ab. 4860                        |  |  | Db. 29160. Dbb. 30375            |  |  |                                    |  |  |
|                                  | F  | 5832  | F. 5832. Fb. 6075               |  |  | Bbb. 36450                       |  |  |                                    |  |  |
| Root 729.                        | F  | 11664 | Db. 7290                        |  |  |                                  |  |  |                                    |  |  |
|                                  | Db | 14580 | Db. 14580                       |  |  |                                  |  |  |                                    |  |  |
|                                  | Bb | 17496 | Bb. 17496. Bbb. 18225           |  |  |                                  |  |  |                                    |  |  |
|                                  |    |       | Gb. 21870                       |  |  |                                  |  |  |                                    |  |  |

# SECOND DIAGRAM. QUINTUPLE.

*Communication between the  
minor and major Scales.*

|                       |     |       |           |                       |           |                        |                         |
|-----------------------|-----|-------|-----------|-----------------------|-----------|------------------------|-------------------------|
| Root 729.             | F # | 17496 | F # 17496 | F # 18225             | A # 21870 | F # 36450              |                         |
|                       | D # | 14580 |           | D # 14580             |           |                        |                         |
|                       | B   | 11664 |           | D # 7290              | D # 29160 | D # 30375              |                         |
| Root 243.             | B   | 5832  | B 5832    | B # 6075              | B # 24300 |                        | D # 60750               |
|                       | G # | 4860  |           | G # 4860              | B # 12150 | B # 48600              | B # 50625               |
|                       | E   | 3888  |           | G # 2430              | G # 9720  | G # 10125              | G # 40500               |
| Root 81.              | E   | 1944  | E 1944    | E # 2025              | E # 8100  |                        | G # 20250               |
|                       | C # | 1620  |           | C # 1620              | E # 4050  | E # 16200              | E # 16875               |
|                       | A   | 1296  |           | C # 810               | C # 3240  | C # 3375               | C # 13500               |
| Root 27.              | A   | 648   | A 648     | A # 675               | A # 2700  |                        | C # 6750                |
|                       | F # | 540   |           | F # 540               | A # 1350  | A # 5400               | A # 5625                |
|                       | D   | 432   |           | F # 270               | F 1080    | F # 1125               | F # 4500                |
| Root 9.               | D   | 216   | D 216     | D # 225               | D # 900   |                        | F # 2250                |
|                       | B   | 180   |           | B 180                 | D # 450   | D # 1800               | D # 1875                |
|                       | G   | 144   |           | B 90                  | B 360     | B # 375                | B # 1500                |
| Root 3.               | G   | 72    | G 72      | G # 75                | G # 300   |                        | B # 750                 |
|                       | E   | 60    |           | E 60                  | G # 150   | G # 600                | G # 625                 |
|                       | C   | 48    |           | E 30                  | E 120     | E # 125                | E # 500                 |
| Root 1.               | C   | 24    | C 24      | C # 25                | C # 100   |                        | Root 125.<br>4. Sect. " |
|                       | A   | 20    |           | A 20                  |           | Root 25.<br>3. Sect. " |                         |
|                       | F   | 16    |           |                       |           |                        |                         |
| Root 1.<br>1. Sect. " |     |       |           | Root 5.<br>2. Sect. " |           |                        |                         |

*The Three Terms, Or fundamental Bases, proper to form three Scales with Minor 3<sup>ds</sup> & three with Major 3<sup>ds</sup>*

|          |   |   |     |   |                 |             |          |
|----------|---|---|-----|---|-----------------|-------------|----------|
| 1 Flat.  | D | A | 3   | 3 | 30. C           | F.          | 1 Flat.  |
|          |   | D | 9   | 2 | 90. F           |             |          |
|          |   | G | 27  | 1 | 270. B $\flat$  |             |          |
| 2 Flats. | G | D | 9   | 3 | 90. F           | B $\flat$ . | 2 Flats. |
|          |   | G | 27  | 2 | 270. B $\flat$  |             |          |
|          |   | C | 81  | 1 | 810. E $\flat$  |             |          |
| 3 Flats. | C | G | 27  | 3 | 270. B $\flat$  | E $\flat$ . | 3 Flats. |
|          |   | C | 81  | 2 | 810. E $\flat$  |             |          |
|          |   | F | 243 | 1 | 2430. A $\flat$ |             |          |

*Fig. 2.*

*The Three Terms, Or fundamental Bases, proper to form three Scales with Major 3<sup>ds</sup> & three with Minor 3<sup>ds</sup>*

|           |   |    |     |   |                 |              |           |
|-----------|---|----|-----|---|-----------------|--------------|-----------|
| 3 Sharps. | A | E  | 243 | 3 | 1620 C $\sharp$ | F $\sharp$ . | 3 Sharps. |
|           |   | A. | 81  | 2 | 540 F $\sharp$  |              |           |
|           |   | D. | 27  | 1 | 180 B           |              |           |
| 2 Sharps. | D | A. | 81  | 3 | 540 F $\sharp$  | B.           | 2 Sharps. |
|           |   | D. | 27  | 2 | 180 B           |              |           |
|           |   | G. | 9   | 1 | 60 E            |              |           |
| 1 Sharp.  | G | D. | 27  | 3 | 180 B           | E.           | 1 Sharp.  |
|           |   | G. | 9   | 2 | 60 E            |              |           |
|           |   | C. | 3   | 1 | 20 A            |              |           |

*Fig. 4.*

FIRST DIAGRAM.

*Illustration of the Pythagorean Numbers.*

| Roots. | Pythagorean Numbers |               | Suspension of $\frac{1}{4}$ 4 <sup>th</sup> | Suspension of $\frac{1}{4}$ 9 <sup>th</sup> | A 4 <sup>th</sup> Term compounded.           | A 4 <sup>th</sup> Term double compounded.    |
|--------|---------------------|---------------|---------------------------------------------|---------------------------------------------|----------------------------------------------|----------------------------------------------|
| 3      | 6                   | { E<br>C<br>A | A 8 <sup>th</sup>                           | E 9 <sup>th</sup><br>A 5 <sup>th</sup>      | D 7 <sup>th</sup>                            | F 9 <sup>th</sup><br>D 7 <sup>th</sup><br>.  |
| 1      | 8                   | { B<br>G<br>E | E 5 <sup>th</sup>                           |                                             | B 5 <sup>th</sup><br>G 3 <sup>d</sup><br>E 1 | B 5 <sup>th</sup><br>G 3 <sup>d</sup><br>E 1 |
| 9      | 9                   | { A<br>F<br>D | D 4 <sup>th</sup>                           | E 3 <sup>d</sup><br>D 1                     |                                              |                                              |
| 3      | 12                  | { E<br>C<br>A | A 1                                         |                                             |                                              |                                              |

SECOND DIAGRAM.

*Illustration of the Pythagorean Numbers.*

|   | Roots |   | Suspension of $\frac{1}{4}$ | Suspension of $\frac{1}{9}$ | A third Term compounded | A third Term double compounded |
|---|-------|---|-----------------------------|-----------------------------|-------------------------|--------------------------------|
|   |       |   |                             |                             |                         | $9^{\text{th}}$ A $\flat$ orb  |
|   |       | G |                             | G $9^{\text{th}}$           |                         |                                |
|   |       | E |                             |                             | F $\flat 7^{\text{th}}$ | F $\flat 7^{\text{th}}$        |
| 3 | 12    | C | C $8^{\text{th}}$           | C $5^{\text{th}}$           |                         |                                |
|   |       | D |                             |                             | D $5^{\text{th}}$       | D $5^{\text{th}}$              |
|   |       | B |                             |                             | B $3^{\text{d}} \#$     | B $3^{\text{d}} \#$            |
| 9 | 9     | G | G $5^{\text{th}}$           |                             | G $1$                   | G $1$                          |
|   |       | C |                             |                             |                         |                                |
|   |       | A |                             | A $3^{\text{rd}}$           |                         |                                |
| 1 | 8     | F | F $4^{\text{th}}$           | F $1$                       |                         |                                |
|   |       | G |                             |                             |                         |                                |
|   |       | E |                             |                             |                         |                                |
| 3 | 6     | C | C $1$                       |                             |                         |                                |

*Pythagorean Numbers.*